MACHINE DESIGN

A Text Presenting Those Fundamentals of Theory and Analysis Which Are Basic to the Field of Machine Design

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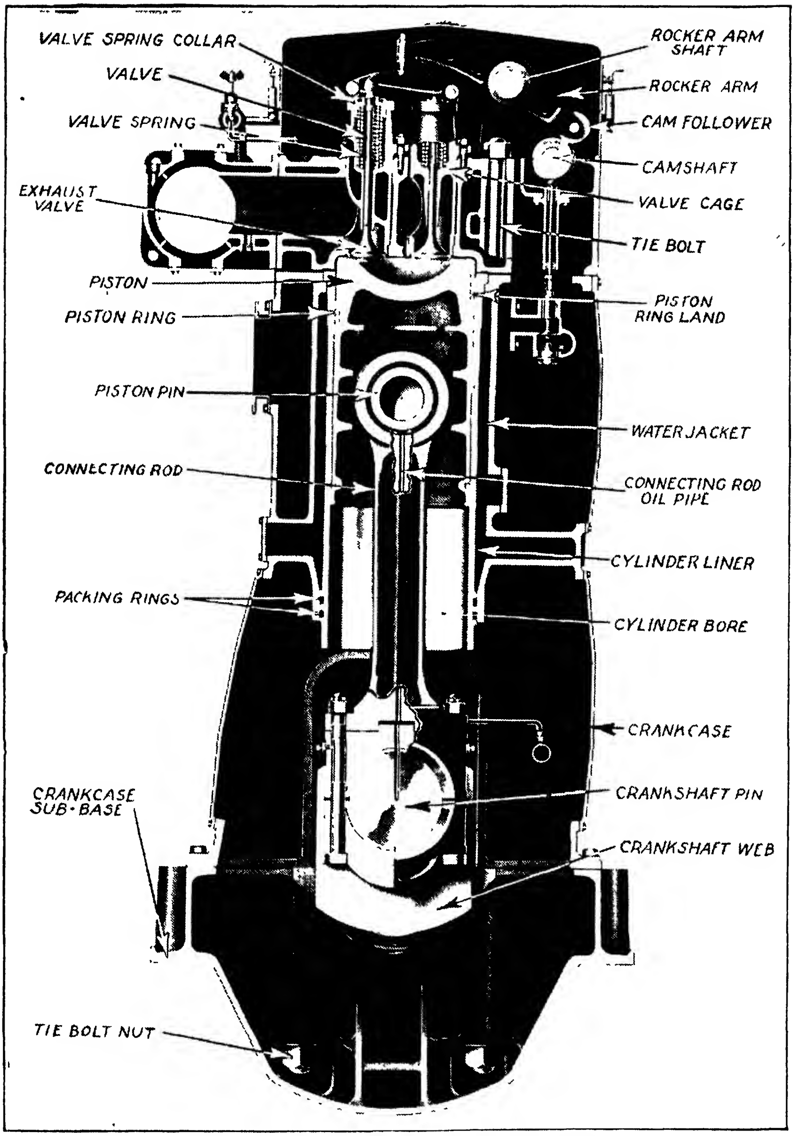
PREFACE

MACHINE DESIGN is an extremely interesting and fascinating field of endeavor. It is very broad in its scope and hence offers many specific lines, any one of which presents the possibility of specialization. Thus there are those who restrict their activity to the design of heat engines or to just one type thereof, to transmission machinery, to pumps, to steam generators, to power plant auxiliaries and accessories, etc.

Underlying the entire field of Machine Design, there are, however, many basic fundamentals of theory and analysis and a great deal of factual information with which any prospective designer of machines or machinery must become acquainted. The author has attempted to select for this text that material which is most basic to the field of Machine Design in general, and which in its treatment will bring to the student the general idea of analysis which permeates the entire field of design. The presentation of this material is based on the assumption that the student has completed the subject of Mechanism and that his mathematical training has extended only through trigonometry and logarithms. Hence the calculus has not been resorted to. This has necessitated the inclusion of several rational formulas for which no derivations are given.

In order to expose the text material in as clear a manner as possible, the solutions of many examples are included in the text. The student is advised to study carefully these solutions in order that he may note the reasons for the various steps therein and obtain the sequence of thought which they present. He should then attempt an independent solution of these same examples, referring to the text only at such times as he needs help.

The author wishes to express his appreciation to the various manufacturers who were so generous in supplying information and illustrations from their catalogs and bulletins. The student's attention is called to the great fund of information available in such industrial literature. Information and data for this text have been drawn from other sources as well, and a sincere effort has been made to acknowledge this courtesy throughout the text.



CROSS SECTION OF NATIONAL-SUPERIOR DIESEL ENGINE

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MACHINE DESIGN

CHAPTER I

FUNDAMENTAL PRINCIPLES — SIMPLE STRESSES, ETC.

A Machine. A machine is a combination of interrelated bodies, links, or elements which are characterized by being resistant and in general rigid. These links are so chosen and arranged that in operation they can collectively either transform other forms of energy into mechanical energy, or receive mechanical energy, power, from some external source, transmit and modify it, thus doing work in performing some task.

The machine that merely transforms energy is generally a heat engine, so named because it receives heat energy which it changes or transforms into another kind of energy, mechanical energy. To this class of machines belong the steam engine, the steam turbine, the Diesel engine, the Otto engine, etc. The electric motor is a machine that changes electric energy into mechanical energy. It is the office of these machines, or engines, to develop power to be used by the other type of machines. However in doing this, they must consume in their own operation some of the power which they generate. Therefore they deliver less energy than they receive. It is the object of the designer then to design an engine so that it will consume as little of its energy as possible in its own operation.

The second and more general type of a machine is the one that receives mechanical energy to be utilized by it in doing some specific task for which it was designed. Such a machine is not designed to transform energy, but rather to modify it. Consider a hoist. Its task is to raise or lower a predetermined maximum load. The mechanical energy received by the hoist must be so modified as to make it possible to do this. Not all the energy received can be used in performing the designated task for, like machines of the first type, some energy must be expended in the operation of the machine itself. The smaller this amount consumed by the machine in its own operation, the higher

will be its mechanical efficiency, in that the mechanical efficiency of a machine is the ratio of the energy used directly in performing its useful task or work to the energy received from the source of the power supplied. As in the case of the design of prime movers, the designer must constantly strive to design a machine so that its efficiency is as high as possible; thus reducing the cost of operation to a minimum.

Machine Design. From that which precedes, it becomes evident that energy is supplied to a machine from some source and that the machine upon receiving this energy transmits and modifies it so that some useful effect is produced. This transmission and modification of energy within the machine requires the inclusion of a certain series of links, a certain train of mechanism, whose office is two-fold as follows:

1. The production of the desired motion to permit the machine to perform its task.

2. The modification and transmission of the forces involved without rupture or undue change of shape of the various links.

The first of these suggests that in the design of a machine the relative motion of machine parts must be considered. Such consideration must be undertaken early if not at the actual beginning of the design and falls under the province of Mechanism.

The second of these suggests that in the design of a machine the various links must be so built up or constructed that they can withstand the forces to which they are subjected and which become the so-called loads upon them. As we shall see, the relative motion must be known before these loads can be ascertained. The analysis of the forces involved and the subsequent design of the machine parts so that they will stand up under these forces without rupture or undue distortion is the province of so-called Machine Design.

The design of a machine is generally so complicated that the student must first acquaint himself with the fundamentals of the design of the individual elements entering therein. Eventually several of these elements can be considered collectively, so that he is gradually drawn into the picture of the design of the machine as a whole. In the study of this subject, the student will find himself constantly applying his knowledge of mechanism, mechanics, strength of materials, and mathematics.

Load. Any external force to which a machine part is subjected

is called a load. One is not thoroughly familiar with a load, and cannot proceed to design the machine part on the basis of it until the following information is known:

1. Magnitude of the load
2. Direction of the load
3. Type of load.

The magnitude of the load is generally given in pounds. The direction of the load acquaints the designer with the internal molecular effect or type of stress set up within the link by the load. There

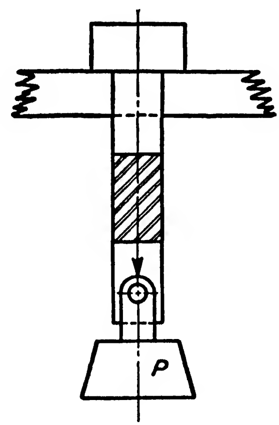


Fig 1 Rod in Tension

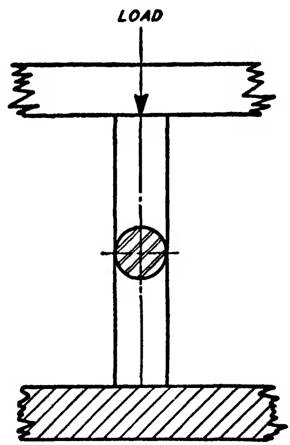


Fig. 2. Post in Compression

are three types of load; namely, dead or steady load, live or varying load, and shock load. A dead load is one that does not change in size or magnitude, that is, it remains constant, while a live load is one that is continually changing. A load that is suddenly applied or removed is known as a shock load. For the same magnitude of load, a machine part resists a dead load more easily than a live load and a live load more easily than a shock load. In other words, for the same magnitude, a machine part must be made progressively heavier in design as the loads vary in type from a dead to a shock load.

Strain and Elasticity. When a load is applied to a body, it produces a deformation or change of size and shape in that body. This deformation of the body is known as Strain. In case the external force, or load, is removed, the body will generally return to its natural

size and shape. This property of regaining its size and shape upon the removal of its load, which most solid bodies have to some degree, is known as Elasticity.

Stress. When a body is strained by the application of an external force (see Figs. 1, 2, and 3), there is a tendency to a greater or lesser degree to break or rupture the body. If rupture is not to occur, the body must be able to have set up within itself along certain involved sections such as the revolved sections in Figs. 1, 2, and 3 an internal resisting force equal in magnitude and opposite in direction to the external force. Such an internal resisting force is called a Stress and is measured in pounds. It is in reality the ability that the molecules of a body have to cling to or to grip each other in order to retain their relative positions while some external force or load tends

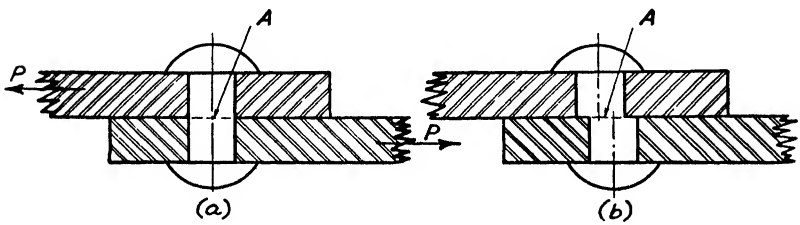


Fig. 3 Rivet in Shear

to dislodge them. The direction and tendency of the load dictates the type of stress and the involved area over which the stress is distributed. Unit stress is the total number of pounds of stress or the resistance over an involved section divided by the area of the section in square inches. Therefore

if S = unit stress in pounds per square inch
A = area of section in square inches

total stress, or the resistance = AS pounds.

As was stated previously, the total resistance or stress set up if rupture does not occur is equal to the load.

If P, as in Figs. 1, 2, and 3, is the load, in general we have

$$P = AS \tag{1}$$

From the above formula, it is evident that for a constant load, P, if the area, A, is increased, the unit stress, S, will decrease. On the other hand, should the area, A, be decreased, the unit stress must increase; that is, every square inch of the involved cross section

must exert itself to a greater extent or have set up over it a greater stress. This of course is due to the statement already made that the total resistance must equal the load. Now the stress or resistance that a square inch of an involved cross section of a body can set up, or have set up over it, is limited depending upon the material and the type of stress. Should the load be increased while the cross section of the body remains the same, the stress will finally reach a point where no more stress or resistance can be offered per square inch of the involved area, and failure or rupture of the member would take place. The stress per square inch at which such failure will take place is called the Ultimate Stress and will be designated by the symbol, U. The load that causes failure is likewise known as the ultimate load. If then P is the ultimate load in pounds, AU will become the total ultimate resistance in pounds; hence we have, in general

$$P = AU \tag{2}$$

By placing a body of a given material in a testing machine by which the different types and magnitudes of loads can be placed upon the body, the ultimate stresses of the material in pounds per square inch can be recorded. Such a list is presented in Table I.

Factor of Safety. When a machine part is subjected to a load and failure of the part does not occur, the stress, S, of formula (1), set up in the member by the load, P, is the Working or Induced Stress. In practice, however, no machine part should be so loaded that the working stress approaches anywhere near the ultimate stress, U; therefore a much lower value than U is chosen by the designer for his Design stress, or Safe stress. The ratio of the ultimate stress to the design or safe stress selected by the designer is called the Factor of Safety, and is designated by F.

Thus
$$F = \frac{\text{ultimate unit stress}}{\text{safe unit stress}} = \frac{U}{S} \tag{3}$$

or
$$S = \frac{U}{F} \tag{4}$$

The selection of a proper value for the factor of safety depends upon the material, the type of machine, the kind of load, and the possibility of an accidental over-load being placed upon the machine part. The selection depends upon and reflects the judgment and

TABLE I

	Ultimate Stress in Pounds per Square Inch (Average)			Modulus of Elasticity	
	U_t	U_c	U_s	Tension, E_t or Compression, E_c	Shear E_s
Cast iron	20,000	80,000	20,000	15,000,000	6,000,000
Wrought iron	48,000	48,000	40,000	27,000,000	10,000,000
‡Medium steel .15% to .4% carbon	60,000	60,000	50,000	29,000,000	12,000,000
Hard steel .3% to .7% carbon	75,000	75,000	55,000	30,000,000	12,000,000
Nickel steel—3.5% nickel	90,000	90,000	68,000	30,000,000	12,000,000
Molybdenum steel	140,000	140,000	105,000	...	
Silico-manganese steel	250,000	250,000	185,000		
Cast aluminum	13,000			9,000,000	...
Rolled aluminum	20,000 up	...	...	10,400,000	...
Cast copper	20,000 to 30,000	...	...	12,000,000	...
Hard drawn copper	50,000 to 70,000	...	...	15,000,000	...
Brass	25,000 to 35,000	18,000 to 20,000	...	9,000,000	...
*Timber (parallel to grain)	......	2,500 to 4,000	400 to 800	...	...
Leather	3,000 to 5,000	...	...	...	...
Brick	......	4,000 to 10,000	...	...	...

‡Bending stress for all steel can be taken equal to tensile stress
*Bending stress for timber in extreme outer fiber, 4000 to 6500 pounds per square inch.

TABLE II — Factor of Safety

Material	For Dead, or Steady Load	For Live, or Varying Load	For Shock
Cast iron	5 to 6	8 to 12	16 to 20
Wrought iron	4	7	10 to 15
Steel	4	8	12 to 16
Soft metals and alloys	6	9	15
Leather	9	12	15
Timber	7	10 to 15	20
Brick	10 to 15	15 to 20	20 to 30
Stone	10 to 15	15 to 20	20 to 30

experience of the designer. In general, 3 to 4 may be selected in the case of a dead load, 6 to 8, in the case of a live load, and 9 to 16 for a shock load. Table II gives more specific information on factors of safety.

Tension and Tensile Stress. In Fig. 1, a rod with a rectangular section is subjected to a load of P pounds. The tendency of the load is to stretch or elongate the rod. In such a case as this the rod is said to be under Tension, the load is said to be a tensile load, and the

stress set up in the involved cross-sectional area such as the revolved section shown in the figure is said to be a Tensile stress. Since the total stress or resistance to the load is always opposite in direction to the direction of the load, it is evident that the sections involved in tension are always at right angles to the load. It is very important that the student note the relative positions of the sections of the machine parts that become involved with each type of stress, so as to use the correct dimensions in the design. Thus in Fig. 1, if we neglect the weight of the rod, the length of the latter plays no part in the design of the rod for the tensile load. If we let

S_t = safe stress in tension in pounds per square inch
U_t = ultimate stress in tension in pounds per square inch
P = safe load in pounds
A = involved area in square inches

then Formula (1) becomes

$$P = AS_t, \text{ the design formula for tension} \tag{5}$$

From formula (4),

$$S_t = \frac{U_t}{F} \tag{6}$$

Substituting this value of S_t in formula (5)

$$P = A\frac{U_t}{F} \tag{7}$$

Likewise, formula (2) becomes for tension,

$$P = AU_t \tag{8}$$

in which P is now the ultimate or breaking load in tension.

Example. A rod of medium steel supports a dead load of 10,000 pounds in tension, as in Fig. 1. Find the dimensions of the cross section of the rod, (*a*) when the cross section is rectangular with the breadth equal to three times the thickness, (*b*) when the section is square, and (*c*) when the section is circular.

Solution. From Table I, U_t, for medium steel, = 60,000 pounds per square inch and from Table II, $F = 4$.

Applying formula (6)

$$S_t = \frac{U_t}{F} = \frac{60{,}000}{4} = 15{,}000 \text{ lb. per sq. in.}$$

Applying formula (5), in which $P = 10{,}000$ pounds.

$$P = AS_t$$
$$10{,}000 = A \times 15{,}000$$

Dividing both members of the equation by 15,000,

$$A=\frac{10,000}{15,000}=0.667 \text{ sq. in.}$$

(*a*) For rectangular section, with breadth, b, and thickness, t.

$$A=b\times t$$

and it is given that $b=3t$

Substituting this in the formula for A

$$A=3t\times t=3t^2$$

But $$A=0.667 \text{ sq. in.}$$

$$\therefore\ 3t^2=0.667$$

$$t^2=\frac{0.667}{3}=0.222$$

$$t=\sqrt{0.222}=0.47 \text{ in. say } \tfrac{1}{2} \text{ in.} \quad \textit{Ans.}$$

$$\therefore\ b=3\times\tfrac{1}{2}=1\tfrac{1}{2} \text{ in.} \quad \textit{Ans.}$$

(*b*) For square section,

$A=t^2$, where t is one side of the square section

$$\therefore\ t^2=0.667$$

$$t=\sqrt{0.667}=0.82 \text{ in., say } \tfrac{7}{8} \text{ in.} \quad \textit{Ans.}$$

(*c*) For circular section,

$A=\frac{\pi d^2}{4}$, in which d is the diameter of the section.

$$\therefore\ \frac{\pi d^2}{4}=0.667$$

Multiplying both members of the equations by 4 and dividing by π

$$d^2=\frac{4\times 0.667}{\pi}=0.849$$

$$\therefore\ d=\sqrt{0.849}=0.92 \text{ in., say } \tfrac{15}{16} \text{ in.} \quad \textit{Ans.}$$

Note. It will be noted by the student that in all of the above answers, the theoretical result has been changed to a common fraction that is slightly larger than the theoretical. Thus the actual answer is a more practicable value that will either suit the scale used by mechanics in which the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, and $\frac{1}{64}$ of an inch are generally available or that is a standard stock size. A designer must be careful to acquaint himself in regard to such sizes

when he is to purchase material, so as to include the right practical dimensions in his Bill of Material, and also because the design of other parts may be affected thereby. It should further be noted that the theoretical values obtained in solutions are the minimum values that may be used. Any larger value adopted in reality increases the factor of safety and lowers the unit working stress. Due to the above statements, a slide rule is very effective in solving most machine design problems as it possesses a sufficient degree of accuracy in most cases. The answers given in this text will be obtained in the main by slide rule computation and the student is advised to acquire the habit of using a slide rule.

Example. A hollow wrought iron rod is subjected to a varying tensile load of 2400 pounds acting along its axis. Its inside diameter is $\frac{3}{4}$ inch and its outside diameter is 1 inch.

(*a*) Find the induced stress set up in the rod.

(*b*) Find the factor of safety employed.

(*c*) Is the rod safely dimensioned for this load?

Solution. Here $P=2400$ lb., D, outside diameter $=1$ in.; d, inside diameter $=\frac{3}{4}$ in.

$$A\text{, area of rod}=\frac{\pi D^2}{4}-\frac{\pi d^2}{4}=\frac{\pi}{4}(D^2-d^2)=\frac{\pi}{4}\left[1^2-\left(\tfrac{3}{4}\right)^2\right]=0.344 \text{ sq. in.}$$

Substituting in Formula (5), $P=AS_t$,

$$2400=0.344\times S_t$$

$$S_t=\frac{2400}{0.344}=7000\sim\text{lb. per sq. in.}\quad \textit{Ans.}$$

(*b*) Using formula (6)

$$S_t=\frac{U_t}{F} \text{ or } F=\frac{U_t}{S_t}$$

From Table I, U_t, for wrought iron, $=48{,}000$ lb. per sq. in.

$$F=\frac{48{,}000}{7000}=6.85\quad \textit{Ans.}$$

(*c*) Yes, since the factor of safety is practically the assigned factor of safety of Table II in this case.

Note. In the case of this example, the student takes the position of the checker rather than the designer. The same formulas apply, but it should be noted that upon solving for S_t, it might be found to be in such error as to exceed even the U_t for the given

material used. Such would be possible if the designer had erred in his work, giving the link too small an area. In the latter event, S_t would be the induced stress on the basis of the design, but it would not be a safe stress as the designer meant it to be.

Compression and Compressive Stress. When an external force acts upon a body tending to shorten or crush that body, the external force is known as a compressive load and the body is said to be under compression. Such a condition is shown in Fig. 2 where a load as shown is supported by a relatively short member called a post. The load acts downward along the axis of the post and hence there must be an internal molecular resistance equal to the load and acting upward in order to secure a condition of equilibrium with the load. Such a resistance is known as a compressive stress. The tendency toward displacement of the molecules of the body is in a plane at right angles to the forces, hence the involved section in the case of compression is perpendicular to the load as in tension. But, to repeat, the tendency in compression is to shorten the body, while in tension the tendency is to stretch or elongate the body. The involved area, A, is shown cross-hatched in Fig. 2, where it happens to be circular in form. If we let

S_c = safe stress in compression in pounds per square inch

U_c = ultimate stress in compression in pounds per square inch

general formula (1) becomes

$$P=AS_c\text{, the design formula for compression,} \tag{9}$$

From formula (4),

$$S_c=\frac{U_c}{F} \tag{10}$$

Substituting this value of S_c in formula (9),

$$P=A\frac{U_c}{F} \tag{11}$$

Likewise, formula (2) becomes for compression

$$P=AU_c \tag{12}$$

in which P is the ultimate or breaking load in compression.

Example. A short cylindrical cast-iron post as in Fig. 2 supports a compressive load of 20 tons. If F is taken equal to 10, find the diameter of the post.

Solution. Here $P=20\times2000$ lb. $=40{,}000$ lb.,

from Table I, $U_c=80{,}000$ pounds per square inch

$F=10$

$$A=\frac{\pi d^2}{4}$$

Substituting these values in formula (**11**),

$$40{,}000=\frac{\pi d^2}{4}\times\frac{80{,}000}{10}$$

Cancelling like factors in the second member of the equation,

$$40{,}000=\pi d^2\times 2000$$

Dividing by 2000π, the coefficient of d^2, we have

$$d^2=\frac{40{,}000}{2000\pi}=\frac{20}{\pi}=6.36$$

$$d=\sqrt{6.36}=2.52 \text{ in., say } 2\tfrac{5}{8} \text{ in.} \quad \textit{Ans.}$$

Note. Raise the theoretical or computed answer more liberally for cast iron than for steel.

Example. What load in compression can a cubical wooden block, whose edge is 1 foot in length, safely sustain if the safe compressive stress, S_c, is taken as 500 pounds per square inch?

Solution. Here $S_c=500$ lb. per sq. in.

$A=12\times 12=144$ sq. in.

Substituting in formula (**9**),

$$P=144\times 500=72{,}000 \text{ lb.} \quad \textit{Ans.}$$

Example. Find the ultimate or breaking load of the wooden block of the previous example, assuming the factor of safety as 7.

Solution. If $S_c=500$ lb. and $F=7$,

and $U_c=F\times S_c$, from formula (**10**)

Evaluating in the above

$U_c=7\times 500=3500$ lb. per sq. in.

since $A=144$ sq. in., applying formula (**12**), we have

$P=AU_c=144\times 3500=504{,}000$ lb. *Ans.*

Shear and Shearing Stress. In Fig. 3(a), two plates are subjected to forces that tend to slip or slide one plate along the other in the direction as shown in the figure. The plates in this case are held in position by a rivet. The load, P, placed upon the rivet, as the latter resists the action of the forces, tends to slip the molecules of the rivet at section A past each other. Fig. 3(b) shows this slipping action as having taken place so as to more plainly set forth the involved area, over which the total stress, or the total resistance offered by the

rivet, is distributed. Since each square inch of this involved area is capable of offering a certain safe resistance of this type, enough square inches must be provided to offer a total safe resistance equal to the load. When a load is placed on a body tending to cut or "shear" it across, the stress at a section along which there is a tendency to cut is called a shearing stress, and the body is said to be under shear. It will be noticed that the involved area in shear over which the cutting or slipping tends to take place is parallel to the direction of the forces involved, instead of being at right angles as in the cases of tension and compression. The student should note this carefully.

Let S_s = safe shearing stress in pounds per square inch and

U_s = ultimate shearing stress in pounds per square inch;

then formula (1) becomes

$$P = AS_s, \text{ the design formula for shear.} \tag{13}$$

From formula (4),
$$S_s = \frac{U_s}{F} \tag{14}$$

Substituting this value for S_s in formula (13).

$$P = A\frac{U_s}{F} \tag{15}$$

Formula (2) becomes for shear

$$P = AU_s \tag{16}$$

in which P is now the ultimate or breaking load in shear.

Example. If the rivet of Fig. 3 is made of medium steel and is $\frac{3}{4}$ inch in diameter, what total safe resistance can it offer with $F=8$? Also to what load can it be subjected safely?

Solution. From Table I, $U_s = 50{,}000$ lb. per sq. in.

$$S_s = \frac{U_s}{F} = \frac{50{,}000}{8} = 6250 \text{ lb. per sq. in.}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(\frac{3}{4})^2}{4} = \frac{\pi\frac{9}{16}}{4}$$

$$= \pi \times \tfrac{9}{64} = 0.442 \text{ sq. in.}$$

The total safe resistance in shear $= AS_s$

$= 0.442 \times 6250$

$= 2760+$ lb. *Ans.*

Since the resistance set up is dependent on and equal to the load or, in other words, since from formula (13) $P = AS_s$,

P, the safe load $= 2760+$ lb. *Ans.*

Example. A punch and die is illustrated in Fig. 4. What force, P, will be required to punch a hole in the plate, if the diameter of the hole is $\frac{3}{4}$ inch, the thickness of plate is $\frac{3}{8}$ inch, and the ultimate shearing strength of the plate is 50,000 pounds per square inch?

Solution. Since this is a case of shear, the involved area must be parallel to the force, P. Since a cylindrical plug will be punched from the plate to form the hole, the involved area, A, will be the lateral area of this plug, or of the hole formed. It will be noted that the elements of the lateral area of this cylindrical plug, or hole, are

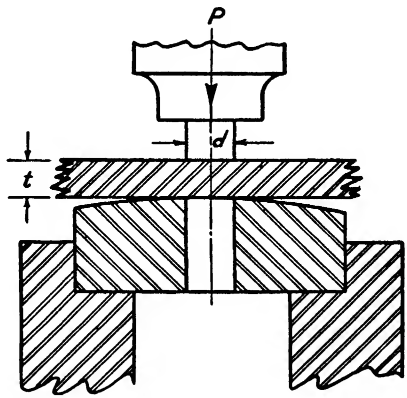


Fig. 4. Punch and Die

parallel to the direction of the force, P. Hence the area is said to be parallel as it must be in shear.

Here $U_s = 50{,}000$ pounds per square inch, P = the ultimate or breaking load in pounds.

Since the lateral area of a cylinder is equal to the circumference of the base multiplied by the altitude,

$$A = \pi dt$$
$$= \pi \times \tfrac{3}{4} \times \tfrac{3}{8} = 0.884 \text{ sq. in.}$$

Using formula (**16**),

$$P = AU_s$$
$$= 0.884 \times 50{,}000 = 44{,}200 \text{ lb.} \quad \textit{Ans.}$$

Example. A rod is fitted into a yoke and secured thereto by a pin, as shown in Fig. 5. The material throughout is medium steel, and a factor of safety of 8 is used. (a) To what load acting along its axis, can the rod be safely subjected, if its diameter, D, is $\frac{1}{2}$ inch?

(*b*) If the assembly is subjected to the load found in part (*a*), what diameter of pin must be used safely to meet the condition of shear therein?

Solution. (*a*) The load, P, acting along the axis of the rod tends to elongate the rod, thereby putting the latter under tension. Hence

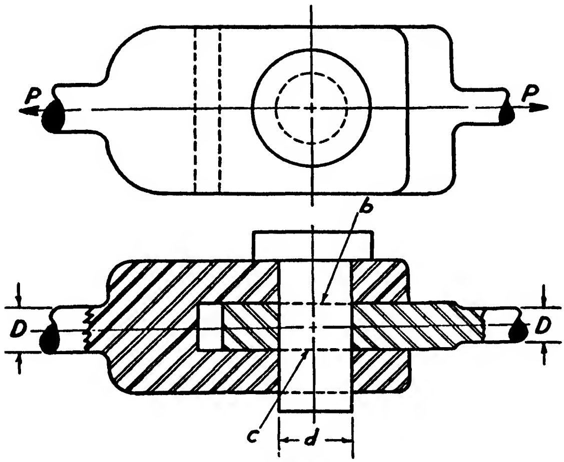


Fig. 5

the involved area, A, of the rod is, as in all cases of tension, perpendicular to the axis, and therefore here

$$A=\frac{\pi D^2}{4}$$
$$=\frac{\pi(\frac{1}{2})^2}{4}=0.1963 \text{ sq. in.}$$

and $\qquad U_t$, from Table I $=60{,}000$ lb. per sq. in.

$$\therefore\ S_t=\frac{60{,}000}{8}=7500 \text{ lb. per sq. in.}$$

Using formula (5), and substituting the above values of A and S_t

$$P=AS_t$$
$$=0.1963\times 7500=1472 \text{ lb.} \quad \textit{Ans.}$$

(*b*) A tensile load on the rods will tend to cut across or shear the pin at b and c simultaneously. Therefore two like circular areas of diameter, d, are involved. Such a condition is spoken of as "double shear" and the area in such a case is always twice the area in single shear. Notice that these involved areas are parallel to the direction

of the load, which is consistent with a condition of shear. If we let

$$a = \text{the area of the pin} = \frac{\pi d^2}{4}$$

then
$$A = 2a = 2 \times \frac{\pi d^2}{4}$$

In shear, U_s, from Table I, = 50,000 lb. per sq. in.

From formula (14), $S_s = \dfrac{U_s}{F}$

$$\therefore\ S_s = \frac{50{,}000}{8} = 6250 \text{ lb. per sq. in.}$$

Using formula (13), $P = AS_s$

$$= 2 \times \frac{\pi d^2}{4} \times S_s$$

Here $P = 1472$ lb., $S_s = 6250$ lb. per sq. in.

Evaluating, $1472 = 2 \times \dfrac{\pi d^2}{4} \times 6250$

Simplifying, $1472 = \pi d^2 \times 3125$

Dividing both members of the equation by $\pi \times 3125$,

$$\frac{1472}{\pi \times 3125} = d^2$$

or
$$d = \sqrt{\frac{1472}{\pi \times 3125}} = \sqrt{0.15} = 0.388 \text{ in., say } \tfrac{7}{16} \text{ in.} \quad \textit{Ans.}$$

Modulus of Elasticity. If a gradually increasing load is applied to a bar of metal, the deformation or strain increases proportionally to the load and hence to the stress set up within the bar, until finally a magnitude of load is reached above which this proportionality between stress and strain no longer exists. This point above which stress and strain are no longer proportional is known as the Elastic Limit. When a body is loaded beyond this limit, the deformation increases much faster than before the elastic limit was reached, and furthermore, the body no longer possesses the property of elasticity. In other words, if the body is loaded beyond the elastic limit and the load is then removed, the body will not recover its former size and shape. It is said to have undergone "permanent set." The student should recognize that this plays an important part in the selection of a factor of safety, for in every case, the latter should be so selected that it will produce a safe working stress well below the elastic limit.

All of the above is shown graphically in the stress-deformation diagram of Fig. 6. A test having been made on a given material, the stresses and strains for various loads have been plotted on their respective axes. The strain is noticed to be very small and to increase proportionally to the stress from O to A, thus producing a straight line that is nearly vertical between these two points. O is the point of zero loading and A is the point known as the elastic limit. Between them the body possesses the property of elasticity. Beyond A, notice how rapidly the strain increases until at point B, failure of the body occurs. The stress at B, shown by the ordinate Ob, is the Ultimate Stress of the material.

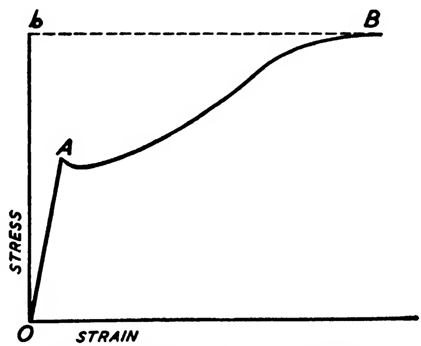


Fig. 6. Stress-Deformation Diagram

Since stress and strain for a given material are proportional within the elastic limit, there is a constant ratio existing between them. This ratio is known as the Modulus of Elasticity and will be designated by the symbol E. Therefore, if Δ (delta) is used as the symbol for unit strain,

$$E=\frac{\text{unit stress}}{\text{unit strain}}=\frac{S}{\Delta} \tag{17}$$

If the stress is one of tension, then S becomes S_t, E becomes E_t, and Δ is equal to the elongation per inch of length. We have then

$$E_t=\frac{S_t}{\Delta} \tag{18}$$

Since formula (5) gives $P=AS_t$

Dividing both members of the equation by A,

$$S_t=\frac{P}{A}$$

Substituting this value of S_t in formula (18),

$$E_t = \frac{P}{A\Delta} \tag{19}$$

Example. In testing a specimen of a material for elongation, a unit deformation of 0.0005 inch was obtained with a load producing a unit tensile stress of 15,000 pounds per square inch. Find the modulus of elasticity in tension.

Solution. From formula (18),

$$E_t = \frac{S_t}{\Delta} = \frac{15{,}000}{0.0005} = 30{,}000{,}000 \quad \textit{Ans.}$$

Example. If the length of the above specimen was 5 feet, what was the total elongation?

Solution. Since total elongation is equal to the unit elongation per inch of length multiplied by the length in inches, in this case,

$$\text{Total elongation} = 5 \times 12 \times 0.0005 = 0.03 \text{ in.} \quad \textit{Ans.}$$

Example. How much will a steel bar be elongated by a load of 15,000 pounds if the bar is 10 feet in length and has a cross-sectional area of $\frac{1}{2}$ square inch? Let $E_t = 30{,}000{,}000$

Solution. Here $E_t = 30{,}000{,}000$; $P = 15{,}000$ lb.; $A = 0.5$ sq. in. It will be necessary to find the unit elongation, Δ, from formula (19), and then multiply Δ by the length of the piece, which is 120 in. Solving formula (19) for Δ, first multiply both members of formula (19) by $A\Delta$; thus

$$E_t \times A \times \Delta = P$$

Dividing by the coefficient of Δ, which is $E_t \times A$, we obtain

$$\Delta = \frac{P}{E_t \times A}$$

Evaluating,

$$\Delta = \frac{15{,}000}{30{,}000{,}000 \times 0.5} = \frac{15{,}000}{15{,}000{,}000}$$

$$= 0.001 \text{ in., elongation per in. of length}$$

$$\text{Total elongation} = 120 \times 0.001 = 0.12 \text{ in.} \quad \textit{Ans.}$$

Conclusion. The three stresses described in this chapter are called the Simple Stresses. A thorough understanding of them is essential, for they form the backbone of the analysis of the design of machine parts. They are referred to as simple stresses to distinguish them from the so-called Compound Stresses, which are, in

reality, a combination of simple stresses produced in some section of a machine part by the forces acting thereon. Such compound stresses occur in beams, shafts, columns, etc., and will be treated in the next chapter.

PROBLEMS

1. What is the difference between stress and strain?

2. Do stress and strain always accompany each other in a machine element subjected to an external force? Why? *Ans.* Yes; because they are produced simultaneously by the load.

3. A cast iron part supports a compressive load of 30 tons.

(*a*) If the area in compression is 6 square inches, what is the induced or working compressive stress in lb. per sq. in.? What is the factor of safety?

(*b*) If the area in compression is doubled, what are the unit compressive stress and the factor of safety?

(*c*) If the load is doubled and the area is 6 square inches as in part (*a*), obtain the unit compressive stress and the factor of safety. *Ans.* (*a*) 10,000 lb. per sq. in.; 8 (*b*) 5000 lb. per sq. in.; 16 (*c*) 20,000 lb. per sq. in.; 4.

4. A steel rod has a rectangular section which is twice as wide as it is thick. It sustains a steady or dead load of 100,000 pounds in tension. Required: the dimension of its cross section assuming U_t to be 60,000 pounds per sq. in. *Ans.* 1.83 in. by 3.66 in. say $1\frac{7}{8}$ in. by $3\frac{3}{4}$ in.

5. A steel rod, as in Fig. 1, supports a live load of 3000 pounds. If the ultimate tensile strength of the rod is 75,000 pounds per square inch and a factor of safety of 8 is used, find the dimensions of the rod when its section is (*a*) rectangular with the breadth three times the thickness, (*b*) square, and (*c*) circular. *Ans.* (*a*) $\frac{3}{8}$ in. by $1\frac{1}{8}$ in. (*b*) $\frac{9}{16}$ in. to $\frac{19}{32}$ in. (*c*) $\frac{11}{16}$ in. diam.

6. A cast-iron post under a compressive load shows a unit deformation of $0.001\frac{1}{3}$ inch. The compressive stress in the member created by the load is 20,000 pounds per square inch. Calculate the modulus of elasticity, E_c. *Ans.* 15,000,000

7. A short piece of extra strong wrought-iron pipe has an outside diameter of 14 inches and an inside diameter of 13 inches. Find the steady compressive load to which this piece of pipe can be safely subjected. *Ans.* 254,500 lb. nearly.

8. Find the ultimate or breaking load in shear of a $\frac{1}{2}$-inch rivet whose ultimate strength in shear is 50,000 lb. per sq. in. *Ans.* 9800 lb.

9. A rod is fitted into a yoke and secured thereto by a pin as shown in Fig. 5. The material throughout is medium steel, and a factor of safety of 12 is used, as mild shock loading is anticipated.

(*a*) To what load acting along its axis, can the rod be subjected safely, if its diameter is $\frac{3}{4}$ in.?

(*b*) If the assembly is subjected to the load found in part (*a*), what diameter of pin must be used safely to meet the condition of shear therein? *Ans.* (*a*) 2209 lb. (*b*) $\frac{5}{8}$ in.

10. Between what points on a stress-strain diagram are stress and strain proportional?

11. What force is required to punch a $\frac{7}{8}$-in. hole in a $\frac{1}{2}$-in. plate if the ultimate shearing strength of the plate is 50,000 pounds per square inch? *Ans.* 69,000 lb. approximately.

12. Under what conditions does permanent set occur in a machine part?

CHAPTER II

FUNDAMENTAL PRINCIPLES (Continued)
COMPOUND STRESSES, ETC.

Compound Stresses. The compound stresses are three in number, and are referred to as Bending, Twisting, and Buckling stresses.

Bending stresses are stresses set up by external forces which act on beams as in Fig. 7. The dotted lines exaggerate the tendency of the load, *P*, to deflect the beam.

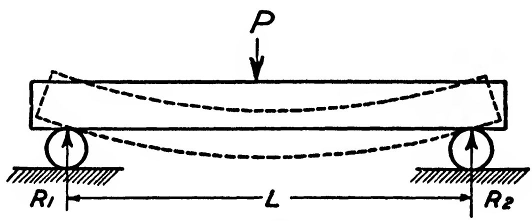


Fig 7 Simple Beam, Bending

Twisting stresses are those to which a bar is subjected when an external force applied to the bar turns or tends to turn the latter about its axis, as in Fig. 8.

Buckling stresses are those introduced into a long bar or rod by an external force which acts along the axis of the rod. This is shown in Fig. 9. The rod must be rather long in comparison to its smallest cross-sectional dimension, otherwise the load upon it will set up a compressive stress.

Beams. A bar, having in general a horizontal position and resting upon or being held by one or more supports is called a beam. It carries loads or is subjected to external forces which act at right angles to its length. Among the different kinds of beams are the following, which are met with very often in machine design.

A Simple Beam, Fig. 7, is one that rests on a support at each end.

A Cantilever Beam, Fig. 10, is a beam that is supported at one

end only. The dotted lines show the tendency of the beam to deflect under the load, P.

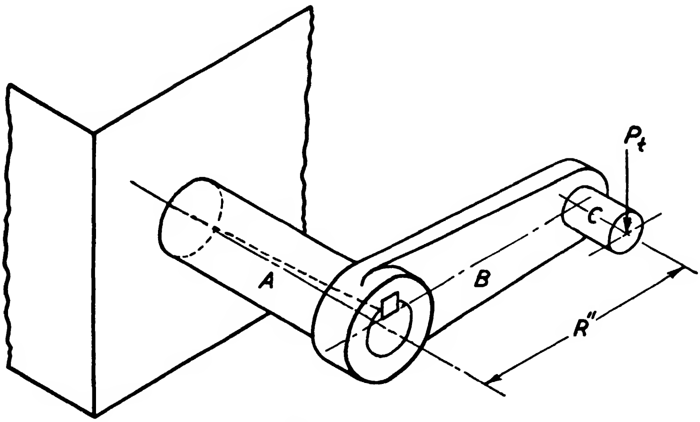


Fig 8 Twisting

An Overhung Beam is shown by Fig. 13 to be one that projects beyond one or both of its two supports in order to sustain loads outside the supports as well as between them.

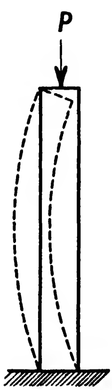


Fig. 9. Buckling

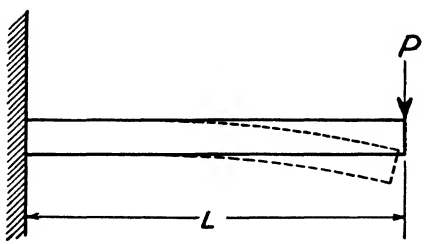


Fig. 10 Cantilever Beam, Bending

A Restrained Beam is a beam that has its ends fixed in position at the supports. This can be done for instance by bolting or riveting the ends of the beam to the supporting members.

Moment of a Force. The moment of a force with respect to a point is the measure of the tendency of the force to produce rotation

about that point. The moment of a force then depends not only on the magnitude of the force but also upon the perpendicular distance from the line of action of the force to the point. Most of us have noted this in the selection of a longer wrench to loosen the nut on a bolt. When with a shorter wrench we had exerted ourselves to the utmost without result, a longer wrench loosened the nut easily. Hence the moment of a force is equal to the product of the force and the perpendicular distance from the line of action of the force to the point. This perpendicular distance through which the force acts is called the moment arm of the force and the point with respect to which the moment is taken is called the origin or center of moments.

In Fig. 11, the force, P, in pounds working through the perpendicular distance, a, in inches, with respect to the origin, O, produces

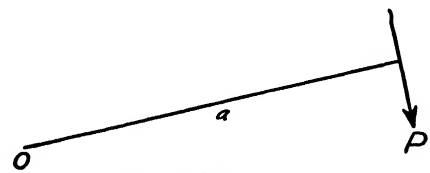


Fig. 11 Moment of a Force

a moment equal to $P \times a$ inch-pounds (in.-lb.). If the arm, a, had been in feet, the moment would have been expressed in foot-pounds. In case several forces were acting in the same plane with respect to a given origin, some attempting to produce rotation in a clockwise direction (in the direction in which the hands of a clock move), and others in a counter-clockwise direction, the moments of the former would be considered positive algebraically, while the moments of the latter would be considered negative.

Example. In Fig. 11, P is a force of 100 pounds and a is an arm 36 inches in length. Find the moment of P in both inch-pounds and foot-pounds.

Solution. Moment of $P = P \times a$

$$= 100 \times 36 = 3600 \text{ in.-lb.} \quad \textit{Ans.}$$

Since 12 in.-lb. = 1 ft.-lb.

$$3600 \text{ in.-lb.} = \frac{3600}{12} = 300 \text{ ft.-lb.} \quad \textit{Ans.}$$

Reactions. The external forces exerted on a beam, such as the

simple beam shown in Fig. 12, are the loads applied to the beam and the resulting forces set up at the supports, A and B, by these loads. These forces at the supports are known as Reactions. Loads and reactions together constitute a system of coplanar forces in equilibrium. Therefore it is evident from mechanics that the summation of the moments of all the forces acting on the beam, taken about either support, is equal to zero. This principle permits one to find the reactions when the loads are known. It is also evident that since the loads act vertically downward and the reactions vertically upward, for the conditions that prevail, the sum of the reactions must equal the sum of the loads. This statement serves to check the reactions after they have been found.

In order to see the utility of these principles, we shall consider as our problem the solution of the simple beam of Fig. 12, for the

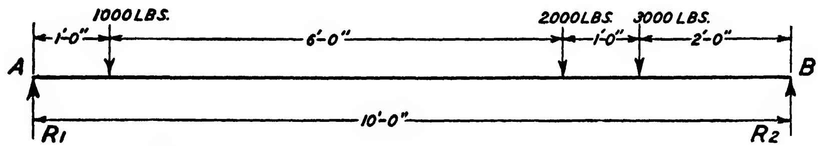


Fig 12. Layout of Simple Beam with Concentrated Loads

reactions, R_1, at the left support, A, and R_2, at the right support, B. It will be noticed that the distance between the supports, called the span, is known as well as the magnitude and relative location of the several so-called concentrated loads. The beam itself is considered without weight. Hence no uniformly distributed loads enter into our computation.

Step 1. Equate the summation of the moments of all the forces with respect to the right support, B, to zero. It will be noticed that the force, R_2, works through a zero moment arm when the center of moments is at B. This eliminates it as an unknown from the equation and leaves only the reaction, R_1, as an unknown. Hereafter this zero moment will not be shown.

$$R_1 \times 10 - 1000 \times 9 - 2000 \times 3 - 3000 \times 2 + R_2 \times 0 = 0$$

$$10R_1 - 9000 - 6000 - 6000 = 0$$

Collecting and transposing, $10R_1 = 21{,}000$

Dividing by 10, the coefficient of R_1, $R_1 = \dfrac{21{,}000}{10} = 2100$ lb.

Step 2. Equate the summation of the moments of all the forces with respect to the left support, A, to zero.

Thus, $-R_2 \times 10 + 3000 \times 8 + 2000 \times 7 + 1000 \times 1 = 0$

$$-10R_2 + 24{,}000 + 14{,}000 + 1000 = 0$$

Collecting and transposing, $-10R_2 = -39{,}000$

Dividing by -10, $R_2 = 3900$ lb.

Step 3. Prove the correctness of the values of R_1, and R_2, by adding them to see if their sum is equal to the sum of the loads.

Sum of loads $= 1000 + 2000 + 3000 = 6000$ lb.

Sum of reactions $= 2100 + 3900 = 6000$ lb. to check above.

The reactions occurring in an overhung beam are found in the same manner as for the simple beam. Consider the following problem of the overhung beam as given by Fig. 13, dealing with the three concentrated loads at C, D, and E.

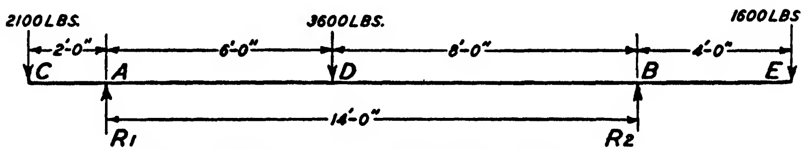


Fig. 13. Layout of Overhung Beam with Concentrated Loads

Solution. Taking moments about B as a center,

$$-2100 \times 16 + R_1 \times 14 - 3600 \times 8 + 1600 \times 4 = 0$$

$$-33{,}600 + 14R_1 - 28{,}800 + 6400 = 0$$

Collecting and transposing, $14R_1 = 56{,}000$

$R_1 = 4000$ lb. *Ans.*

Taking moments about A as a center,

$$-2100 \times 2 + 3600 \times 6 - R_2 \times 14 + 1600 \times 18 = 0$$

$$-4200 + 21{,}600 - 14R_2 + 28{,}800 = 0$$

$$14R_2 = 46{,}200$$

$R_2 = 3300$ lb. *Ans.*

To check: $R_1 + R_2 = 4000 + 3300 = 7300$ lb., the sum of the loads.

The loads that were considered above were in each case concentrated loads. Such a load is one that is applied to the beam at a certain point. Other loads are spread along the beam over part or all of its length. Such loads are said to be uniformly distributed and are generally given as so many pounds per foot of length. It is evident

that should the weight of a beam of constant cross section be considered, it would necessarily be a uniformly distributed load. As a rule in the design of a machine, the weight of a part acting as a beam may be neglected, and only the concentrated loads considered. Since uniformly distributed loads do enter into the design at times, however, let us solve the following example in order to show how to deal with them.

Example. It is required to find the reactions of a simple beam which has a span of 10 feet, is subjected to a concentrated load of 3000 pounds located 4 feet from the right support, and carries a uniformly distributed load throughout its length of 200 pounds per foot. The latter may be thought of as including the weight of the beam.

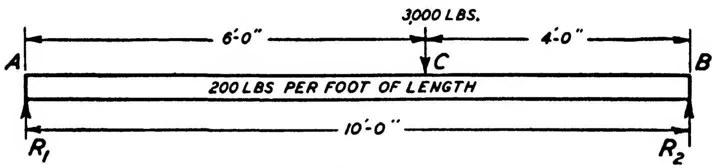


Fig 14

Solution. The first step always is to draw a diagram showing the relative location of the concentrated loads and using a double line to indicate the uniformly distributed load. See Fig. 14.

In finding R_1, as in the previous examples, the summation of the moments of all forces with respect to the support *B*, is taken. Here this will include the entire uniformly distributed load because the forces along the entire length are considered and the uniformly distributed load of this problem is existent throughout the length of the beam.

Rule. When considering all (or a part) of a uniformly distributed load in a given step, consider it as being concentrated at the midpoint of the length of that part of the beam over which it is distributed. This rule holds not only in the case of the reactions but also later on in securing bending moments.

Since in this step all of it is to be considered, it will amount to $200 \times 10 = 2000$ pounds, which according to the rule will be considered as if it were acting at a point 5 feet out from the origin of moments, *B*. Thus our equation for the reaction, R_1, will read:

$$R_1 \times 10 - 2000 \times 5 - 3000 \times 4 = 0$$

or

$$10R_1 - 22{,}000 = 0$$

$$10R_1 = 22{,}000$$

$$R_1 = 2200 \text{ lb.} \quad \textit{Ans.}$$

To secure R_2, we have,

$$-R_2 \times 10 + 3000 \times 6 + 2000 \times 5 = 0$$

$$-10R_2 + 28{,}000 = 0$$

$$10R_2 = 28{,}000$$

$$R_2 = 2800 \text{ lb.} \quad \textit{Ans.}$$

To check. $R_1 + R_2 = 2800 + 2200 = 5000$ lb., the sum of all loads.

Bending Moment. It will be brought out later on that in the design of a beam the Bending Moment plays an important part. The bending moment at or for a section of a loaded beam is by definition the algebraic sum of the moments of all the external forces acting on the beam either to the left or to the right of the section with respect to any point in the section. Every section of a beam has a bending moment. Its value may be positive, negative, or zero. It must be remembered that the external forces acting on a beam include all the loads both concentrated and uniformly distributed, as well as the reactions.

The rule of signs used in obtaining the reactions is followed in obtaining the bending moment. The absolute value of the latter will be the same irrespective of whether the forces to one side of the section or to the other side are used, but the algebraic sign will be different. This is of no consequence later on in the design of a beam, for the bending moment chosen for use at that time will in any event be considered positive. One naturally selects that side of the section which has the fewer forces so as to shorten the work involved. But in the following examples, the same side of the section will be used so that the algebraic signs will be consistent. Such procedure is necessary also in plotting a bending moment diagram in Strength of Materials.

Example. Compute the bending moments for sections 1 foot apart in the beam of Fig. 12, in which the reactions, R_1, and R_2, have been found to be 2100 and 3900 pounds respectively.

The bending moment at a section will be designated as M with a subscript which shows either the name of the section or the distance from the left support. Thus M_a is the bending moment at support,

A, while M_7 is the bending moment under the 2000-pound load, located 7 feet from the left support.

Applying the definition of a bending moment,

$M_a = 0$
$M_1 = 2100 \times 1 = +2100$ ft.-lb.
$M_2 = 2100 \times 2 - 1000 \times 1 = +3200$ ft.-lb.
$M_3 = 2100 \times 3 - 1000 \times 2 = +4300$ ft.-lb.
$M_4 = 2100 \times 4 - 1000 \times 3 = +5400$ ft.-lb.
$M_5 = 2100 \times 5 - 1000 \times 4 = +6500$ ft.-lb.
$M_6 = 2100 \times 6 - 1000 \times 5 = +7600$ ft.-lb.
$M_7 = 2100 \times 7 - 1000 \times 6 = +8700$ ft.-lb.
$M_8 = 2100 \times 8 - 1000 \times 7 - 2000 \times 1 = +7800$ ft.-lb.
$M_9 = 2100 \times 9 - 1000 \times 8 - 2000 \times 2 - 3000 \times 1 = +3900$ ft.-lb.
$M_b = 2100 \times 10 - 1000 \times 9 - 2000 \times 3 - 3000 \times 2 = 0$

(**Note:** The student should solve this example by considering the forces to the right of each section.)

Example. Compute the bending moments for sections 1 foot apart in the example of Fig. 14.

Solution. Since the example has been solved for the reactions, these can now be utilized. Otherwise, since all the forces must be known before it is possible to obtain the bending moments, it would be necessary to solve first for the reactions.

$$M_a = 0$$
$$M_1 = 2200 \times 1 - 200 \times \tfrac{1}{2} = 2100 \text{ ft.-lb.}$$

(Since the section is 1 foot from the left support, over this 1 foot of beam, there are 200 pounds of uniformly distributed load. This part of the total uniformly distributed load is all of the latter that is involved to the left of this section. It is then momentarily considered as if it were concentrated at the mid-point of the length of that part of the beam. Therefore its moment arm is $\frac{1}{2}$ foot and its moment is $-200 \times \frac{1}{2}$.)

$$M_2 = 2200 \times 2 - 400 \times 1 = 4000 \text{ ft.-lb.}$$

(Here the uniform load to be considered is over 2 feet of the beam and therefore is 400 pounds. The latter, assumed to be located at the mid-point of this 2 feet would then have a moment arm with respect to this section of 1 foot. Therefore its bending moment is -400×1).

$M_3 = 2200 \times 3 - 600 \times 1\tfrac{1}{2} = +5700$ ft.-lb.
$M_4 = 2200 \times 4 - 800 \times 2 = +7200$ ft.-lb.
$M_5 = 2200 \times 5 - 1000 \times 2\tfrac{1}{2} = +8500$ ft.-lb.

$$M_6 = 2200\times 6 - 1200\times 3 = +9600 \text{ ft.-lb.}$$
$$M_7 = 2200\times 7 - 3000\times 1 - 1400\times 3\tfrac{1}{2} = +7500 \text{ ft.-lb.}$$
$$M_8 = 2200\times 8 - 3000\times 2 - 1600\times 4 = +5200 \text{ ft.-lb.}$$
$$M_9 = 2200\times 9 - 3000\times 3 - 1800\times 4\tfrac{1}{2} = +2700 \text{ ft.-lb.}$$
$$M_b = 2200\times 10 - 3000\times 4 - 2000\times 5 = 0$$

Maximum Bending Moment. The preceding examples are evidence of the fact that at every section of a beam there is a bending moment created by the forces to which the beam is subjected. The one among these which has the largest absolute value is called the Maximum Bending Moment, and will be designated henceforth by the symbol, M, with no subscript. The value of M must be known before a beam can be designed.

The section at which the maximum bending moment occurs is known as the Dangerous Section. It is the section at which the effect of the external forces acting upon a beam is felt to the greatest extent. The dangerous section in a simple beam which is subjected to concentrated loads only, will be under one of those loads. Hence to find the maximum bending moment, one need only find the bending moments under each concentrated load and take the largest absolute value for M. In case uniformly distributed loads are present, the dangerous section is not necessarily under one of the concentrated loads and the maximum bending moment must be obtained by computing bending moments at frequent intervals throughout the length of the beam or through the use of a shear diagram as given in texts on Strength of Materials. In all cantilever beams the dangerous section and hence the maximum bending moment is located at the support, no matter to what kind of loading the beam is subjected.

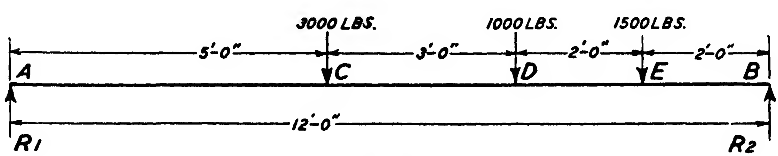


Fig. 15

Example. Find the maximum bending moment in in.-lb. of a simple beam with concentrated loads as shown in Fig. 15.

Solution. Before any bending moment can be obtained, all external forces must be known. Therefore it will be necessary at the start to find the reactions.

To find R_1, take center of moments at support B. This gives

$$R_1 \times 12 - 3000 \times 7 - 1000 \times 4 - 1500 \times 2 = 0$$
$$12R_1 - 28{,}000 = 0$$
$$12R_1 = 28{,}000$$
$$R_1 = 2333\tfrac{1}{3} \text{ lb.}$$

To find R_2, take center of moments at support A. This gives

$$-R_2 \times 12 + 1500 \times 10 + 1000 \times 8 + 3000 \times 5 = 0$$
$$-12R_2 + 38{,}000 = 0$$
$$12R_2 = 38{,}000$$
$$R_2 = 3166\tfrac{2}{3} \text{ lb.}$$

$R_1 + R_2 = 2333\tfrac{1}{3} + 3166\tfrac{2}{3} = 5500$ lb., to check with the sum of the loads.

Now that all external forces are known including loads and reactions, we can proceed to find the bending moments. Since there are only concentrated loads, the maximum bending moment will be either at section C, D, or E. So it will be necessary to find only M_c, M_d, and M_e and from them select the largest absolute value as M, the maximum bending moment.

$$M_c = R_1 \times 5$$
$$= 2333\tfrac{1}{3} \times 5 = 11{,}666\tfrac{2}{3} \text{ ft.-lb.}$$
$$M_d = R_1 \times 8 - 3000 \times 3$$
$$= 2333\tfrac{1}{3} \times 8 - 3000 \times 3$$
$$= 9{,}666\tfrac{2}{3} \text{ ft.-lb.}$$
$$M_e = -R_2 \times 2$$
$$= -3166\tfrac{2}{3} \times 2 = -6333\tfrac{1}{3} \text{ ft.-lb.}$$

Therefore $\quad M = M_c = 11{,}666\tfrac{2}{3}$ ft.-lb.

$M = 11{,}666\tfrac{2}{3} \times 12 = 140{,}000$ in.-lb. *Ans.*

and the dangerous section is at C.

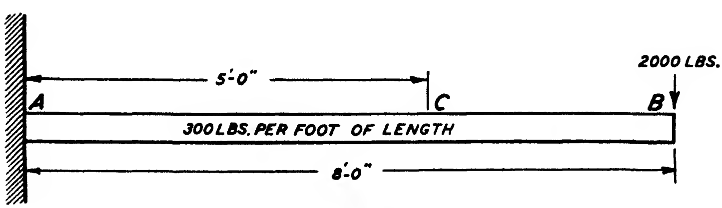


Fig. 16

Example. A cantilever beam extends 8 feet out from its support in a wall and carries an end load of 2000 pounds along with a uni-

formly distributed load of 300 pounds per foot of length. (*a*) Where is the dangerous section? (*b*) Find the maximum bending moment. (*c*) Find the bending moment at a section of the beam 5 feet out from the wall. (*d*) What is the bending moment at the outer end of the cantilever?

Solution. A diagram, Fig. 16, is always the first step.

(*a*) The dangerous section is at A, the support, in any cantilever.

(*b*) Since the maximum bending moment is always at the support in this type of beam, it is equal to the sum of the bending moments of all the forces on the beam taken with respect to the support.

Distributed load $=8\times300=2400$ lb.

The moment of the distributed load $=2400\times4=9600$ ft.-lb.

The moment of the end load $=2000\times8=16{,}000$ ft.-lb.

$M=9600+16{,}000=25{,}600$ ft.-lb. or 307,200 in.-lb. *Ans.*

(*c*) To find the bending moment at any section of a beam such as C, consider only the loads between it and the end of the beam. Therefore $M_c=2000\times3+900\times1\frac{1}{2}$ (since there are only 900 pounds of distributed load outside of C)

$=6000+1350=7350$ ft.-lb. or 88,200 in.-lb. *Ans.*

(*d*) Zero.

Special Cases of Loading. There are several special cases of loading in the design of beams which are often recurring in Machine Design. In these the maximum bending moment can be found by the general processes already shown, or formulas for the maximum bending moment can be derived by these processes, the use of which will lessen the labor of the solution of such problems. Formulas for M for several cases will now be derived.

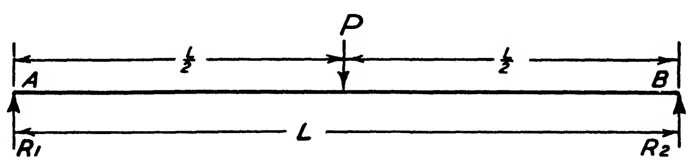


Fig. 17

Case 1. A simple beam loaded by a single concentrated load, P, at the center of its span, as in Fig. 17.

From the symmetry of loading it is evident that

$$R_1=R_2=\frac{P}{2}$$

Since M must occur under the single concentrated load, P,

$$M=R_1\times\frac{L}{2}$$

Substituting for R_1, its equal $\frac{P}{2}$, we obtain

$$M=\frac{P}{2}\times\frac{L}{2}=\frac{PL}{4} \tag{20}$$

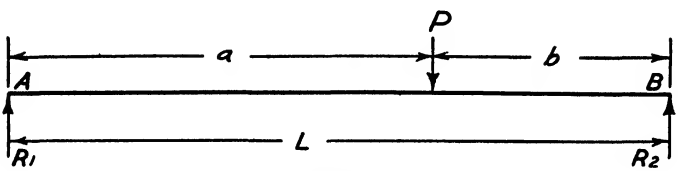


Fig. 18

Case 2. A simple beam loaded by a single concentrated load, P, which is not at the center of its span, as in Fig. 18.
To find R_1, take B as the origin.

$$R_1\times L-P\times b=0$$
$$R_1\times L=Pb$$
$$R_1=\frac{Pb}{L}$$

Since M must occur under the only concentrated load, P,

$$M=R_1\times a$$

Substituting for R_1, its equal $\frac{Pb}{L}$, we obtain

$$M=\frac{Pb}{L}\times a=\frac{Pab}{L} \tag{21}$$

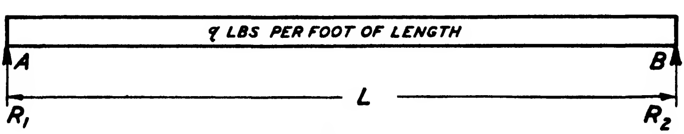


Fig 19

Case 3. A simple beam loaded only with a uniformly distributed load along its entire span, as in Fig. 19.

Let Q = total distributed load
and q = distributed load per foot of length of beam
Then $Q = L \times q$

To find R_1, take summation of the moments of all the external forces about support B as an origin. The entire distributed load, Q, must be considered, in this step, as concentrated at the mid-point of the beam. Hence Q will act through a moment arm equal to $\frac{L}{2}$, while the moment arm of R_1 will be L. Thus

$$R_1 \times L - Q \times \frac{L}{2} = 0$$

$$R_1 \times L = Q \times \frac{L}{2} = \frac{QL}{2}$$

Dividing by L, $$R_1 = \frac{QL}{2L} = \frac{Q}{2}$$

which is also evident from the symmetry of loading occurring in the above. This symmetry of loading also suggests that the maximum bending moment will be midway between the supports. Hence to get M at this point, we will consider the forces to the left of the mid-section. These will be R_1 and $\frac{1}{2}$ of Q. The latter then may be considered as acting at a point midway between the center (mid-section) of the beam and the left support or through a moment arm of $\frac{L}{4}$ with respect to the mid-section. Therefore

$$M = R_1 \times \frac{L}{2} - \frac{Q}{2} \times \frac{L}{4}$$

Substituting for R_1, its equal, $\frac{Q}{2}$, we have

$$M = \frac{Q}{2} \times \frac{L}{2} - \frac{Q}{2} \times \frac{L}{4}$$

$$= \frac{QL}{4} - \frac{QL}{8} = 2\frac{QL}{8} - \frac{QL}{8}$$

$$= \frac{QL}{8} \tag{22}$$

Case 4. A cantilever loaded with a single concentrated load, P, at its end point, as in Fig. 20.

Here M occurs at the support and hence we have,

$$M = PL \tag{23}$$

Case 5. A cantilever loaded with only a uniformly distributed load, Q, as in Fig. 21.

Let Q = total distributed load $= L \times q = Lq$.

Here M, as in every cantilever, must occur at the support. Q will be considered as concentrated at the mid-section, so will work through a moment arm of $\frac{L}{2}$ with respect to the support. This gives,

$$M = Q \times \frac{L}{2} = \frac{QL}{2} \tag{24}$$

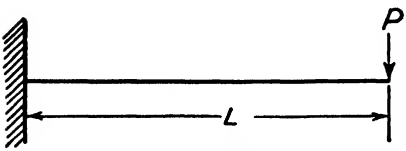


Fig 20

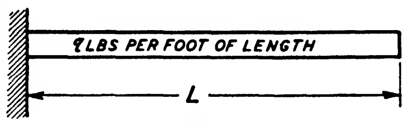


Fig. 21

Moment of Inertia. In the discussion to follow, a quantity, known as the Moment of Inertia of a plane area will become involved. This quantity was introduced to the student in Strength of Materials but it may be well at this time to review its definition. If a plane area be divided into an infinite (very large) number of infinitesimal (very small) parts, then the sum of the products obtained by multiplying the area of each part by the square of its distance from some line is called the Moment of Inertia of the area with respect to that line. In the design of beams, the line used is taken through the center of gravity of the cross section of the beam and is always horizontally located when the external forces on the beam are vertical. Such a line is referred to later on as the neutral axis of the section. Since a moment of inertia is a sum of products, each one of which products is obtained by multiplying an area in square inches by the square of a distance given in inches, we have for the unit of a moment of inertia, inches $^2 \times$ inches^2 = inches^4, or biquadratic inches.

Resisting Moment of a Beam. The loads to which a beam is subjected set up stresses acting over a cross section of the beam which are not the same at all points of that section. The fibers on one side are stretched and therefore under tension, while those on the other side are shortened and under compression. In a simple beam as in

Figs. 7 and 22, the upper fibers are compressed and shortened, and the lower fibers lengthened. In a cantilever, as in Fig. 10, the upper fibers are in tension while the lower ones are in compression, for the loads on such a beam tend to deflect it gradually downward from the

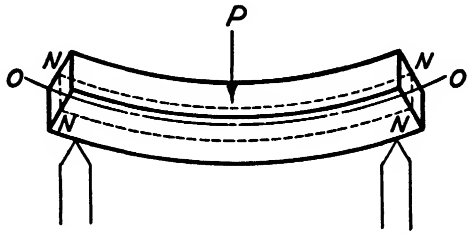


Fig 22. Resistance of Simple Beam to Bending

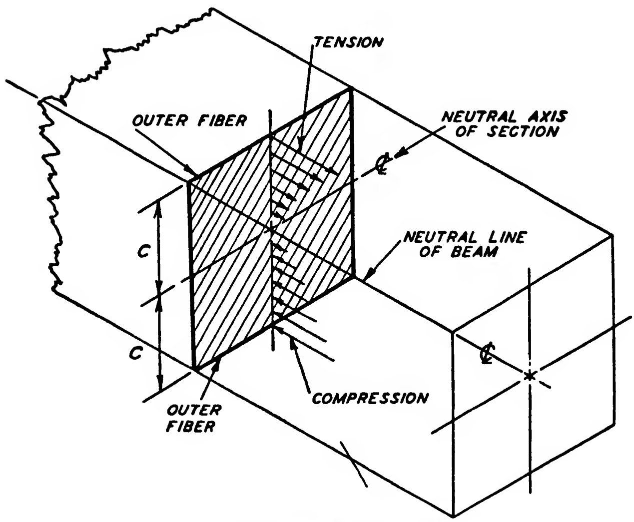


Fig 23 Section of Cantilever Beam

support to the end of the beam. This change in length of the fibers, and therefore in the intensity of the stress, proceeds uniformly from the bottom to the top of the section of a beam as shown by the vectors in the section of a cantilever beam pictured in Fig. 23. Since the character of stress changes from tension to compression, in passing from the convex side to the concave side of a beam, there must be a point or layer in the beam which has neither tension nor compression exerted therein. This layer or surface extends from one end of the

beam to the other and is called the Neutral Surface. It is represented in Fig. 22 by a plane, $NNNN$. The central line, OO, of this plane is called the Neutral Line of the beam. It passes through the center of gravity of every section of the beam. Every cross section of the beam, such as that shown in Fig. 23, cuts from this neutral plane a straight line, called the Neutral Axis of the section. It is evident that this neutral axis is in a plane at right angles to the direction of the external forces, that it intersects at right angles the neutral line of the beam, and hence passes through the center of gravity of the section. The relative placement of this neutral axis with respect to the dimensions of its section is of great importance in the design of beams.

The external forces of a beam set up a resistance in every section and the moment of that resistance must safely resist, and hence be equal to, the bending moment at that section. The moment of the safe resistance of any section is equal to $S\times\frac{I}{C}$, in which S is the safe unit stress in the outermost fiber, I is the rectangular moment of inertia of the section with respect to the neutral axis of the section, and C is the distance from the neutral axis to the outermost fiber. Therefore the bending moment at any section,

$$M_{(any\ section)}=S\times\frac{I}{C}=S\frac{I}{C} \qquad (25)$$

The above formula may be used to design any section of a beam to meet the conditions of the particular bending moment at that section, or it may be used in checking a given section for the stress, S. Since the cross section of a beam as a rule is continuous throughout the length of the beam, evidently the section that is to dictate the uniform section of the beam is the one at which the bending moment is a maximum. If then the beam is designed to be safe at this section, it will be still safer or subjected to still lower working stresses at all other sections. In other words, the section at which the bending moment is a maximum, although perfectly safe, will be the weakest section in the beam. This is the reason it is called the Dangerous Section. From the preceding, it is evident that the design formula for beams then becomes

$$M=S\frac{I}{C} \qquad (26)$$

in which M is the maximum bending moment.

Section Modulus. The ratio $\frac{I}{C}$ of formula (26) is called the rectangular Section Modulus. Therefore by definition, the section modulus of a section is equal to its moment of inertia divided by the distance from the neutral axis to the outermost fiber. Since moments of inertia are given in biquadratic inches, inches^4, or as abbreviated, in.^{4}, section moduli must be in cubic inches, in.^{3} Why? Letting Z represent the rectangular section modulus and substituting it for its equal, $\frac{I}{C}$, in formula (26), we obtain,

$$M=SZ \tag{27}$$

where M = maximum bending moment in inch-pounds.

S = safe bending stress in pounds per square inch

and Z = rectangular section modulus of the section used, in in.^{3}

When checking the stress in a section of a designed beam, formula (27) of course can be used, but it is more convenient to have the formula in the form wherein it stands solved for S. Such a form can be obtained by merely dividing both members of formula (27) by Z. This produces

$$S=\frac{M}{Z} \tag{28}$$

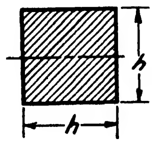
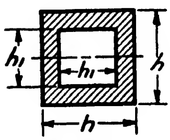
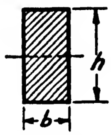
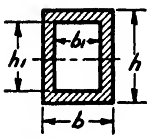
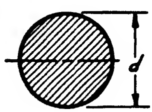
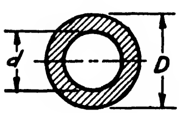
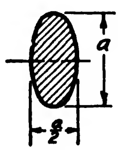
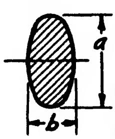
Formulas (26) and (27) are the most important formulas relating to beams. Table III gives values of both I and Z for the typical sections used in the design of machine elements, which are generally of such a type that the center of gravity coincides with the geometric center. It will be noted that values of both I and Z are given in terms of the dimensions of the section, so through these the latter are introduced into the design formula. Formula (27) is more often used than formula (26), since it contains one less term.

Example. A wooden beam, square in section, extends 8 feet out from the wall by which it is supported. It sustains an end load of 2000 pounds. If the safe stress is assumed to be 750 pounds per square inch, what are the dimensions of its cross section? Neglect the weight of the beam.

Solution. At the beginning of the solution of every beam problem, a sketch should be made illustrating the dimensions, the magnitudes of loads, and the involved section. For this solution, such a sketch is given in Fig. 24.

TABLE III — Rectangular Moments of Inertia, Section Moduli, and Radii of Gyration

(In each case, the neutral axis is horizontal and passes through the center of gravity.)

Section	Moment of Inertia, I	Section Modulus, Z	Radius of Gyration
	$\frac{h^4}{12}$	$\frac{h^3}{6}$	$\frac{h}{\sqrt{12}}$ or $0.289h$
	$\frac{h^4-h_1^4}{12}$	$\frac{h^4-h_1^4}{6h}$	$\sqrt{\frac{h^2+h_1^2}{12}}$
	$\frac{bh^3}{12}$	$\frac{bh^2}{6}$	$\frac{h}{\sqrt{12}}$ or $0.289h$
	$\frac{bh^3-b_1h_1^3}{12}$	$\frac{bh^3-b_1h_1^3}{6h}$	$\sqrt{\frac{bh^3-b_1h_1^3}{12(bh-b_1h_1)}}$
	$\frac{\pi d^4}{64}$ or $0.049d^4$	$\frac{\pi d^3}{32}$ or $0.098d^3$	$\frac{d}{4}$
	$\frac{\pi(D^4-d^4)}{64}$ or $0.049(D^4-d^4)$	$\frac{\pi(D^4-d^4)}{32D}$ or $0.098\frac{D^4-d^4}{D}$	$\frac{\sqrt{D^2+d^2}}{4}$
	$\frac{\pi a^4}{128}$ or $0.0245a^4$	$\frac{\pi a^3}{64}$ or $0.049a^3$	$\frac{a}{4}$
	$\frac{\pi a^3 b}{64}$	$\frac{\pi a^2 b}{32}$	$\frac{a}{4}$

The dangerous section will be at the support where the bending moment is a maximum as in all cantilever beams.

Applying formula (23), in which $P=2000$ lb. and $L=96$ in.,

$$M=PL=96\times2000=192{,}000 \text{ in.-lb.}$$

(**Note**: In all beam design, be sure to obtain M in inch-pounds for substitution later in the design formula for beams.)

Since the section is a square, from Table III, we find that $Z=\frac{h^3}{6}$. Substituting this value of Z in formula (27), together with the known values for S and M, we have,

$$M=SZ$$

$$192{,}000=750\times\frac{h^3}{6}$$

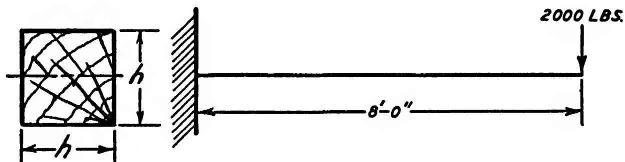


Fig 24

Multiplying both members of the equation by 6 and dividing by 750,

$$h^3=\frac{192{,}000\times6}{750}=1536$$

$$h=\sqrt[3]{1536}=11.5 \text{ in., say } 11\tfrac{1}{2} \text{ in.}$$

$\therefore$ the section of the beam is $11\frac{1}{2}$ in.$\times11\frac{1}{2}$ in. *Ans.*

Example. Design the beam of the preceding example for a rectangular section in which the dimension perpendicular to the neutral axis is 3 times the other dimension.

Solution. The sketch for the section of the beam is as shown in Fig. 25. Notice the relation of the longer dimension with respect to the neutral axis. In this example, M will necessarily be the same as in the preceding. The difference between this solution and the previous one will be in the section modulus to be used. Here, from Table III,

$$Z=\frac{bh^2}{6}$$

and since $h=3b$, $h^2=(3b)^2=9b^2$

$$\therefore\ Z=\frac{b\times 9b^2}{6}=\frac{9b^3}{6}=\frac{3b^3}{2}, \text{ so that formula (27) becomes}$$

$$M=S\times\frac{3b^3}{2}$$

Substituting the values of M and S from the preceding example, we have,

$$192{,}000=750\times\frac{3b^3}{2}$$

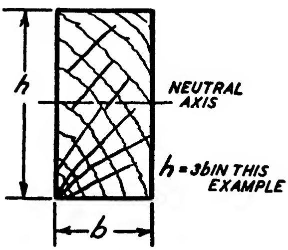


Fig. 25

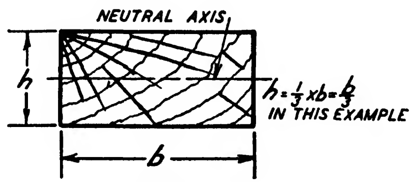


Fig. 26

Multiplying each member of the equation by 2 and dividing by (750×3), we obtain,

$$b^3=\frac{192{,}000\times 2}{750\times 3}=170.7$$

$$b=\sqrt[3]{170.7}=5.55 \text{ in., say } 5\tfrac{5}{8} \text{ in.}$$

$$h=3b=3\times 5\tfrac{5}{8}=16\tfrac{7}{8} \text{ in.}$$

$\therefore$ the section is $5\frac{5}{8}$ in.$\times 16\frac{7}{8}$ in. *Ans.*

Example. Design the beam of the two preceding examples for a rectangular section in which the dimension perpendicular to the neutral axis is $\frac{1}{3}$ the dimension parallel to that axis. See Fig. 26.

Solution. Again we have,

$$M=192{,}000 \text{ in.-lb.}$$

$$S=750 \text{ lb. per sq. in.}$$

$$Z=\frac{bh^2}{6} \text{ from Table III}$$

But in this case, $h=\frac{b}{3}$, so that

$$h^2=\frac{b^2}{9}$$

Therefore $$Z=\frac{b\times\frac{b^2}{9}}{6}=\frac{\frac{b^3}{9}}{6}$$

$$=\frac{b^3}{9}\times\frac{1}{6}=\frac{b^3}{54}$$

Substituting this value of Z in formula (27), $M=SZ$,

$$M=S\times\frac{b^3}{54}$$

Evaluating, $$192{,}000=750\times\frac{b^3}{54}$$

Multiplying each member of the above equation by 54 and dividing by 750,

$$b^3=\frac{192{,}000\times 54}{750}=13{,}824$$

$$b=\sqrt[3]{13{,}824}=24 \text{ in.}$$

$$\therefore\ h=\frac{b}{3}=\frac{24}{3}=8 \text{ in.}$$

$\therefore$ the section is 8 in.$\times$24 in. *Ans.*

(**Note:** From the last two examples, it may be noticed that less material is needed in a beam of rectangular cross section when the longer edge of the section is perpendicular to the neutral axis. This is important in the design of machine parts for it reduces both the weight and the cost of the machine.)

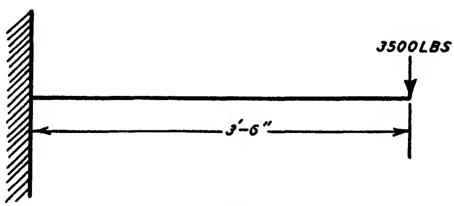


Fig. 27

Example. A steel I-beam, used as a cantilever, projects $3\frac{1}{2}$ feet from its support. The weight of the beam is to be neglected and the section modulus is given in a structural handbook as 20.4 inches^3.

(*a*) What is the maximum bending stress set up in the beam when it supports an end load of 3500 pounds as shown in Fig. 27?

(*b*) What is the maximum bending stress set up in the beam when it supports a total uniformly distributed load of 3500 pounds as shown in Fig. 28?

(*c*) In what section of the beam do these maximum stresses occur and what are the factors of safety in parts (*a*) and (*b*) if the ultimate bending stress is 60,000 pounds per square inch?

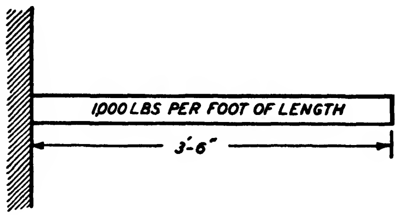


Fig. 28

Solution. (*a*) It will be noticed that the numerical values of the section moduli of standard structural steel shapes are given in structural steel handbooks. These numerical values are substituted directly for Z in the design formula. It will be possible in this example to find the value of the maximum bending moment by evaluating in formula (23). Here $P=3500$ lb. and $L=3\frac{1}{2}\times 12=42$ in., so that

$$M=PL$$
$$=3500\times 42=147{,}000 \text{ in.-lb.}$$

With this value of M, and with $Z=20.4$ in.^{3}, using formula (28),

$$S=\frac{M}{Z}$$
$$=\frac{147{,}000}{20.4}=7200+ \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(*b*) Here $Q=3500$ lb. Use formula (24) to obtain M, remembering that this formula considers Q as a concentrated load located midway between support and end of beam.

$$M=Q\times\frac{L}{2}$$
$$=3500\times\frac{42}{2}=3500\times 21=73{,}500 \text{ in.-lb.}$$

Now with this value of M and with $Z=20.4$ in.^{3}, we shall again evaluate in formula (28) to obtain,

$$S=\frac{73{,}500}{20.4}=3600 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(*c*) The above maximum bending stresses occur at the dangerous section only or in other words at the section where the bending moment is a maximum. *Ans.*

In part (*a*), $F=\dfrac{U}{S}$

$$=\frac{60{,}000}{7200}=8.33 \quad \textit{Ans.}$$

In part (*b*), $F=\dfrac{60{,}000}{3600}=16.66 \quad \textit{Ans.}$

(**Note**: Does it seem reasonable that the factor of safety in part (*b*) should be exactly twice the factor of part (*a*)? Why?)

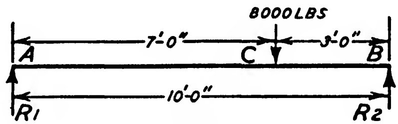


Fig 29

Example. A cylindrical steel beam whose safe bending stress is 10,000 pounds per square inch rests on supports 10 feet apart. The beam carries a single concentrated load of 8000 pounds located 3 feet from the right support. Neglecting the weight of the beam, find its diameter.

Solution. To find R_1, equate to zero the summation of moments of all external forces with respect to the origin at B. See Fig. 29.

$$R_1\times 10-8000\times 3=0$$
$$10R_1=24{,}000$$
$$R_1=2400 \text{ lb.}$$

To find R_2, in like manner, use A as the origin.

$$-R_2\times 10+8000\times 7=0$$
$$10R_2=56{,}000$$
$$R_2=5600 \text{ lb.}$$

To check, $R_1+R_2=2400+5600=8000$ lb., the loading.

The dangerous section and hence the maximum bending moment, will be under the load at C. Therefore considering the force to the right of C,

$$M=-R_2\times3\times12\text{ in.-lb.}$$
$$=-5600\times3\times12=-201{,}600\text{ in.-lb.}$$

M is considered as $+201{,}600$ in.-lb.
Since the section is circular, Table III gives

$$Z=\frac{\pi d^3}{32}$$

and $S=10{,}000$ lb. per sq. in.
Substituting these values in formula (27), $M=SZ$,

$$201{,}600=10{,}000\times\frac{\pi d^3}{32}$$

Multiplying both members of the equation by 32 and dividing by $10{,}000\pi$,

$$d^3=\frac{201{,}600\times32}{10{,}000\times\pi}=205$$

$$d=\sqrt[3]{205}=5.87\text{ in., say }5\tfrac{7}{8}\text{ or }5\tfrac{15}{16}\text{ in.}\quad \textit{Ans.}$$

(**Note**: The general method of solution was used for finding the maximum bending moment in the above. Since it is a simple beam subjected to one concentrated load only, the special case of formula (21) could have been used here to get M. Thus by formula (21),

$$M=\frac{Pab}{L}$$

in which $P=8000$ lb.; $a=7\times12=84$ in.; $b=3\times12=36$ in.; and $L=10\times12=120$ in.
Evaluating in the above formula,

$M=\dfrac{8000\times84\times36}{120}=201{,}600$ in.-lb., which checks the general solution.)

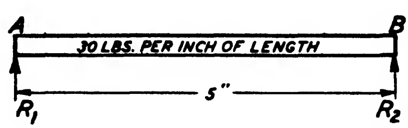


Fig. 30

Example. A steel pin supported at each end is subjected to a uniformly distributed load of 30 pounds per inch of length. If the length of the pin is 5 inches and the safe bending stress allowed in the design is 8000 lb. per sq. in., what diameter of pin need be employed? See Fig. 30.

Solution. As this is a special case of loading, the maximum bending moment can be found by the general method of solution or by using formula (22). Using the latter formula,

$$M=\frac{QL}{8}$$

in which $Q=30\times5=150$ lb., and $L=5$ in.
Evaluating in our formula,

$$M=\frac{150\times5}{8}=\frac{750}{8}=94-\text{ in.-lb.}$$

Using formula (27), in which $M=94$ in.-lb., $S=8000$ lb. per sq. in., and $Z=\frac{\pi d^3}{32}$, we have,

$$M=SZ$$

$$94=8000\times\frac{\pi d^3}{32}$$

Multiplying each member of the above equation by 32 and dividing by $8000\times\pi$, $d^3=\frac{94\times32}{8000\times\pi}=0.12$

$$d=\sqrt[3]{0.12}=0.492 \text{ in., say } \tfrac{1}{2} \text{ in.} \quad \textit{Ans.}$$

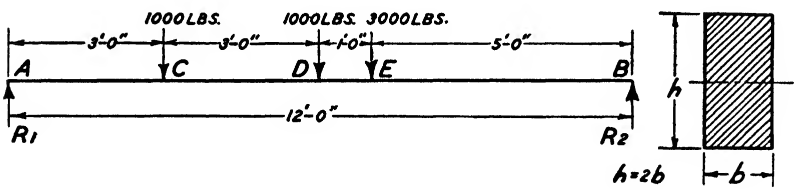


Fig 31

Example. A wrought iron beam rests on supports 12 feet apart. The section of the beam is rectangular with its breadth twice its thickness. The beam supports three concentrated loads as shown in Fig. 31. Required: the dimensions of its cross section assuming the safe bending stress equal to 6000 pounds per square inch. Neglect the weight of the beam.

Solution. Step 1. Find and check the reactions, R_1 and R_2.

Taking the moment of all the external forces with respect to the right support, and equating their summation to zero,

$$R_1\times12-1000\times9-1000\times6-3000\times5=0$$

Simplifying, $12R_1-9000-6000-15{,}000=0$

Collecting and transposing, $12R_1=30,000$

$$R_1=2500 \text{ lb.}$$

Taking the moments of all the external forces with respect to the left support, and equating their summation to zero,

$$-R_2\times12+3000\times7+1000\times6+1000\times3=0$$

Simplifying, $-12R_2+21,000+6000+3000=0$

Collecting and transposing, $-12R_2=-30,000$

$$12R_2=30,000$$

$$R_2=2500 \text{ lb.}$$

To check, $R_1+R_2=2500+2500=5000$ lb., the sum of the loads.

Step 2. Find the maximum bending moment, M, in inch-pounds.

Since there are only concentrated loads upon this simple beam, the maximum bending moment will be at the section under one of them. Therefore proceed to obtain M_c, M_d, and M_e and take the largest absolute value as M and consider it to be positive.

$$M_c=R_1\times3\times12=2500\times36=90,000 \text{ in.-lb.}$$

$$M_d=R_1\times6\times12-1000\times3\times12$$
$$=2500\times72-36,000=180,000-36,000$$
$$=144,000 \text{ in.-lb.}$$

For M_e consider forces to the right since it will be shorter to do so.

$$M_e=-2500\times5\times12=-150,000 \text{ in.-lb.}$$

$$\therefore\ M=M_e=+150,000 \text{ in.-lb.}$$

Step 3. Use the general design formula (**27**) and solve for the dimensions of the dangerous section.

$$M=SZ$$

$$Z=\frac{bh^2}{6}, \text{ Table III}$$

Here $h=2b$

$$\therefore\ Z=\frac{b\times(2b)^2}{6}=\frac{b\times4b^2}{6}=\frac{2b^3}{3}$$

$$S=6000 \text{ lb. per sq. in.}; M=150,000 \text{ in.-lb.}$$

Substituting in the design formula given above,

$$150,000=6000\times\frac{2b^3}{3}$$

Multiplying each member of this equation by 3 and dividing by 6000×2, we have

$$b^3 = \frac{150{,}000 \times 3}{6000 \times 2} = 37.5$$

$$b = \sqrt[3]{37.5} = 3.34 \text{ in., say } 3\tfrac{3}{8} \text{ in.}$$

$$\therefore\ h = 2b = 2 \times 3\tfrac{3}{8} = 6\tfrac{3}{4} \text{ in.}$$

$\therefore$ the section of the beam is $3\frac{3}{8}$ in.$\times 6\frac{3}{4}$ in. *Ans.*

Twisting. Fig. 8, presented at the beginning of this chapter and reproduced here for convenience, illustrates a cylindrical bar, A, secured to and supported by the wall as shown. At the end of A is a crank, B, and pin, C. The length of the crank, measured from the

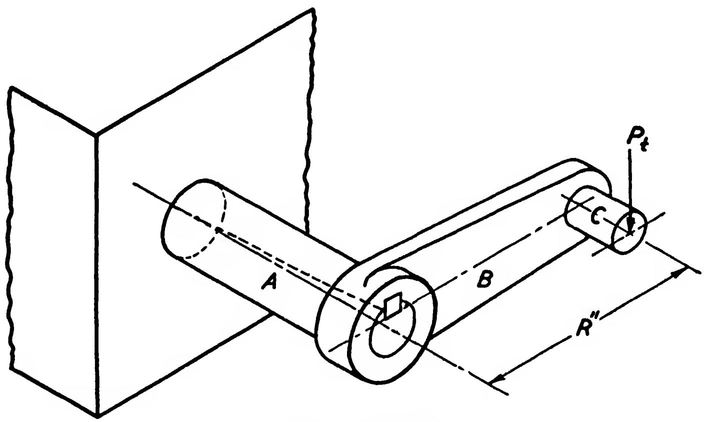


Fig 8 Twisting

center-line of A to the center-line of C, is given by the dimension, R. In such a case as this, a force, P_t, acting at right angles to the center-line of the crank, B, will tend to turn or twist the rod, A, about its axis. The measure of this tendency is the moment of the force, P_t, with respect to the axis of A and is equal to the product of the force and the perpendicular distance through which the force acts with respect to that axis. This distance is, by definition, the moment arm of the force and is given in the figure as R inches. Therefore the moment of P_t is equal to $P_t \times R$ inch-pounds when P_t is given in pounds, and R in inches. The bar, A, is said to be subjected to the twisting moment, $P_t \times R$, or is said to be under Torsion. The effect of this is to displace the elements of the cylindrical bar as shown by the dotted lines in the accompanying figure, and in general to set up a shearing stress within the bar.

In the figure, the bar is shown secured to or held fast by the wall in order to provide the resistance to rotation. If there were no resistance, there could be no possibility of twisting, or the twisting moment, P_tR, would have to be equal to zero. In Machine Design, such a bar is in most cases supplied its resistance to rotation by being connected to a machine through some mechanism such as a pair of gears. The student can visualize this by picturing in his mind a machine located on the back side of the wall and geared or belted to the bar which is then supported in the wall by a smooth, practically frictionless hole, or bearing. It is the office of the bar to supply power and motion to the machine so that the latter can do the work it is designed to do.

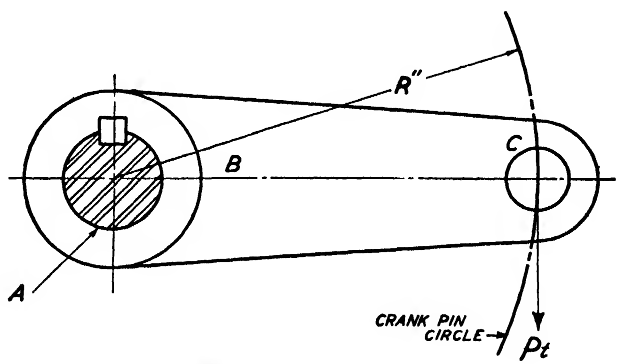


Fig 32. End View of Fig 8

In such a case, the bar, now called a shaft, is supplied energy by some force like unto P_t. The bar must transmit this energy to the machine. The force, P_t, must then have a moment that is large enough to overcome the resistance to motion of the shaft due to the connected machine, so that motion may ensue and power be transmitted. In the event of rotation of the bar (or shaft), it is still subjected to a twisting moment or is under torsion, but now the force, P_t, actually causes rotation, and as a result the bar is said to transmit Torque. A rotating bar that is subjected to a twisting moment is called a Shaft. A discussion of the latter will be given later on in this text.

An end view of the crank and shaft of Fig. 8 is shown in Fig. 32. It will be noticed that the force, P_t, is located tangent to the path taken by its point of application, the pin, C. Therefore P_t is a tangential force and hence the use of the subscript, t. If, then, $P_t=$ a

tangential force in pounds, R = radial moment arm in inches through which the tangential force, P_t, acts in causing rotation of the shaft, A, and T = torque, or twisting moment or torsional moment in inch-pounds, then

$$T = P_t R \tag{29}$$

Example. A tangential force of 100 pounds is applied at the crank-pin of Fig. 32. If the radius, R, of the crank-pin circle is 6 inches in length, what is the twisting moment or torque set up in the crank-shaft?

Solution. Here $P_t = 100$ lb.
$R = 6$ in.

Using formula (29), $T = P_t \times R$
$= 100 \times 6 = 600$ in.-lb. *Ans.*

Resisting Moment of a Shaft. When a shaft is subjected to torsion or transmits torque, the stresses set up in the shaft are known as Torsional Stresses. As previously stated these are in reality shearing stresses, and they occur in a right or transverse section of a shaft. For at any section of a shaft there is a tendency due to the twisting action, for one part to slide past the other. To prevent or resist this slipping or shearing, there arise these shearing stresses at all parts of the cross section. This shearing stress that is set up is not uniformly distributed over the involved area. Its value increases from zero at the center to a maximum at the outside of the shaft.

The Resisting Moment of a shaft is the sum of the moments of these shearing stresses about the center of the shaft. It is given algebraically by the formula,

$$\text{Resisting Moment of a shaft} = \frac{S_s \times I_p}{C} \text{ in.-lb.}$$

where S_s = the maximum shearing stress in pounds per square inch in the extreme outer fiber, set up by the torque transmitted; I_p = the polar moment of inertia of the involved section in inches^4; and C = the distance in inches from the neutral axis to the extreme outer fiber.

Since this resisting moment is caused by the twisting moment, T, of formula (29), it must be equal thereto. This statement gives us

$$T = S_s \frac{I_p}{C} \tag{30}$$

In the latter, $\frac{I_p}{C} = Z_p$, the polar section modulus in inches^3. Sub-

stituting this value in formula (30), we obtain,

$$T = S_s Z_p, \text{ the design formula for torsion} \qquad (31)$$

Formulas (30) and (31) find use in Machine Design in two ways, namely, (1) as a check formula to see if a designed shaft is safe, and (2) as a design formula to secure the proper diameter (or diameters if the shaft is hollow), so that the shaft may safely transmit its torque. In the first case, either formula is solved for the induced stress, S_s, by inserting therein numerical values for all the other factors. If the induced stress, thus determined, is considered a safe stress by the checker, then the shaft is properly designed. In the second case, either formula is solved for the diameter, using in the solution a numerical value for S_s that is considered by the designer to be a safe stress for the given material used. So in actual design, S_s becomes the safe stress in lb. per sq. in.

The above statements apply equally well for rods circular in section or otherwise, which are subjected to a twisting moment, but which, since they are stationary, do not transmit torque. Table IV gives the polar moments of inertia and section moduli for the various sections more often employed.

Example. How much torque can safely be transmitted by a $1\frac{7}{16}$-inch shaft if $S_s = 7000$ pounds per square inch?

Solution. Here $S_s = 7000$ lb. per sq. in., and $Z_p = \frac{\pi d^3}{16} = \pi \times \frac{(1\frac{7}{16})^3}{16}$
Using formula (31),

$$T = S_s Z_p$$

$$= 7000 \times \frac{\pi \times (1\frac{7}{16})^3}{16} = 4100+ \text{ in.-lb.} \quad \textit{Ans.}$$

Example. It is required to check the design of a 2-inch medium steel shaft subjected to a turning moment of 40,000 inch-pounds.

Solution. Here $T = 40{,}000$ in.-lb.

From Table IV, Z_p, for a circular section, $= \frac{\pi d^3}{16}$

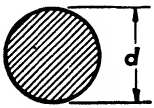
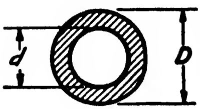
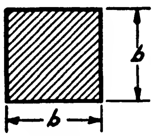
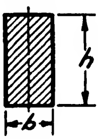
since $d = 2$ in., $Z_p = \frac{\pi \times 2^3}{16} = \frac{\pi \times 8}{16} = \frac{\pi}{2}$ in.^{3}

Evaluating in formula (31),

$$T = S_s Z_p$$

$$40{,}000 = S_s \times \frac{\pi}{2}$$

TABLE IV — Polar Moments of Inertia and Section Moduli

Section	Moment of Inertia, I_p	Section Modulus, Z_p
	$\dfrac{\pi d^4}{32}$ or $0.098d^4$	$\dfrac{\pi d^3}{16}$ or $0.196d^3$
	$\dfrac{\pi}{32}(D^4-d^4)$ or $0.098(D^4-d^4)$	$\dfrac{\pi}{16}\cdot\dfrac{(D^4-d^4)}{D}$ or $0.196\dfrac{(D^4-d^4)}{D}$
	$\dfrac{b^4}{6}$	$\dfrac{2b^3}{9}$ or $0.222b^3$
	$\dfrac{bh(b^2+h^2)}{12}$	$\dfrac{2b^2h}{9}$ or $0.222b^2h$

Solving for S_s by dividing each member of the equation by $\frac{\pi}{2}$,

$$S_s=\frac{40{,}000}{\frac{\pi}{2}}=40{,}000\times\frac{2}{\pi}$$

$$=\frac{80{,}000}{\pi}=25{,}400+\text{lb. per sq. in.}$$

From Table I, U_s for medium steel $=50{,}000$ lb. per sq. in.

Therefore, $$F=\frac{U_s}{S_s}=\frac{50{,}000}{25{,}400}=2-$$

Since this factor of safety is too low, the shaft is not safely designed. *Ans.*

Example. Find the diameter of a steel shaft to transmit 25,400 in.-lb. of torque if the safe shearing stress is assumed to be 7000 lb. per sq. in.

Solution. Here $T=25{,}400$ in.-lb., $S_s=7000$ lb. per sq. in., Z_p, from Table IV, $=\dfrac{\pi d^3}{16}$

Substituting these values in formula (31),

$$T=S_sZ_p$$

$$25{,}400=7000\times\frac{\pi d^3}{16}$$

Multiplying by 16, and dividing by $7000\times\pi$,

$$d^3=\frac{25{,}400\times16}{7000\times\pi}=18.5$$

$$d=\sqrt[3]{18.5}=2.64 \text{ in., say } 2\tfrac{11}{16} \text{ in. or } 2\tfrac{3}{4} \text{ in.} \quad \textit{Ans.}$$

Example. A wrought iron rod, square in section, is subjected to a twisting moment which is set up in the rod by a force of 500 pounds working through a moment arm of 10 inches with respect to the axis of the rod. If a factor of safety of 7 is assumed, what are the dimensions of the cross section of the rod?

Solution. Since $P_t=500$ lb. and $R=10$ in., from formula (29), the twisting moment to which the rod is subjected is,

$$T=P_tR=500\times10=5000 \text{ in.-lb.}$$

From Table I, U_s for wrought iron $=40{,}000$ lb. per sq. in.

Therefore $S_s=\dfrac{40{,}000}{7}$ lb. per sq. in.

From Table IV, for a square section, $Z_p=\frac{2}{9}\times b^3=0.222b^3$

Evaluating in the design formula for twisting,

$$T=S_sZ_p$$

$$5000=\frac{40{,}000}{7}\times0.222b^3$$

Solving for b^3,

$$b^3=\frac{5000\times7}{40{,}000\times0.222}=3.94$$

$$b=\sqrt[3]{3.94}=1.58- \text{ in., say } 1\tfrac{5}{8} \text{ in.} \quad \textit{Ans.}$$

Torque and Power. Should the tangential force, P_t pounds, be applied continuously to the pin of the rotating crank (See Fig. 32), there will be an amount of work done in one revolution of the crank equal to $P_t\times2\pi R$ in.-lb. This is because Work equals Force times Distance, and the distance in this case is equal to the circumference of the crank-pin circle which is $2\pi R$ inches. In any number of revolutions, N, an amount of work will be done equal to N times the work done in one revolution, which will be $P_t\times2\pi R\times N$ in.-lb. Suppose

now that these N revolutions of the crank are swept through in one minute. Evidently the work done in N revolutions will then be done in one minute. So that,

work done per minute by $P_t = P_t \times 2\pi R \times N$ in.-lb.

But Power is the rate at which work is done and the unit of power is the so-called horsepower. One horsepower is the doing of 33,000 foot-pounds of work per minute or the doing of 12×33,000 in.-lb. per minute. Therefore if we divide the work done in in.-lb. per min. by 12×33,000, we will obtain the horsepower, H. This produces the formula,

$$H = \frac{P_t 2\pi RN}{12 \times 33{,}000} \tag{32}$$

where P_t = tangential force in pounds
R = moment arm of P_t in inches
and N = the number of revolutions per minute, the r.p.m.

Remembering from mathematics that in a continued product a sequence of factors can be rearranged in any order, rewriting the numerator of the second member of formula (32), we have,

$$H = \frac{P_t \times R \times 2 \times \pi \times N}{12 \times 33{,}000}$$

Since formula (29) states that $P_t \times R = T$, we may substitute for $P_t \times R$ in the above, its equal, T, obtaining,

$$H = \frac{T 2\pi N}{12 \times 33{,}000}, \text{ where } T \text{ is in in.-lb.} \tag{33}$$

Multiplying both members of formula (33) by 12×33,000 and dividing by $2 \times \pi \times N$,

$$T = \frac{12 \times 33{,}000 H}{2\pi N} \text{ in.-lb.} \tag{34}$$

Formulas (33) and (34) are the most fundamental formulas involving the torque, the horsepower, and the r.p.m.

Returning to formula (32), it will be noticed that in the numerator of its second member, there occurs the sequence of factors, $2\pi RN$. In Mechanism, this was found to be equal to the linear velocity, V_L, of a point of a rotating body. Therefore, in formula (32), $2\pi RN$ is the linear velocity in inches per minute (because R is in inches) of the point of application, C, of Fig. 32. The linear velocity of C be-

comes $\frac{2\pi RN}{12}$ feet per minute. Rearranging the factors of the second member of formula (**32**), as follows,

$$H=\frac{P_t}{33,000}\times\frac{2\pi RN}{12}$$

we have

$$H=\frac{P_t\times V_L}{33,000}=\frac{P_tV_L}{33,000} \tag{35}$$

where V_L is the linear velocity in feet per minute.

Formula (**35**) is the most basic horsepower formula. All other formulas for determining the horsepower are in reality derived from it.

Example. A shaft transmits 10 horsepower at 800 revolutions per minute. Required: the torque in the shaft.

Solution. Here $H=10$ hp., $N=800$ r.p.m.
Evaluating in formula (**34**),

$$T=\frac{12\times 33,000H}{2\pi N}$$

$$=\frac{12\times 33,000\times 10}{2\times\pi\times 800}=788 \text{ in.-lb.} \quad \textit{Ans.}$$

Example. A shaft is subjected to a torsional moment of 10,000 in.-lb. while rotating at 100 r.p.m. What horsepower is being transmitted by the shaft?

Solution. Here $T=10,000$ in.-lb. and $N=100$ r.p.m.
Substituting these values in formula (**33**),

$$H=\frac{\mathrm{T}2\pi N}{12\times 33,000}$$

$$=\frac{10,000\times 2\times\pi\times 100}{12\times 33,000}=15.9 \text{ hp.} \quad \textit{Ans.}$$

Example. What horsepower is required to lift a load of 5000 pounds by means of a cable wrapped around the drum of a hoist, the drum being 40 inches in diameter and making 24 revolutions per minute?

Solution. Let it be assumed that the linear velocity of the drum, V_L, is the linear velocity of the cable, or in other words, the speed of lift. From Mechanism,

$V_L=2\pi RN=\pi DN$ f.p.m. if R or D is in feet. From this formula, the linear velocity of the drum is

$$V_L=\pi\times\frac{40}{12}\times24=251.3+\text{ f.p.m.}$$

Since the load on the cable is a tangential force at the drum, $P_t=5000$ lb.

Evaluating in formula (**35**),

$$H=\frac{P_tV_L}{33{,}000}$$

$$=\frac{5000\times251.3}{33{,}000}=36\text{ hp.}\quad \textit{Ans.}$$

Example. What is the torque in the shaft to which the drum of the preceding example is keyed?

Solution. Here $P_t=5000$ lb. and $R=\frac{40}{2}=20$ in.

From formula (**29**),

$$T=P_tR$$

Evaluating in this formula,

$$T=5000\times20=100{,}000\text{ in.-lb.}\quad \textit{Ans.}$$

Example. A spur gear, A, whose pitch diameter is 20 inches is keyed to a shaft which transmits a torque of 5000 inch-pounds. This gear, A, drives another gear, B, on a parallel shaft. What tooth pressure (tangential force) is exerted by gear, A, upon gear, B?

Solution. Here $T=5000$ in.-lb., and $R=\frac{20}{2}=10$ in.

From formula (**29**), $T=P_tR$

Solving this formula for the tooth pressure, P_t,

$$P_t=\frac{T}{R}$$

Evaluating in the above,

$$P_t=\frac{5000}{10}=500\text{ lb.}\quad \textit{Ans.}$$

Torque Multiplication. In the study of the subject of Mechanism, several rules were stated and proved dealing with the angular velocity, or r.p.m. ratio, of a pair of shafts connected by gears or pulleys. Among these rules were the following:

(1) When two shafts are connected by a pair of gears, the r.p.m.'s of the shafts (or gears) are inversely proportional to the numbers of teeth. This statement, it will be remembered, holds for

all gears no matter what type is used. Therefore the shafts upon which the gears are located may have parallel axes, intersecting axes, or non-parallel and non-intersecting axes.

(2) When two parallel shafts are connected by gears, the r.p.m.'s of the shafts are inversely proportional to the pitch diameters or inversely proportional to the pitch radii. This principle holds true for those gears that can be used to connect shafts with parallel axes and shafts with intersecting axes.

(3) When two shafts are connected by a belt running over a pair of pulleys, the r.p.m.'s of the shafts are inversely proportional to the diameters or inversely proportional to the radii of the pulleys.

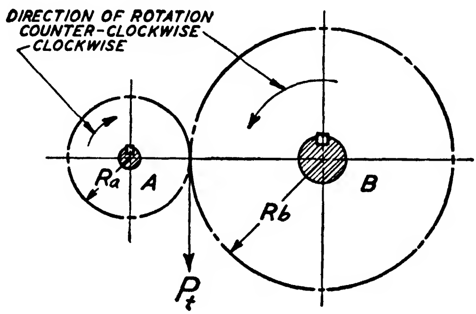


Fig. 33

In general it is noted from these rules that the speed in a mechanism is reduced when driving from a small gear (or pulley) to a larger one and that the speed is increased when driving from a large gear (or pulley) to a smaller one.

In Fig. 33, two shafts are shown connected by gears, A and B. The gears are shown by their tangent pitch circles as is customary, and it should be remembered at this time that the so-called radius of a gear is the radius of its pitch circle. If N_a, R_a, and t_a are the r.p.m., the radius and the number of teeth respectively of gear A, and N_b, R_b, and t_b are the r.p.m., the radius and the number of teeth respectively of gear B, we have from the previous statement,

$$\frac{N_a}{N_b}=\frac{R_b}{R_a} \text{ or } \frac{N_b}{N_a}=\frac{R_a}{R_b} \tag{36}$$

and

$$\frac{N_a}{N_b}=\frac{t_b}{t_a} \text{ or } \frac{N_b}{N_a}=\frac{t_a}{t_b} \tag{37}$$

Now since the teeth of the driving gear A, press down on the teeth of the driven gear, B, the tooth pressure of A, namely P_{ta}, is equal to the tooth pressure of B, namely, P_{tb}. Therefore,

$$P_t(\text{of Fig. 33}) = P_{ta} = P_{tb} \tag{38}$$

and the tooth pressure is assumed to act at the point of tangency of the pitch circles of the gears. We shall call the torques in the shafts of A and B, T_a and T_b respectively. Solving formula (29) for P_t, we have

$$P_t = \frac{T}{R} \tag{39}$$

Therefore for the case of Fig. 33,

$$P_{ta} = \frac{T_a}{R_a} \tag{40}$$

and
$$P_{tb} = \frac{T_b}{R_b} \tag{41}$$

But $P_{ta} = P_{tb}$, hence their equals are in turn equal to each other,

so that
$$\frac{T_a}{R_a} = \frac{T_b}{R_b}$$

or
$$T_a : R_a = T_b : R_b$$

Since in any proportion the first element is to the third as the second is to the fourth,

$$T_a : T_b = R_a : R_b$$

or
$$\frac{T_a}{T_b} = \frac{R_a}{R_b} \text{ or } \frac{T_b}{T_a} = \frac{R_b}{R_a} \tag{42}$$

Formula (42) states that the torques of a pair of shafts connected by gears are directly proportional to the pitch radii of the gears. This statement is true also for pulleys.

Now from formula (36) in the form, $\frac{N_b}{N_a} = \frac{R_a}{R_b}$ it will be noticed by comparing the above with formula (42) that both $\frac{N_b}{N_a}$ and $\frac{T_a}{T_b}$ are equal to the same ratio, $\frac{R_a}{R_b}$. Therefore they are equal to each other.

Hence
$$\frac{T_a}{T_b} = \frac{N_b}{N_a} \text{ or } \frac{T_b}{T_a} = \frac{N_a}{N_b} \tag{43}$$

Hence the torques in a pair of shafts connected by gears or pulleys are inversely proportional to the r.p.m.'s of the shafts. In other

words, if the speed is decreased, the torque is increased, or by decreasing the speed throughout a train of mechanism, a multiplication of torque is obtained. The speed reduction factor of a single kinematic pair (one driver and its immediate follower) or of an entire train of mechanism thus becomes the Torque Multiplication Factor.

Example. Two shafts are connected by spur gears as in Fig. 33. The pitch radii of gears A and B are 4 inches and 20 inches respectively. If shaft A makes 800 revolutions per minute and is subjected to a twisting moment of 1000 inch-pounds, what is (*a*) the r.p.m. of B? (*b*) the torque in shaft B? (*c*) the speed reduction factor? (*d*) the torque multiplication factor? (*e*) the tooth pressure of A and B?

Solution. (*a*) Here $R_a=4$ in., $R_b=20$ in., and $N_a=800$ r.p.m. Substituting these values in formula (36),

$$\frac{N_b}{N_a}=\frac{R_a}{R_b}$$

$$\frac{N_b}{800}=\frac{4}{20}=\frac{1}{5}$$

Multiplying both members of the equation by 800,

$$N_b=\frac{800}{5}=160 \text{ r.p.m.} \quad \textit{Ans.}$$

(*b*) In addition to the given data as stated in part (*a*), $T_a=1000$ in.-lb. Applying formula (42) in the form,

$$\frac{T_b}{T_a}=\frac{R_b}{R_a}$$

and evaluating therein,

$$\frac{T_b}{1000}=\frac{20}{4}$$

$$T_b=\frac{20\times1000}{4}=5000 \text{ in.-lb.} \quad \textit{Ans.}$$

(*c*) The speed reduction factor, $\dfrac{N_a}{N_b}=\dfrac{800}{160}=5.$ *Ans.*

(*d*) The torque multiplication factor $=\dfrac{T_b}{T_a}=\dfrac{5000}{1000}=5$ *Ans.*

(*e*) From formulas (40), and (41)

$$P_{ta}=\frac{T_a}{R_a}=\frac{1000}{4}=250 \text{ lb.} \quad \textit{Ans.}$$

and $$P_{tb}=\frac{T_b}{R_b}=\frac{5000}{20}=250 \text{ lb.} \quad \textit{Ans.}$$

Example. A gear train mechanism as shown in Fig. 34, is used to transmit 25 horsepower from the first driving shaft, *I*, to the last driven shaft *III*. To shaft *I* is keyed a spur gear, *A*, which drives spur gear, *B*, keyed to shaft *II*. Another spur gear, *C*, is keyed to shaft *II*, and drives gear, *D*, on shaft *III*. The number of

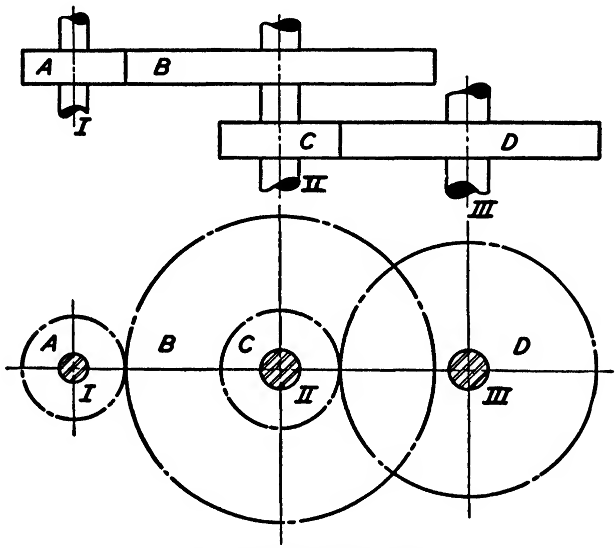


Fig. 34. Gear Train Mechanism

teeth, *t*, on the several gears is as follows: $t_a=20$ teeth, $t_b=100$ teeth, $t_c=16$ teeth, $t_d=96$ teeth. Shaft *I* makes 1200 r.p.m. (*a*) Find the angular velocity, or r.p.m. ratio between shafts *I* and *II*. (*b*) Find the angular velocity or r.p.m. ratio between shafts *II* and *III*. (*c*) What is the speed reduction factor of the entire train of mechanism? (*d*) What is the torque in shaft, *I*? (*e*) What are the r.p.m. and torque of shaft *II*? (*f*) What are the r.p.m. and torque of shaft *III*? (*g*) What is the torque multiplication factor of the entire train? Check the answer of part (*f*) by using this factor.

Solution. (*a*). Considering r.p.m. ratio $\frac{N_I}{N_{II}}$ in the kinematic pair in which *A* is the driver and *B*, the follower or driven member,

we have $t_a=20$, $t_b=100$, and $N_I=1200$ r.p.m.$=N_a$ (since the gear is keyed to the shaft, it must have the speed thereof).

Evaluating in formula (37),

$$\frac{N_a}{N_b}=\frac{N_I}{N_{II}}=\frac{t_b}{t_a}=\frac{100}{20}=\frac{5}{1} \quad Ans.$$

(*b*) Here $t_c=16$ and $t_d=96$. Applying formula (37),

$$\frac{N_c}{N_d}=\frac{N_{II}}{N_{III}}=\frac{t_d}{t_c}=\frac{96}{16}=\frac{6}{1} \quad Ans.$$

(*c*) To find $\frac{N_a}{N_d}$ which is also $\frac{N_I}{N_{III}}$, we have from steps (*a*) and (*b*),

$$\frac{N_a}{N_b}=\frac{5}{1} \text{ and } \frac{N_c}{N_d}=\frac{6}{1}$$

Since if equals are multiplied by equals, their products are equal,

$$\frac{N_a}{N_b}\times\frac{N_c}{N_d}=\frac{5}{1}\times\frac{6}{1}$$

But in the above $N_c=N_b$, for gears *B* and *C* are keyed to the same shaft. Cancelling these like factors in the first member of the above equation,

$$\frac{N_a}{N_d}=\frac{30}{1} \text{ or } \frac{N_I}{N_{III}}=\frac{30}{1} \quad Ans.$$

(*d*) To secure the torque in shaft *I*, we shall use formula (34), in which $H=25$ hp. and $N_I=1200$ r.p.m. Thus

$$T_I=\frac{12\times 33{,}000\,H}{2\pi N_I}$$

$$=\frac{12\times 33{,}000\times 25}{2\times\pi\times 1200}=1313 \text{ in.-lb.} \quad Ans.$$

(*e*) In order to find N_{II}, or what is the same thing, N_b, we have $N_I=N_a=1200$ r.p.m., $t_a=20$, and $t_b=100$. Substituting these in formula (37),

$$\frac{N_b}{N_a}=\frac{t_a}{t_b}$$

$$\frac{N_b}{1200}=\frac{20}{100}=\frac{1}{5}$$

$$N_b=1200\times\frac{1}{5}=240 \text{ r.p.m.} \quad Ans.$$

To find the torque in shaft II, use formula (43), in which $T_I = 1313$ in.-lb., and $\frac{N_I}{N_{II}} = \frac{N_a}{N_b} = 5$

$$\frac{T_{II}}{T_I} = \frac{N_I}{N_{II}}$$

$$\frac{T_{II}}{1313} = 5$$

$$T_{II} = 5 \times 1313 = 6565 \text{ in.-lb.} \quad \textit{Ans.}$$

or T_{II} could be found by the general formula (34), as follows:

$$T_{II} = \frac{12 \times 33{,}000\, H}{2\pi N_{II}}$$

$$= \frac{12 \times 33{,}000 \times 25}{2 \times \pi \times 240} = 6565 \text{ in.-lb.} \quad \textit{Ans.}$$

(*f*) Here we have $N_{II} = N_c = N_b = 240$ r.p.m., $t_c = 16$, $t_d = 96$, and $N_{III} = N_d$. Solving for N_d by applying formula (37),

$$\frac{N_d}{N_c} = \frac{t_c}{t_d}$$

$$\frac{N_d}{240} = \frac{16}{96} = \frac{1}{6}$$

$$N_d = 240 \times \frac{1}{6} = 40 \text{ r.p.m.} \quad \textit{Ans.}$$

To find T_{III}, the torque in shaft III, we shall apply formula (43) again, which will be in this case,

$$\frac{T_{III}}{T_{II}} = \frac{N_{II}}{N_{III}} = \frac{N_c}{N_d}$$

Evaluating,

$$\frac{T_{III}}{6565} = \frac{240}{40} = \frac{6}{1}$$

$$T_{III} = 6565 \times 6 = 39{,}390 \text{ in.-lb.} \quad \textit{Ans.}$$

(*g*) Since the speed reduction factor, $\frac{N_I}{N_{III}} = \frac{30}{1}$, from part (*c*), the torque multiplication factor, $\frac{T_{III}}{T_I}, = \frac{30}{1}$ *Ans.*

Since $T_I = 1313$ in.-lb., substituting this in the preceding statement,

$$\frac{T_{III}}{1313} = \frac{30}{1}$$

To check above, $T_{III} = 30 \times 1313 = 39{,}390$ in.-lb.

Example. If a 20-in. drum is placed successively on shafts *I*, *II*, and *III* of the preceding example, what loads can be raised by a cable placed about the drum?

Solution. Here we have from the preceding solution, $T_I=1313$ in.-lb., $T_{II}=6565$ in.-lb., $T_{III}=39{,}390$ in.-lb.; and $R=10$ in. Keying the drum to shaft *I*, from formula (**39**),

$$P_t=\frac{T}{R}=\frac{1313}{10}=131.3 \text{ lb.} \quad \textit{Ans.}$$

For shaft *II*, with the same drum in use,

$$P_t=\frac{T}{R}=\frac{6565}{10}=656.5 \text{ lb.} \quad \textit{Ans.}$$

Likewise, for shaft *III*,

$$P_t=\frac{T}{R}=\frac{39{,}390}{10}=3939 \text{ lb.} \quad \textit{Ans.}$$

(**Note**: From this example, it is to be seen that a 25-horsepower motor using a 20-inch drum can lift a load of only 131.3 pounds when the drum is placed on the motor or armature shaft, but the same motor using the mechanism as given can theoretically lift a load 30 times as large, or 3939 pounds, when the same drum is installed on the last driven shaft. This would be reduced somewhat in practice because of the lower mechanical efficiency of the machine with its increased mechanism. Notice also that this greater lifting power has been obtained at a sacrifice of speed.)

Buckling. When comparing the compound stresses at the beginning of this chapter, it was stated that a fairly long timber, a bar of steel, etc., when used to sustain end loads which act lengthwise of the pieces, are subjected to what is known as Buckling. Such members are in general called columns, posts, or struts, and are of such a length that they tend to bend laterally under their loads as shown by the dotted lines of Fig. 9. If they were relatively short, such members would not act as columns, and would not tend to bend under their lengthwise load, but would, on the other hand, be under the simple stress of compression, and hence designed by the simple stress formula, $P=AS_c$. The design of columns, however, cannot be so simply made, and another design formula must be introduced for them.

The strength of a column, and therefore its design, depends to

quite an extent on the manner in which its ends bear on or are joined to the other parts of the machine. The two kinds of end conditions met with in the design of machine parts are as follows:

1. "Pin" ends
2. "Fixed" ends

When the end of a machine element of a column classification is fastened to its supporting links by means of pins that would permit the element to rotate if the one end were free, it is said to be "pinned" at its ends. When this element is rigidly connected to its end supports, it is said to be "fixed ended."

A quantity called the "radius of gyration" appears in the design formula for columns that we shall use. By definition, the radius of gyration of the section of a column with respect to any line, is such a length that its square, if multiplied by the area of section, gives the rectangular moment of inertia of the section with respect to the given line. In a design, this given line is that axis drawn through the center of gravity of the section with respect to which the radius of gyration as well as the moment of inertia, will be the least.

From the above,

if r = least radius of gyration
I = least rectangular moment of inertia in inches^4
A = area of section in square inches

then
$$r^2 \times A = I$$
or
$$r^2 = \frac{I}{A} \tag{44}$$

Values of r^2, as well as values of I, are given in Table V. It will be noticed that the values of I are the same as in Table III. However I for a rectangular section appears to be different. This is because the axis is taken parallel to h instead of perpendicular to h as in Table III.

The solution of most problems of this type in Machine Design may be effected by the use of the following column formulas, known as the Gordon-Rankine equations.

For columns having pinned ends,
$$P = \frac{S_c A}{1 + 4k\dfrac{L^2}{r^2}} \tag{45}$$
and for columns with fixed ends,

$$P = \frac{S_c A}{1 + k\frac{L^2}{r^2}} \tag{46}$$

in which formulas,

P = safe load in pounds

A = area of cross section in square inches

r^2 = the square of the least radius of gyration of the section

S_c = safe compressive stress in pounds per square inch

k = a constant, the values of which are given in Table VI

L = length of column in inches.

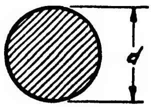
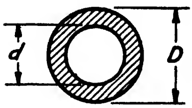
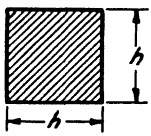
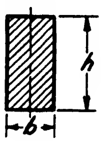
In these formulas, the use of U_c, the ultimate compressive stress in pounds per square inch, will cause P to become the ultimate or breaking load.

It is evident from Table V that the formula for r^2 carries the dimension (or dimensions) of the cross section. Hence in using the Gordon-Rankine equation for direct design, the unknown would be the dimension placed into the formula by the value of r^2, and all other terms would be known. Thus the equation could be solved for the dimension. This as a rule is a trifle unwieldy so that often it is better to assume the size of section to be used and solve the Gordon-Rankine equation for P. The actual load in the problem being known, a comparison between it and the value of P from the equation will tell whether the assumed section is all right, and if not, how much it should be modified or changed. Or instead of solving for P as stated, the value of the known load on the member can be inserted in the formula, and the latter solved for S_c. The factor of safety involved in the design on the basis of the assumed section can then be computed and a decision thus made as to the advisability of using the section. To get a fair idea of the dimensions of the area to be assumed, one can make a trial calculation based on pure compression with the formula, $P = AS_c$, and then be sure to increase those dimensions somewhat before using in the Gordon-Rankine equation, for buckling insists on a larger area for a given load than pure compression.

Example. Show that the value of r^2 as given in Table V for a rectangular section is correct.

Proof. It is to be noted that the axis is parallel to the longer dimension, h, in which case, $I = \frac{hb^3}{12}$

TABLE V

Section	Least I	Least r^2
	$\frac{\pi d^4}{64}$, or $0.049d^4$	$\frac{d^2}{16}$
	$\frac{\pi}{64}(D^4-d^4)$ or $0.049\ (D^4-d^4)$	$\frac{D^2+d^2}{16}$
	$\frac{h^4}{12}$	$\frac{h^2}{12}$
	$\frac{hb^3}{12}$	$\frac{b^2}{12}$

Since A of a rectangular section is equal to the product of its two dimensions, $A=hb$

By definition, $r^2=\frac{I}{A}=\frac{\frac{hb^3}{12}}{hb}=\frac{hb^3}{12}\times\frac{1}{hb}=\frac{b^2}{12}$, which checks Table V.

Example. What thrust can be carried safely by a wrought iron connecting-rod of a steam engine, if the rod is 5 feet in length and has a cross section of 2 inches by 4 inches? Assume pinned ends and a factor of safety of 8.

Solution. Here $S_c=\frac{U_c}{F}=\frac{48{,}000}{8}=6000$ lb. per sq. in.; $L=5\times12=60$ in.; $A=b\times h=2\times4=8$ sq. in.; $4k$, from Table VI, $=\frac{1}{9000}$; r^2, from Table V, $=\frac{b^2}{12}=\frac{4}{12}=\frac{1}{3}$.

Substituting in formula (45),

$$P=\frac{S_cA}{1+4k\frac{L^2}{r^2}}$$

$$=\frac{6000\times 8}{1+\frac{1}{9000}\times\frac{(60)^2}{\frac{1}{3}}}=\frac{48{,}000}{1+\frac{1}{9000}\times\frac{3600}{1}\times\frac{3}{1}}$$

$$=\frac{48{,}000}{1+1.2}=\frac{48{,}000}{2.2}=21{,}800 \text{ lb.} \quad \textit{Ans.}$$

TABLE VI

Material	U_c	k	$4k$
Cast iron	80000	$\frac{1}{6000}$	$\frac{1}{1500}$
Wrought iron	48000	$\frac{1}{36000}$	$\frac{1}{9000}$
Medium steel	60000	$\frac{1}{30000}$	$\frac{1}{7500}$
Hard steel	75000	$\frac{1}{20000}$	$\frac{1}{5000}$
Timber	2500 to 4000	$\frac{1}{3000}$	$\frac{1}{750}$

Combined Stresses. Nearly all cases of design of machine elements involve directly, or may be assumed to involve, one of the three simple stresses or one of the three compound stresses. Hence it becomes essential to have perfect command of all the fundamental formulas included so far in this text.

But in other cases of design, rather complex combinations of stresses occur, which will not permit simple and direct application of the basic design formulas. These are cases of Combined Stresses, in which the relations become somewhat more complicated and difficult of analysis and solution; the study of which, however, involves the same fundamentals as the more direct or simpler cases.

One of the most common cases of combined stresses is that which occurs in a shaft while the latter is transmitting power. A combination of bending and torsion is here involved, and the bending may be of such a magnitude that it must be considered in the design of the shaft. The chapter dealing with shafts will discuss this condition. Other combinations of stresses such as tension or compression with bending, etc., may be introduced from time to time as is made necessary by the analysis of the design of the typical machine elements included herein.

PROBLEMS

(**Note.** Consider all beams in this list of problems as without weight unless otherwise stated.)

1. A beam which is 10 feet in length rests on end supports and is subjected to three concentrated loads of 2000 pounds, 3000 pounds, and 4000 pounds, located at 1 foot, 3 feet, and 6 feet respectively from the left support. Compute and check the reactions for this beam. *Ans.* 5500 lb.; 3500 lb.

2. In the overhung beam of Fig. 13, change the loads to 1000 pounds at C, 3000 pounds at D, and 2000 pounds at E. Compute and check the reactions. *Ans.* 2286 − lb.; 3714 + lb.

3. A simple beam has a span of 12 feet. It carries a uniformly distributed load of 300 pounds per foot of length and a single concentrated load of 2000 pounds located 5 feet from the right support. Compute and check the reactions. *Ans.* 2633 + lb.; 2967 − lb.

4. Find the bending moments for sections 1 foot apart in the beam of Problem 1.

Answers are given reading from the left support to the right, M_0 being the bending moment at the former, and M_{10} the bending moment at the latter support.

$M_0 =$ 0 ft.-lb.	$M_6 =$ 14,000 ft.-lb.
$M_1 =$ 5,500 ft.-lb.	$M_7 =$ 10,500 ft.-lb.
$M_2 =$ 9,000 ft.-lb.	$M_8 =$ 7,000 ft.-lb.
$M_3 =$ 12,500 ft.-lb.	$M_9 =$ 3,500 ft.-lb.
$M_4 =$ 13,000 ft.-lb.	$M_{10} =$ 0 ft.-lb.
$M_5 =$ 13,500 ft.-lb.	

5. A cantilever beam 6 feet in length carries an end load of 3000 pounds. Compute the maximum bending moment. Where will it occur? *Ans.* 18,000 ft.-lb.

6. If the cantilever beam of the preceding problem carried an additional uniformly distributed load of 400 lb. per ft. of length, find

(*a*) the maximum bending moment,

(*b*) the bending moment 3 feet out from the support.

Ans. (*a*) 25,200 ft.-lb.; (*b*) 10,800 ft.-lb.

7. A simple beam carries three concentrated loads of 1000 pounds, 1500 pounds, and 2000 pounds located at distances of 2 feet, 5 feet, and 9 feet respectively from the left support. The span of the beam is 15 feet.

(*a*) Find and check the reactions.

(*b*) Find the value of the maximum bending moment.

(*c*) Is it always necessary to obtain the reactions in a simple beam before finding the maximum bending moment?

(*d*) If a simple beam carries nothing but concentrated loads, where will the dangerous section be located?

Ans. (*a*) 2667 − lb.; 1833 + lb. (*b*) 10,998×12 in.-lb. (*c*) Yes, unless it is a special case.

8. A simple wooden beam, square in cross section, carries a single concentrated load of 2000 pounds, which is located 3 feet from the left support. If the span is 9 feet and the safe bending stress is 800 pounds per square inch, what are the dimensions of its cross section? (**Note.** Find the maximum bending moment by the general method, then check it by using formula (21) of special case No. 2.) *Ans.* 7.1 in. say $7\frac{1}{4}$ in. x $7\frac{1}{4}$ in.

9. A steel cantilever beam projects 10 feet from its support. It carries a uniformly distributed load of 500 lb. per ft. of length. The safe bending stress is to be assumed as 10,000 lb. per sq. in. The section of the beam is rectangular with its longer dimension perpendicular to its vertical axis and equal to three times the shorter. Required: the dimensions of the cross section. *Ans.* $2\frac{3}{4}$ in. by $8\frac{1}{4}$ in.

10. A simple cylindrical steel beam with a span of 8 feet carries two concentrated loads of 2000 pounds and 3000 pounds located 2 feet and 7 feet respectively from the left support. Find the diameter of this beam if the safe bending stress is 8000 lb. per sq. in. *Ans.* 3.84 − in. say $3\frac{7}{8}$ in.

11. A steel I-beam, used as a cantilever, projects 5 feet from its support. It has a section modulus of 20.4 inches^3.

(*a*) What is the maximum stress set up in the beam when it supports an end load of 2500 pounds?

(*b*) What is the maximum stress set up in the beam when it supports a total uniformly distributed load along its entire length of 2500 pounds with no other loading?

(*c*) In what section of the beam do these maximum stresses occur?

(*d*) What are the factors of safety in parts (*a*) and (*b*), if the ultimate bending stress is 60,000 lb. per sq. in.?

Ans. (*a*) 7350+ lb. per sq. in.; (*b*) 3675+ lb. per sq. in.; (*d*) 8.2 −; 16.4 −.

12. A rectangular steel rod, 2 inches in thickness and 10 inches in breadth, is used as a simple beam with the supports located 5 feet apart. The safe bending stress is 10,000 lb. per sq. in.

(*a*) To what concentrated load located midway between the supports can the beam be safely subjected, when its breadth is perpendicular to the neutral axis?

(*b*) To what concentrated load located midway between its supports can the beam be safely subjected, when its thickness is perpendicular to the neutral axis?

(**Note.** In solving this problem, as a first step, use formula (27) to obtain M in inch-pounds. Next, place this value of M along with the given value of the span, L, in formula (20), thus obtaining the load, P.) *Ans.* (*a*) 22,200+ lb.; (*b*) 4450 − lb.

13. A steel pin, supported at each end, is subjected to a uniformly distributed load of 100 lb. per inch of length. The length of the pin is 6 inches, and the safe bending stress allowed in the design is 5000 lb. per sq. in. What diameter of pin need be employed? *Ans.* 0.96 in. Say 1 in.

14. A tangential force of 150 pounds is applied at the pin of a crank. The length of the crank, R, is 8 inches. Find the twisting moment set up in the shaft to which the crank is keyed. *Ans.* 1200 in.-lb.

15. 25,000 in.-lb. of torque are put into a shaft by means of a spur gear. The pitch diameter of the gear on this shaft is 40 inches. What is its tooth pressure? *Ans.* 1250 lb.

16. (*a*) What is the safe resisting moment of a $1\frac{15}{16}$-inch shaft whose $U_s =$ 68,000 lb. per sq. in., $F = 8$, and polar moment of inertia $= \frac{\pi d^4}{32}$ inches^4?

(*b*) What torque can be safely transmitted by this shaft?

Ans. (*a*) 12,123 in.-lb. approximately; (*b*) 12,123 in.-lb. approximately.

17. How much torque can be safely transmitted by a 3-inch shaft if the allowable working stress in shear is 7000 lb. per sq. in.? *Ans.* 37,000+ in.-lb.

18. A wrought iron rod, square in section, is subjected to a twisting moment of 3500 in.-lb. Find the dimensions of the section of the rod, assuming $F=7$. *Ans.* $1\frac{7}{16}$ in. x $1\frac{7}{16}$ in.

19. A shaft transmits 25 hp. at 1000 r.p.m. Find the torque in the shaft in in.-lb. *Ans.* 1575 in.-lb., approximately.

20. What will be the torque in the shaft of the preceding problem

(*a*) if the number of revolutions per minute is doubled, and

(*b*) if the number of revolutions per minute is one-half as large as in Problem 19? *Ans.* (*a*) $(1575 \div 2)$ in.-lb. (*b*) (1575×2) in.-lb.

21. A shaft is subjected to a torsional moment of 15,000 in.-lb. while rotating at 75 r.p.m. What horsepower is being transmitted by the shaft? *Ans.* 17.8+ hp.

22. What horsepower is required to lift a load of 6000 pounds by means of a cable wrapped around the drum of a hoist? The drum is 36 inches in diameter and makes 20 revolutions per minute. *Ans.* 34.3− hp.

23. Two shafts are connected by spur gears as in Fig. 33. The pitch radii of the gears, *A* and *B*, are 5 inches and 30 inches, respectively. If shaft *A* makes 300 revolutions per minute and is subjected to a twisting moment of 2000 inch-pounds, what is

(*a*) the r.p.m. of *B*

(*b*) the torque in shaft *B*, in in.-lb.

(*c*) the speed reduction factor, $\frac{N_a}{N_b}$

(*d*) the torque multiplication factor

(*e*) the tooth pressures of the two gears?

Ans. (*a*) 50 r.p.m.; (*b*) 12,000 in.-lb.; (*c*) 6; (*d*) 6; (*e*) 400 lb.

24. Referring to Fig. 34, it is given that the numbers of teeth on gears *A*, *B*, *C*, and *D* are 40, 120, 20, and 120 respectively. Shaft *I* transmits 10 horsepower at 800 revolutions per minute.

(*a*) Required: the angular velocity, or r.p.m., ratio between shafts *I* and *II*.

(*b*) Required: the angular velocity, or r.p.m., ratio between shafts *II* and *III*.

(*c*) What is the speed reduction factor $\frac{N_I}{N_{III}}$, of the entire train of mechanism?

(*d*) What is the torque in shaft *I*, in in.-lb.?

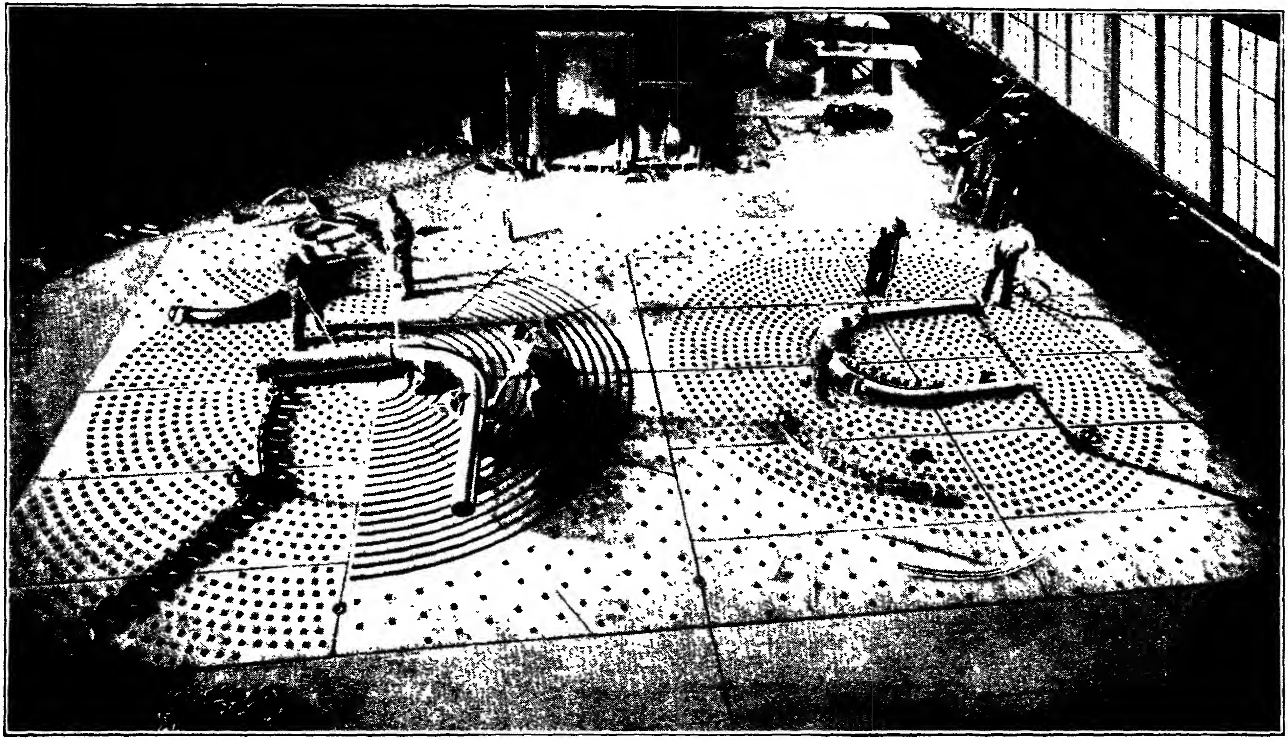
(*e*) What are the r.p.m. and torque in shaft *II*?

(*f*) What are the r.p.m. and torque in shaft *III*?

(*g*) What is the torque multiplication factor of the entire train of mechanism?

Ans. (*a*) $\frac{3}{1}$. (*b*) $\frac{6}{1}$. (*c*) 18. (*d*) 787.8 in.-lb. (*e*) $266\frac{2}{3}$ r.p.m.; 2364.4 in.-lb. (*f*) 44.44 r.p.m.; 14,186.4 in.-lb. (*g*) 18.

25. What thrust can be safely carried by a round steel connecting-rod of a steam engine if its diameter is $2\frac{1}{2}$ inches, length is 50 inches, and the allowable stress is 8,000 pounds per square inch? Assume pinned ends with the value of 4k equal to $\frac{1}{7500}$. *Ans.* 21,200 lb. approximately.



OPERATION OF BENDING PIPE

Courtesy of Crane Co , Chicago, Illinois

CHAPTER III

BOLTS AND SCREWS

There are many kinds of machine fastenings in use for different purposes. Bolts and screws are employed to fasten or clamp machine parts together in such a manner that the parts can be disconnected readily. Certain types of bolts and screws also function in the transmission of power, and in various other ways. They are among the most useful of machine elements.

Definitions. All threads are helical in form and are in reality helical ridges formed by cutting or rolling a groove into the surface of a cylindrical bar, thus producing that which is known as a Screw. Also threads are likewise cut in cylindrical holes, which holes are then said to be Tapped.

The Pitch of the thread is the distance from a point on one thread to a corresponding point on the next thread, no matter what shape a thread may have, the distance to be measured parallel to the axis of the screw.

The Lead of a thread is the lead of the helix of the thread, that is, it is the distance that the helix advances parallel to its axis in one turn about that axis. The lead then is the distance a nut will advance on its bolt in one turn or revolution. From this it becomes evident that with a single thread, the lead is equal to the pitch, or

$$L=p \tag{47}$$

in which L = lead in inches

p = pitch in inches

With a double thread, the lead is equal to twice the pitch, or

$$L=2p \tag{48}$$

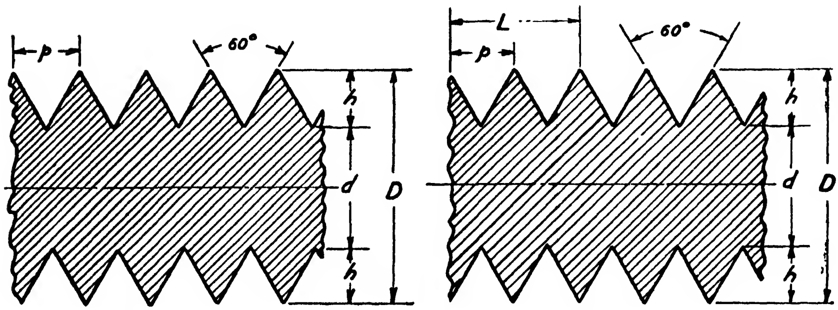
and with a triple thread it is three times the pitch, or

$$L=3p \tag{49}$$

Types of Threads Used on Screw Fastenings. Fig. 35 (*a*) shows the single **V** thread, while Fig. 35 (*b*) shows the double **V** thread. It will be noticed that the sides of the **V** form an angle of 60 degrees, the thread coming to a point at top and bottom so that the section of

the thread itself becomes an equilateral triangle. The following notation is used throughout this material on threads:

h = depth of thread in inches
p = pitch of thread in inches
L = lead of thread in inches
n = number of threads per inch
D = outside or nominal diameter in inches
d = inside or root diameter in inches.



$L=p$
(a) Single V Thread

$L=2p$
(b) Double V Thread

Fig. 35

From the definition of pitch, it is evident that the pitch is the reciprocal of the number of threads per inch. This statement is true for any type of thread. Therefore,

$$p=\frac{1}{n} \tag{50}$$

Since the depth of the thread, h, is the altitude of an equilateral triangle whose side $=p$,

$$h=\frac{p}{2}\sqrt{3}=0.866p \tag{51}$$

and Fig. 35 shows that $d=D-2h$

Substituting, in the above, the value of h from formula (**51**),

$$\begin{aligned} d&=D-2\times0.866p \\ &=D-1.732p \end{aligned} \tag{52}$$

Substituting, in formula (**52**), the value of p from formula (**50**),

$$\begin{aligned} d&=D-1.732\times\frac{1}{n} \\ &=D-\frac{1.732}{n} \end{aligned} \tag{53}$$

The American (National) Standard Thread, formerly called the U. S. Standard, or Seller's Standard is illustrated in Fig. 36. This thread is an outgrowth of the sharp **V** thread, in which the 60-degree angle was retained but the top and root were flattened. Through this alteration, the thread itself becomes more rugged and the screw employing it receives added strength through the larger root diameter provided. This thread has been standardized by the American Standards Association, the A. S. A., under the so-called coarse and fine series, and is now the proper standard to use in this country. The coarse series replaces the old U. S. standard, while the fine series replaces the S. A. E. standard. Table VII gives complete data relative to the coarse (N. C.) series, which is the one more generally used.

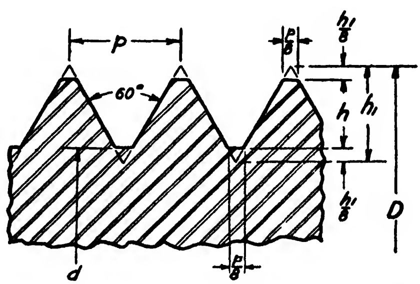


Fig 36. American (National) Standard Thread

In Fig. 36, h_1, is the altitude of the equilateral triangle, while h is the depth of the thread, and it is evident from the dimensions given that

$$h = h_1 - \frac{h_1}{8} - \frac{h_1}{8}$$

$$= h_1 - \frac{h_1}{4} = \frac{4}{4}h_1 - \frac{1}{4}h_1$$

$$= \frac{3}{4}h_1$$

$$h_1 \text{ (like } h \text{ of Fig. 35)} = \frac{p}{2}\sqrt{3}$$

$$= \frac{p}{2} \times 1.732$$

$$= 0.866p$$

Since $$h = \frac{3}{4}h_1,$$

$$h=\frac{3}{4}\times 0.866p=0.65p \tag{54}$$

It is evident that $d=D-2h,$

and therefore from formula (**54**), we have

$$d=D-2\times 0.65p$$
$$=D-1.3p \tag{55}$$

Substituting for p its equal, $\frac{1}{n}$, from formula (**50**),

$$d=D-\frac{1.3}{n} \tag{56}$$

The Whitworth Thread is shown in Fig. 37. Its angle of 55 degrees decreases to some extent the bursting pressure on the nut. Its rounded top, or crest, is not so easily abraded as in the **V** thread,

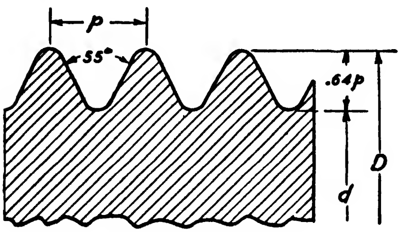


Fig. 37. Whitworth Thread

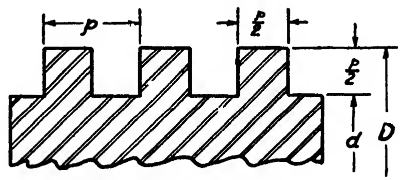


Fig. 38. Square Thread

and the rounded groove bottom reduces the stress concentration that occurs at sharp corners. It is more difficult, however, to cut such a thread to its true form. It is used in England where it is the English Standard.

Types of Threads Used for Power Transmission. *The Square Thread* of Fig. 38, is probably the most typical transmission screw thread, as its mechanical efficiency is considerably higher than that of such threads as the **V** thread. Due to the fact that the sides of its section are parallel rather than at an angle, as in the **V** thread for instance, there is little if any bursting tendency on its nut. Its strength in shear over its root area, which has a thickness per thread equal to $\frac{p}{2}$, is less than that of other threads whose sides are at an angle to each other. As a rule it is formed by being cut in a lathe or milled, as it is a rather difficult procedure to cut it with taps and dies It is evident from the figure that

$$d = D - p \text{ or } D = d + p \tag{57}$$

The Acme Thread of Fig. 39 is a modification of the square thread. The slight slope given to its sides tends to lower its efficiency to some extent, and to introduce some bursting pressure on the nut, but increases its area in shear. It is important to note that dies may be used in the cutting of this thread and hence it is more easily manufactured than the square thread.

In the Acme thread, $$h = \frac{p}{2} + 0.01 \tag{58}$$

Therefore
$$d = D - 2h$$
$$= D - 2\left(\frac{p}{2} + 0.01\right)$$
$$= D - (p + 0.02) \tag{59}$$

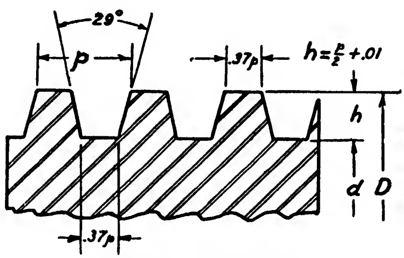


Fig 39. Acme Thread

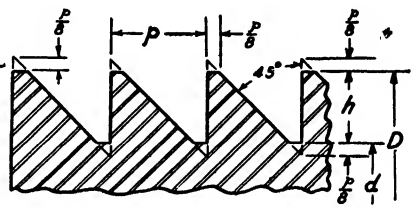


Fig 40. Buttress Thread

Substituting, in the above, the values of p in terms of n from formula (50)

$$d = D - \left(\frac{1}{n} + 0.02\right) \tag{60}$$

The Buttress Thread, shown in Fig. 40, is another transmission screw thread. It replaces the square thread when transmission of power or the production of a thrust in only one direction is desired. It has the efficiency of the square thread, but is much stronger in shear over the root area of its thread, as may be seen in the figure. In this thread,

$$h = \frac{3p}{4} \tag{61}$$

Since $$d = D - 2h,$$

Substituting for h its equal, $\frac{3p}{4}$, from formula (61),

$$d = D - 2 \times \frac{3p}{4}$$

TABLE VII — American National Standard Threads

General Dimensions of Coarse Thread Series

Size	Threads per Inch, n	Major or Outside Diameter, D, in Inches	Minor, or Root Diameter, d, in Inches	Pitch p in Inches
1	64	0.0730	0.0527	0.0156
2	56	.0860	.0628	.0179
3	48	.0990	.0719	.0208
4	40	.1120	.0795	.0250
5	40	.1250	.0925	.0250
6	32	.1380	.0974	.0313
8	32	.1640	.1234	.0313
10	24	.1900	.1359	.0417
12	24	.2160	.1619	.0417
1/4	20	.2500	.1850	.0500
5/16	18	.3125	.2403	.0556
3/8	16	.3750	.2938	.0625
7/16	14	.4375	.3447	.0714
1/2	13	.5000	.4001	.0769
9/16	12	.5625	.4542	.0833
5/8	11	.6250	.5069	.0909
3/4	10	.7500	.6201	.1000
7/8	9	.8750	.7307	.1111
1	8	1.0000	.8376	.1250
1 1/8	7	1.1250	.9394	.1429
1 1/4	7	1.2500	1.0644	.1429
1 3/8	6	1.3750	1.1585	.1667
1 1/2	6	1.5000	1.2835	.1667
1 3/4	5	1.7500	1.4902	.2000
2	4 1/2	2.0000	1.7113	.2222
2 1/4	4 1/2	2.2500	1.9613	.2222
2 1/2	4	2.5000	2.1752	.2500
2 3/4	4	2.7500	2.4252	.2500
3	4	3.0000	2.6752	.2500
3 1/4	4	3.2500	2.9252	.2500
3 1/2	4	3.5000	3.1752	.2500
3 3/4	4	3.7500	3.4252	.2500
4	4	4.0000	3.6752	.2500

Compiled from Bulletin A. S. A., B1 1, 1935

$$=D-1.5p \tag{62}$$

From formula (50), $p=\dfrac{1}{n}$,

so that
$$d=D-\frac{1.5}{n} \tag{63}$$

Example. According to Table VII, a $1\frac{1}{2}$-inch bolt carries 6 American Standard threads per inch. It is required to compute the pitch, the root diameter, the root area, and the body area.

Solution. Formula (50) states that $p=\dfrac{1}{n}$

Therefore $p=\dfrac{1}{6}=0.167$ in. *Ans.*

From formula (55), we have,

$$d=D-1.3p$$

$$=1.5-1.3\times0.167=1.283 \text{ in.} \quad \textit{Ans.}$$

Since the area of a circle $=\frac{\pi d^2}{4}$,

the root area $=\frac{\pi\times1.283^2}{4}=1.293$ sq. in. *Ans.*

and the body area $=\frac{\pi D^2}{4}$

$$=\frac{\pi\times1.5^2}{4}=1.767 \text{ sq. in.} \quad \textit{Ans.}$$

Example. Find the pitch, root diameter, root area, and body area of a 1-inch bolt, which has 8 **V** threads per inch.

Solution. From formula (**50**), we have

$$p=\frac{1}{n}=\frac{1}{8} \text{ in.} \quad \textit{Ans.}$$

From formula (**52**), we have

$$d=D-1.732p$$
$$=1-1.732\times0.125=0.7835 \text{ in.} \quad \textit{Ans.}$$

Hence the root area $=\frac{\pi d^2}{4}$

$$=\pi\times\frac{0.7835^2}{4}=0.482 \text{ sq. in.} \quad \textit{Ans.}$$

The body area $=\frac{\pi D^2}{4}$

$$=\frac{\pi\times1^2}{4}=0.785 \text{ sq. in.} \quad \textit{Ans.}$$

Example. A 2-inch transmission screw has a square thread and a lead of $\frac{3}{4}$ inch. What are the pitch and root diameter if (*a*) the screw is single-threaded, (*b*) the screw is double-threaded, (*c*) the screw is triple-threaded?

Solution. (*a*) From formula (**47**), we have

$$p=L=\tfrac{3}{4} \text{ in.} \quad \textit{Ans.}$$

Using formula (**57**), $d=D-p$

$$=2-\tfrac{3}{4}=1\tfrac{1}{4} \text{ in.} \quad \textit{Ans.}$$

(*b*) Applying formula (**48**)

$$L=2p$$

or $p=\frac{L}{2}=\frac{\frac{3}{4}}{2}=\frac{3}{8}$ in. *Ans.*

and from formula (**57**), $d=D-p$

$$=2-\tfrac{3}{8}=1\tfrac{5}{8} \text{ in.} \quad \textit{Ans.}$$

(*c*) From formula (49),

$$L = 3p$$

or

$$p = \frac{L}{3} = \frac{\frac{3}{4}}{3} = \frac{1}{4} \text{ in.} \quad Ans.$$

and

$$d = D - p$$

$$= 2 - \tfrac{1}{4} = 1\tfrac{3}{4} \text{ in.} \quad Ans.$$

(**Note.** This example shows that the so-called multiple threaded screws have larger root diameters and hence are stronger than a single-threaded screw of the same outside diameter and same lead.)

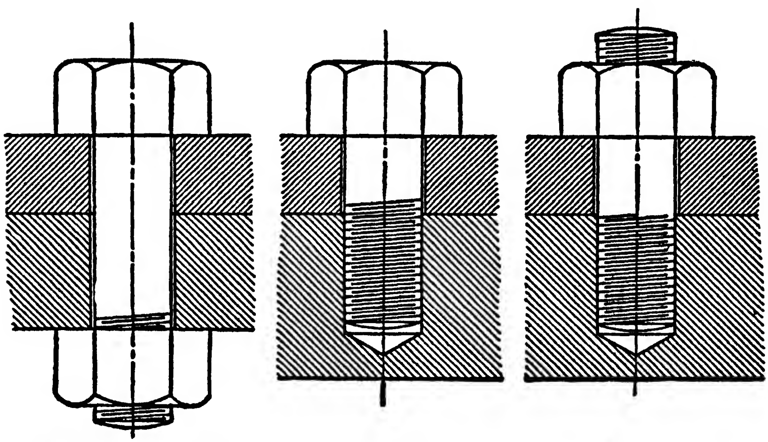
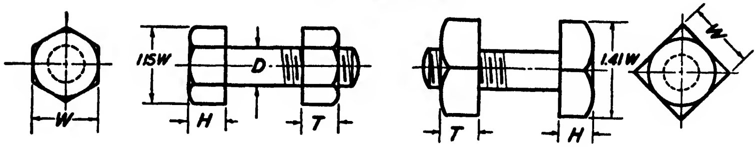


Fig. 41. Through Bolt　　Fig 42. Tap Bolt　　Fig 43. Stud

Common Types of Screw Fastenings. *A Through Bolt* is shown in Fig. 41. It is a round bar with a head on one end and threaded at the other end to take a nut. It is passed through drilled holes in the two parts to be fastened together and clamps them securely to each other as the nut is screwed onto the threaded end. Through bolts may or may not have a machined finish and are made with either square or hexagonal heads whose standard dimensions are contained in Table VIII. When put under tension by a load along its axis, a through bolt should be an easy fit in the holes. If the load acts perpendicular to the axis tending to slide one of the connected parts along the other and thus subjecting it to shear, the holes should be reamed so that the bolt shank fits snugly therein. However, the threaded end must pass freely through the holes. Different minor characteristics which through bolts may possess to fit them to a specific usage

TABLE VIII — American Standard Bolt-Heads and Nuts[1]

Regular Series



Diameter of Bolt D	Unfinished, Square and Hexagon				Semi-finished,[2] Square and Hexagon				Finished,[3] Hexagon only			
	Heads		Nuts		Heads		Nuts		Heads		Nuts	
	W	*H*	*W*	*T*	*W*	*H*	*W*	*T*	*W*	*H*	*W*	*T*
1/4	3/8	11/64	7/16	7/32	3/8	5/32	7/16	13/64	7/16	3/16	7/16	7/32
5/16	1/2	13/64	9/16	17/64	1/2	3/16	9/16	1/4	9/16	15/64	9/16	17/64
3/8	9/16	1/4	5/8	21/64	9/16	15/64	5/8	5/16	5/8	9/32	5/8	21/64
7/16	5/8	19/64	3/4	3/8	5/8	9/32	3/4	23/64	3/4	21/64	3/4	3/8
1/2	3/4	21/64	13/16	7/16	3/4	19/64	13/16	27/64	13/16	3/8	13/16	7/16
9/16	7/8	3/8	7/8	1/2	7/8	11/32	7/8	15/32	7/8	27/64	7/8	1/2
5/8	15/16	27/64	1	35/64	15/16	25/64	1	17/32	1	15/32	1	35/64
3/4	1 1/8	1/2	1 1/8	21/32	1 1/8	15/32	1 1/8	41/64	1 1/8	9/16	1 1/8	21/32
7/8	1 5/16	19/32	1 5/16	49/64	1 5/16	9/16	1 5/16	3/4	1 5/16	21/32	1 5/16	49/64
1	1 1/2	21/32	1 1/2	7/8	1 1/2	19/32	1 1/2	55/64	1 1/2	3/4	1 1/2	7/8
1 1/8	1 11/16	3/4	1 11/16	1	1 11/16	11/16	1 11/16	31/32	1 11/16	27/32	1 11/16	1
1 1/4	1 7/8	27/32	1 7/8	1 3/32	1 7/8	25/32	1 7/8	1 1/32	1 7/8	15/16	1 7/8	1 3/32
1 3/8	2 1/16	29/32	2 1/16	1 13/64	2 1/16	27/32	2 1/16	1 9/64	2 1/16	1 1/32	2 1/16	1 13/64
1 1/2	2 1/4	1	2 1/4	1 5/16	2 1/4	15/16	2 1/4	1 1/4	2 1/4	1 1/8	2 1/4	1 5/16
1 5/8	2 7/16	1 3/32	2 7/16	1 27/64	2 7/16	1 1/32	2 7/16	1 23/64	2 7/16	1 7/32	2 7/16	1 27/64
1 3/4	2 5/8	1 5/32	2 5/8	1 17/32	2 5/8	1 3/32	2 5/8	1 15/32	2 5/8	1 5/16	2 5/8	1 17/32
1 7/8	2 13/16	1 1/4	2 13/16	1 41/64	2 13/16	1 3/16	2 13/16	1 37/64	2 13/16	1 13/32	2 13/16	1 41/64
2	3	1 11/32	3	1 3/4	3	1 7/32	3	1 11/16	3	1 1/2	3	1 3/4
2 1/4	3 3/8	1 1/2	3 3/8	1 31/32	3 3/8	1 3/8	3 3/8	1 29/32	3 3/8	1 11/16	3 3/8	1 31/32
2 1/2	3 3/4	1 21/32	3 3/4	2 3/16	3 3/4	1 17/32	3 3/4	2 3/32	3 3/4	1 7/8	3 3/4	2 3/16
2 3/4	4 1/8	1 53/64	4 1/8	2 13/32	4 1/8	1 11/16	4 1/8	2 5/16	4 1/8	2 1/16	4 1/8	2 13/32
3	4 1/2	2	4 1/2	2 5/8	4 1/2	1 7/8	4 1/2	2 17/32	4 1/2	2 1/4	4 1/2	2 5/8

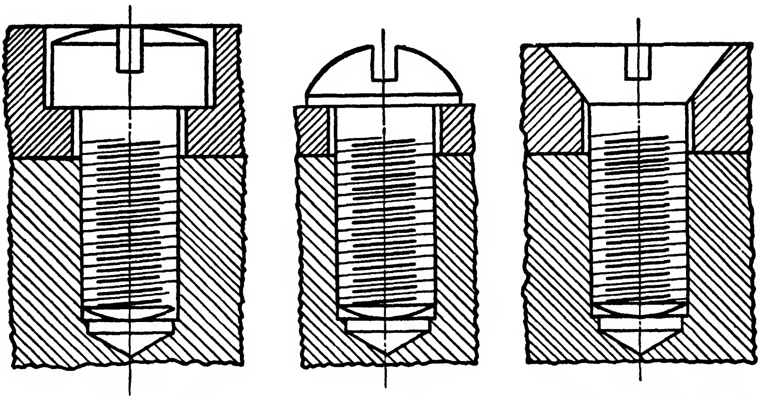
[1] *A S A. B18 2—1933.* [2] Finished under head only, plain or washer faced.
[3] Finished all over, bearing surfaces washer-faced.

create different names by which they become known, such as machine bolts, carriage bolts, automobile bolts, eye bolts, etc.

The Tap Bolt of Fig. 42, secures the parts together by passing through a drilled hole in one part and by being screwed into a tapped hole in the other part. Hence it uses no nut. Its head may be either square or hexagonal, although the latter is the preferred. Tap bolts, whose heads are saw-slotted for use with a screw-driver, are known as Cap Screws. The threaded portion of a tap bolt is about three-fourths of its length.

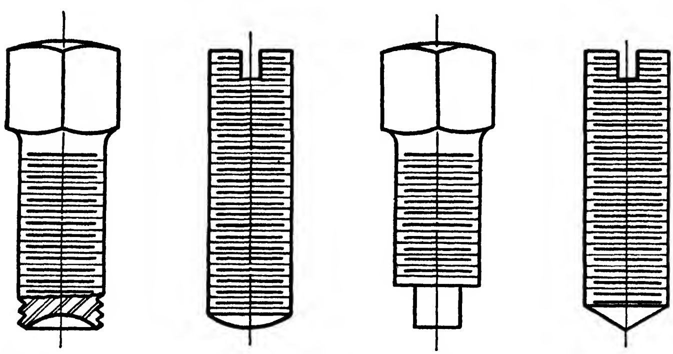
The Bolt as shown in Fig. 43, is a round bar, threaded at both ends, employing a nut but no bolt head. It is screwed into a tapped hole in one of the parts and passes freely through a drilled hole in the other. The two parts are then clamped together by tightening the nut against the upper part. Studs are manufactured either

rough or finished, and generally carry coarse threads. The stud bolt should not be used where a through bolt can serve, but the stud bolt as a rule takes preference over a cap screw.



Round Fillister Head Button Head Flat Countersunk Head

Fig 44 Machine Screw



(a) Cup Point (b) Oval Point (c) Dog or Pivot Point (d) Conical Point

Fig. 45. Set Screws

Machine Screws. The term "Machine Screws" is one applied to very small cap screws. Many different forms of slotted heads are in use with these small screws, three of the most common forms being shown in Fig. 44.

Set Screws. A Set Screw is a screw which keeps two machine parts from having any relative motion by being screwed through a tapped hole in one part until the end or so-called point of the screw is

TABLE IX — Safe Holding Power of Set Screws

Diameter of Set Screw	Holding Power in Pounds	Diameter of Set Screw	Holding Power in Pounds
1/4	100	5/8	840
5/16	168	3/4	1280
3/8	256	7/8	1830
7/16	366	1	2500
1/2	500	1 1/8	3288
9/16	658	1 1/4	4198

forced or pressed against the other part. Fig. 45 illustrates set screws with heads in (*a*) and (*c*) and the headless type in (*b*) and (*d*). The latter are preferred and in some states are required by law in all installations on moving parts. The various points shown may be used on either variety. The screw thread employed is usually the American standard coarse series.

The holding power of a set screw is the frictional resistance set up at the point. Due to this, set screws can resist only light loads. They may be used in transmitting power if the latter is very low in value. An example of such an installation is where a set screw is screwed through the hub of a gear or pulley and by its pressure against the shaft, the gear or pulley is caused to rotate with the shaft. The frictional resistance which it sets up at the shaft is a tangential force, P_t. The moment arm through which this tangential force works in such a case is evidently the radius of the shaft. If the torque in the shaft is known, the necessary holding power, P_t, can be found as in the case of any tangential force, by using formula (29). Or if the holding power is known, the horsepower can be determined through the use of formula (32). According to experiments conducted by Mr. B. H. D. Pinkney and reported in the *American Machinist*, the holding power of set screws is as given in Table IX.

Example. What horsepower can safely be transmitted by a $\frac{1}{2}$-inch set screw employed in a pulley installation if the rotative speed of the pulley is 850 revolutions per minute and the diameter of the shaft is $1\frac{1}{2}$ inches?

Solution. From Table IX, the holding power, $P_t=500$ lb., and from the statement of the problem, $R=\frac{3}{4}$ in., and $N=850$ r.p.m.

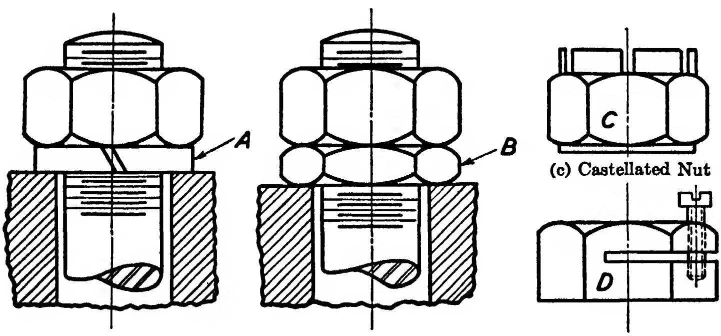
Substituting these known values in formula (32),

$$H=\frac{P_t 2\pi RN}{12\times 33{,}000}$$

$$H=\frac{500\times 2\times \pi\times \frac{3}{4}\times 850}{12\times 33{,}000}=5+\quad \textit{Ans.}$$

Locking Devices for Nuts. The tendency of nuts to loosen under vibration or changing forces acting thereon makes it necessary in a great many installations to secure the nut in position by some kind of locking device. Several of these are shown in Fig. 46.

Fig. 46 (*a*) shows the *Spring Lock Washer* in use at *A*. As the nut tightens the washer against the piece below, one edge of the washer is caused to dig itself into that piece, thus increasing the resistance so that the nut will not loosen so easily. There are many different kinds of spring lock washers manufactured, some of which are fairly effective.



(a) Spring Lock Washer (b) Locknut (c) Castellated Nut (d) Split Nut

Fig. 46

Fig. 46 (*b*) illustrates the use of the *Locknut, B,* which is made about one-half the height of the standard nut with which it is used. Theoretically, the locknut should be placed as shown in the figure. In practice, it is usually placed on the outside so that it can be manipulated more easily. The locknut is tightened against the standard nut. The extent to which locking is secured in this manner is questionable.

Fig. 46 (*c*) illustrates a *Castellated* nut. After this nut is screwed tightly into position, a split cotter pin is inserted through the groove of the nut and a hole through the bolt. Although the nut must be backed off a trifle so as to line up the groove and hole, not much

loosening is introduced because fine threads with their accompanying small leads are used. This is a positive locking device and is used to a great extent in the automobile industry.

The *Split* nut of Fig. 46 (*d*) is one which has a slot sawed about half way through. After the nut is screwed down, the small screw is tightened so as to produce more friction between the nut and the bolt, thus making it more difficult for the nut to become loosened.

A thread-locking device, by which a nut is secured in its position on the bolt, is shown in Fig. 47, which illustrates the *Dardelet Thread.* This thread is designed to be self-locking. The top of the thread on

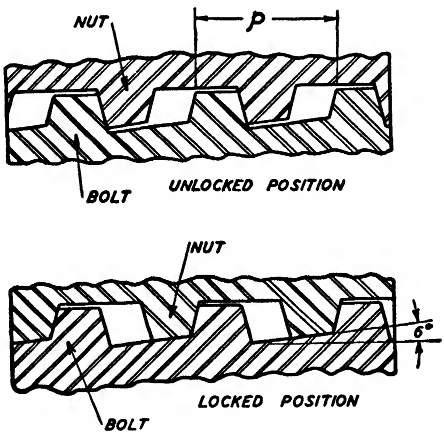


Fig. 47. Dardelet Thread

the nut and the root of the thread on the bolt are tapered at an angle of 6 degrees and the width of the space between the threads is much greater than the thickness of the thread. As the nut is screwed onto the bolt it moves to the right (in Fig. 47) very easily until the tapered surface of the top of the thread of the nut comes into contact with the tapered surface of the bottom of the thread of the bolt, developing a pressure between them. It is this pressure that produces the locking effect. In the final or locked position, the threads are in contact as shown in the lower view.

The Dardelet thread can be manufactured by hot or cold rolling, or by taps and dies. The dimensions of the thread section leave the body of the bolt stronger than that provided by the American Standard coarse thread.

Design of Bolts Supporting Tensile Loads Only. When a bolt supports a load that acts along its axis and tends to stretch it as in Fig. 48, the bolt is under tension, according to the definition of tension given in the first chapter. In every section of the bolt taken at right angles to the axis, there is set up a total tensile stress that is equal to the load. Evidently then, if any involved section of the bolt has a smaller area than other sections, the amount of stress set up over each square inch of such smaller area must be greater than the amount

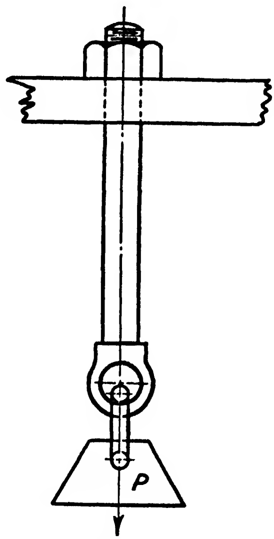


Fig 48 Eye Bolt with Tensile Load

of stress per square inch of the other sections. In other words, this smaller section is weaker than the others, and, in the case of a bolt, is the section at the root of the threads. This is a circular section with a diameter equal to the root diameter, d. Hence in designing a bolt, loaded as in Fig. 48, the section at the root of the threads is made large enough safely to resist the tensile load, P. Other sections, such as those with a diameter, D, the outside diameter of the bolt, will then be still safer. Evidently formula (5), $P=AS_t$, applies here, and A of the formula becomes equal to $\frac{\pi d^2}{4}$. Substituting this in the formula,

$$P=\frac{\pi d^2}{4}S_t.$$

Multiplying each member of the equation by 4, and dividing each member by $\pi \times S_t$, we obtain,

$$d^2 = \frac{4P}{\pi S_t}$$

or

$$d = \sqrt{\frac{4P}{\pi S_t}}, \qquad (64)$$

where d is the root diameter of the bolt

The factor of safety to be used depends upon the type of load and hence Table II can be consulted. Having found either the root area from formula (**5**) or the root diameter from formula (**64**), the standard bolt can be selected from Table VII or from a similar table if another standard thread is used. The one selected should always be the standard diameter that is just above the theoretical value.

Example. The load, P, in Fig. 48, is a dead load of 18,000 pounds. The bolt is steel with an ultimate tensile strength of 60,000 pounds per square inch. American standard coarse threads are used. Find the nominal diameter.

Solution. Here $P = 18{,}000$ lb., $U_t = 60{,}000$ lb. per sq. in., and from Table II, $F = 4$.

Therefore $$S_t = \frac{60{,}000}{4} = 15{,}000 \text{ lb. per sq. in.}$$

Substituting the values of S_t, and P, in formula (**64**),

$$d = \sqrt{\frac{4 \times 18{,}000}{\pi \times 15{,}000}}$$

Canceling like factors in numerator and denominator of the second member of the above equation,

$$d = \sqrt{\frac{4 \times 6}{5\pi}} = \sqrt{\frac{24}{5\pi}} = 1.24 \text{ in.}$$

Consulting Table VII, we find that our root diameter lies between the standard root diameters, 1.1585 in. and 1.2835 in. Hence 1.2835 in. is selected, and the corresponding nominal or outside diameter, D, $= 1\frac{1}{2}$ in. *Ans.*

Example. What safe tensile load may a 2-inch American standard bolt carry if S_t is equal to 6000 pounds per square inch?

Solution. Here $S_t = 6000$ lb. per sq. in. and from Table VII, the root diameter of a 2-in. bolt $= 1.7113$ in. Therefore the root area $= \frac{\pi \times 1.7113^2}{4}$. Substituting these values in formula (**5**),

$$P = AS_t$$
$$= \frac{\pi \times 1.7113^2}{4} \times 6000$$
$$= 13{,}800 \text{ lb.} \quad \textit{Ans.}$$

Design of Bolts Subjected to Initial Tightening. When two pieces are clamped together by a screw or bolt as in Figs. 41, 42, and 43, there is more to consider relative to the stresses involved than in such a case as that of the eye bolt of Fig. 48. In the latter, it is evident that the condition set up within the bolt is one of pure tension caused entirely by the well-defined tensile load, P. Hence there is no doubt in the mind of the designer as to the extent of the loading of the bolt and the total stress set up therein. It is fixed and definitely determined. The designer knows the exact load, P, and that the tensile resistance, AS_t, is equal to it. He needs only then to consider the material of the bolt and the type of load and he can arrive at the safe unit stress which should be used. In this manner he is in a position to solve immediately for the area of the bolt. In the former case, in which machine parts are clamped together, the general procedure in design will be identical to the latter, but there is great uncertainty as to the total loading of the bolt or screw. A knowledge of the possibilities involved must be known so that a sufficiently large factor of safety or safe stress may be used.

The clamping of two machine parts together as in Figs. 41, 42, and 43, introduces into the bolt a so-called loading thereon, known as the Initial Tightening. The induced stress set up within the bolt by this initial tightening is a tensile stress. This initial tightening effect or loading is difficult to measure since (among other things) it is dependent upon the strength of the mechanic and the length of wrench being used. The total tensile load on the bolt will be as listed in the following cases:

Case 1. An uncertain initial tightening effect only,

Case 2. A known external force or load only,

Case 3. An amount greater than the known load but less than the sums of *Cases 1* and *2*.

Case 1 of the above, occurs when two machine parts are fastened together and no external load tends to separate them. Experiments relative to this effect have produced the following formula:*

*From "Elements of Machine Design" by Kimball and Barr, by permission, John Wiley and Sons, Inc. New York

$$P = 16{,}000D \tag{65}$$

where P = the total load, or initial tightening, in pounds.
D = nominal or outside diameter of bolt in inches.

Since from formula (5),

$P = AS_t$, we have,

$$16{,}000D = AS_t$$

Solving for S_t, the unit tensile stress in lb. per sq. in.

$$S_t = \frac{16{,}000D}{A} = \frac{16{,}000D}{\frac{\pi d^2}{4}} \tag{66}$$

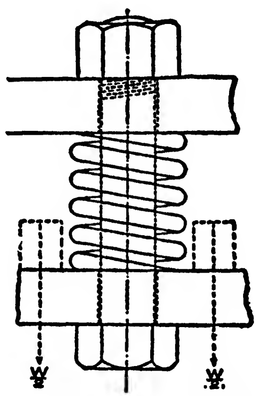


Fig 49 Action of Elastic Packing

Cases 2 and 3 of the above possible loadings are brought about as a rule in securing a fluid-tight joint between the parts connected. The relative elasticities of the materials of the joint (or elastic packing at the joint) and the bolt play an important part here. This condition may be represented diagrammatically by Fig. 49, in which a spring has been introduced to portray the elastic element at the joint and hence takes the compression due to screwing up the nut. Evidently the tension in the bolt is equal to the force necessary to compress the spring and is the initial tension, I, due to the initial tightening. Now suppose that two weights, each equal to $\frac{W}{2}$ and representing external loads on the bolt, are placed symmetrically on either side of the bolt as shown, then the tension in the bolt will be increased by an amount equal to the added weights, becoming equal

to $W+I$ if the bolt is perfectly rigid, that is, not at all elastic. The bolt, however, is elastic and stretches; hence some of the compression on the spring is relieved and the total tension in the bolt is less than $W+I$, by an amount which depends on the relative elasticity of the bolt and spring. This is *Case 3* and is the condition of loading for leak-proof joints.

Suppose that the stud in Fig. 43, is one of those used in connecting the cover to the cylinder of a steam engine, and that the studs have a small initial tension; then the pressure of the steam loads each stud, and if the studs stretch enough to relieve the pressure between the two surfaces due to their initial tightening, then the stress of the studs is due to the steam pressure only. At this point, an increase in the load would cause a parting at the joint. Naturally for a leak-proof joint this could never occur. Therefore when W is nearly equal to I, we have the condition of *Case 2,* which is considered as satisfactory to prevent separation of the parts without making the joint fluid-tight or leak-proof.

With *Case 2,* we may use the design formula in tension, $P=AS_t$. In so doing, P will be the external force on the bolt referred to in the above as W. But to prevent separation of the parts, although the joint need not be fluid-tight, a value of S should be chosen so that there will be a good margin of safety. For ordinary wrought iron and medium steel, S_t may here be taken around 6000 lb. per sq. in. This gives a so-called factor of safety of 8 to 10.

In using the formula $P=AS_t$ for fluid-tight joints, the total load which is greater than W must be taken equal to the latter since no other loading is known. For this reason S_t must be much less than it otherwise would be. Values from 3000 lb. per sq. in. to 5000 lb. per sq. in. may be used for wrought iron and medium steel, the lower value being used for bolts of less than $\frac{3}{4}$-inch diameter.

Example. Two machine parts are fastened together tightly by means of a 1-inch tap bolt as shown in Fig. 42. If the load tending to separate these parts is neglected, compute the unit working stress that is probably set up in this bolt by the initial tightening.

Solution. Here $D=1$ in., and from Table VII, d, the root diameter, = 0.837 in. Substituting these values in formula (**66**),

$$S_t=\frac{16{,}000D}{A}$$

$$=\frac{16{,}000D}{\frac{\pi d^2}{4}}$$

$$=\frac{16{,}000\times 1}{\frac{\pi\times 0.837^2}{4}}=29{,}000 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

Example. If a $\frac{1}{2}$-inch bolt is used as in the preceding example, what is the working stress that is set up due to initial tightening?

Solution. Here $D=\frac{1}{2}$ in. and d, from Table VII=0.400 in. Substituting these values in formula (66),

$$S_t=\frac{16{,}000D}{\frac{\pi d^2}{4}}$$

$$=\frac{16{,}000\times\frac{1}{2}}{\pi\times\frac{0.400^2}{4}}=63{,}600 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(**Note.** The result of this example clearly shows that regular screw stock cannot be used in this case, as the working stress is above the ultimate for the material. It also explains the reason that small bolts are often ruptured while screwing down the nut. The application of formula (66) in both of these examples establishes the fact that the unit working stress decreases as the diameter of the bolt increases and hence larger screws should be used unless severe initial tightening can be avoided.)

Example. A cylinder head of a steam engine is held on by 14 bolts. The diameter of the cylinder is 14 inches and the steam pressure is 125 pounds per square inch. What size of bolts is required if S_t is equal to 3000 lb. per sq. in.?

Solution. The pressure of the steam acts over the entire area of the cylinder head. Therefore, the total steam pressure=pressure in lb. per sq. in.×area of cylinder head in square inches

$$=125\times\frac{\pi\times 14^2}{4}$$

$$=19{,}250 \text{ lb.}$$

The above answer is the total force or external load on the 14 bolts since it is the force tending to push the cylinder head from the cylinder. The load per bolt, P, will be:

$$P=\frac{19{,}250}{14}=1375 \text{ lb.}$$

With $S_t=3000$ lb. per sq. in., $P=1375$ lb., solve for d, the root diameter of the bolts, by using formula (5).

$$P=AS_t=\frac{\pi d^2}{4}\times S_t$$

Evaluating, $$1375=\frac{\pi d^2}{4}\times 3000$$

$$\therefore d=\sqrt{\frac{1375\times 4}{3000\times \pi}}=0.76+ \text{ in.}$$

Table VII gives 0.8376 in. as the standard root diameter that is just above our value of d. Therefore the actual root diameter will be 0.8376 in.

$\therefore D$, the outside diameter $=1$ in. *Ans.*

Other Considerations in Bolt Design. For shocks or blows, as in the case of the bolts found on some connecting-rod ends of engines, the stretch of the bolts acts like a spring to reduce the resulting tensions. Here then is another case where elasticity plays an important part in design. The unit elongation or stretch, Δ, is equal to the total elongation in inches divided by the length of the piece in inches, provided that length has a constant or the same cross-sectional area throughout. Therefore the greater such a length, the less the unit strain or unit stretch will be. And the less the unit stretch, the smaller the unit stress over that constant cross-sectional area. This is proven by referring to formula (18), which is

$$E_t=\frac{S_t}{\Delta}$$

or $$S_t=E_t\times\Delta$$

The above formula states that the unit stress induced over the cross section is equal to the modulus of elasticity times the unit strain. Hence for a given modulus of elasticity, the smaller the value of Δ, the smaller the value of S_t will be. Now we appreciate that along the length of a bolt there are two different areas, the body area, $\frac{\pi D^2}{4}$, and the root area, $\frac{\pi d^2}{4}$. To make the total length of a bolt effective in lowering the unit stretch, Δ, it therefore is necessary to see that a uniform cross section prevails throughout. This reduction of the area of the unthreaded part of the bolt to the root area of the threaded part is done in several ways as shown in Fig. 50. In Fig. 50,

(*a*), the area of the shank is reduced to the root area by drilling a hole as shown, down to the threaded end, while the reduction in Fig. 50, (*b*), is accomplished by turning down the shank. In such a manner, the capacity of the bolt to resist a shock load is increased.

In tightening up a bolt the friction at the surface of the thread produces a twisting moment, which increases the stress in the bolt, just as in the case of shafting under combined tension and torsion; but the increase in stress is small in amount and is taken care of by the factors of safety already considered in bolt design.

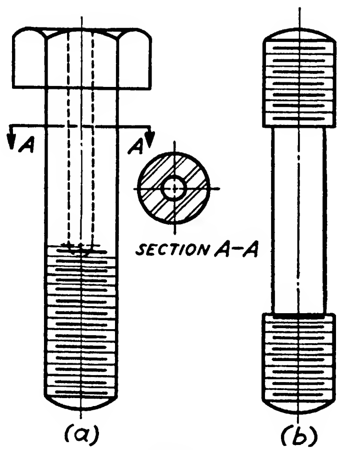


Fig. 50

A tensile load on a bolt subjects both the bolt-head and nut to shear. Reference to Fig. 41 will show that if the two parts connected tend to separate along or parallel to the axis of the bolt, the tensile load thus created on the bolt will tend to strip the threads of the nut and to pull a cylindrical plug of metal through the bolt-head. The areas involved in these cases would have their elements parallel to the load acting. Hence such an action is shear and the tensile load on the bolt becomes a shearing load of equal magnitude on both the nut and bolt-head. This is taken care of in the standard dimensions assigned to bolt-heads and nuts, as given in Table VIII. With these dimensions, the unit shearing stress induced in either bolt-head or nut by a safe tensile load along the axis of the bolt is very low.

When a bolt is acted upon by a force perpendicular to its axis and tending to cut it across its right section, the bolt is under shear.

In such a case the involved area should be the body area and not the root area, and the bolt should fit snugly in its hole.

Example. What size of hole must be drilled in a standard 2-inch bolt to make the body area equal to the root area?

Solution. The body area of a 2-inch bolt $=3.142$ sq. in. and the root area $=2.299$ sq. in. If the body area is to be reduced in size to the root area, a section of material must be taken from the body area which is equal to $3.142-2.299=0.84$ sq. in. Since this is the area of a circle, $\frac{\pi d^2}{4}=0.84$, where in this problem d is the diameter of the drilled hole.

Multiplying both members of the equation by 4 and dividing by π, we have, $d^2=\frac{4\times0.84}{\pi}$

or $$d=\sqrt{\frac{4\times0.84}{\pi}}=1.04-\text{ in. } \textit{Ans.}$$

Example. A standard $1\frac{1}{2}$-inch bolt is subjected to a tensile load that sets up within it an induced tensile stress of 6000 pounds per square inch. (*a*) What tensile load is carried by the bolt? (*b*) What factor of safety is involved in tension if the ultimate tensile strength is equal to 60,000 pounds per square inch? (*c*) To what shearing load is the bolt-head subjected? (*d*) If a standard semi-finished hexagonal bolt-head is used, what is the unit shearing stress induced therein? (*e*) What factor of safety is involved in shear of the bolt-head if the ultimate shearing stress is 50,000 lb. per sq. in.?

Solution. (*a*) Here $S_t=6000$ lb. per sq. in. and from Table VII, the root diameter of a $1\frac{1}{2}$-in. bolt is 1.2835 in. Therefore $A=\frac{\pi d^2}{4}=\frac{\pi\times1.2835^2}{4}=1.295$ sq. in.

Using formula (5),

$$P=AS_t$$

and evaluating therein,

$$P=1.295\times6000=7770 \text{ lb. } \textit{Ans.}$$

(*b*) From formula (4) or (6), with $U_t=60{,}000$ lb. per sq. in. and $S_t=6000$ lb. per sq. in.,

$$F=\frac{U_t}{S_t}$$

$$=\frac{60{,}000}{6{,}000}=10 \quad \textit{Ans.}$$

(*c*) Since the shearing load on the bolt-head is equal to the tensile load on the bolt,

$$P, \text{ in shear}, = 7770 \text{ lb.} \quad \textit{Ans.}$$

(*d*) Let the height, H, of this bolt-head $=1$ in. The area in shear is the lateral area of a cylinder whose diameter is equal to the nominal diameter of the bolt and whose altitude is the height of the bolt-head. Therefore, the area in shear,

$$A = \pi DH$$
$$= \pi \times 1\tfrac{1}{2} \times 1 = 4.71 \text{ sq. in.}$$

Using formula (**13**), in which $P=7770$ lb. and $A=4.71$ sq. in., we have

$$P = AS_s$$
$$7770 = 4.71 \times S_s$$

Dividing by 4.71,

$$S_s = \frac{7770}{4.71} = 1650 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(*e*) Here $U_s = 50{,}000$ lb. per sq. in. and $S_s = 1650$ lb. per sq. in. Applying formula (4) or (**14**),

$$F = \frac{U_s}{S_s}$$
$$= \frac{50{,}000}{1650} = 30+ \quad \textit{Ans.}$$

Efficiency of Transmission Screw. One of the many applications of the transmission screw is the screw jack, Fig. 51. Here a frame or supporting link, A, is threaded with a square thread and acts as the nut for the transmission screw, B. A collar, C, at the upper end of the transmission screw contains several radially drilled holes for taking the rod or lever, E. A link, D, supports the load, W. A tangential force, P_t, applied to the rod causes the screw to rotate about its axis and thus the screw and its load, W, are raised through a distance equal to the lead of the screw, L, for each revolution. The link, D, does not rotate, so it and the load, W, have motion parallel to the axis only.

Now with any machine such as this, there is an energy put into it so that some useful purpose can be fulfilled, in other words, so that work can be done. But not all the energy input can be utilized in doing the useful work because some of it must be used up in other ways as in overcoming the friction between the parts of the machine

that rub on each other. Now a frictional resistance set up between two parts in contact works through the same distance that the parts move relative to each other. And since work in general is the product

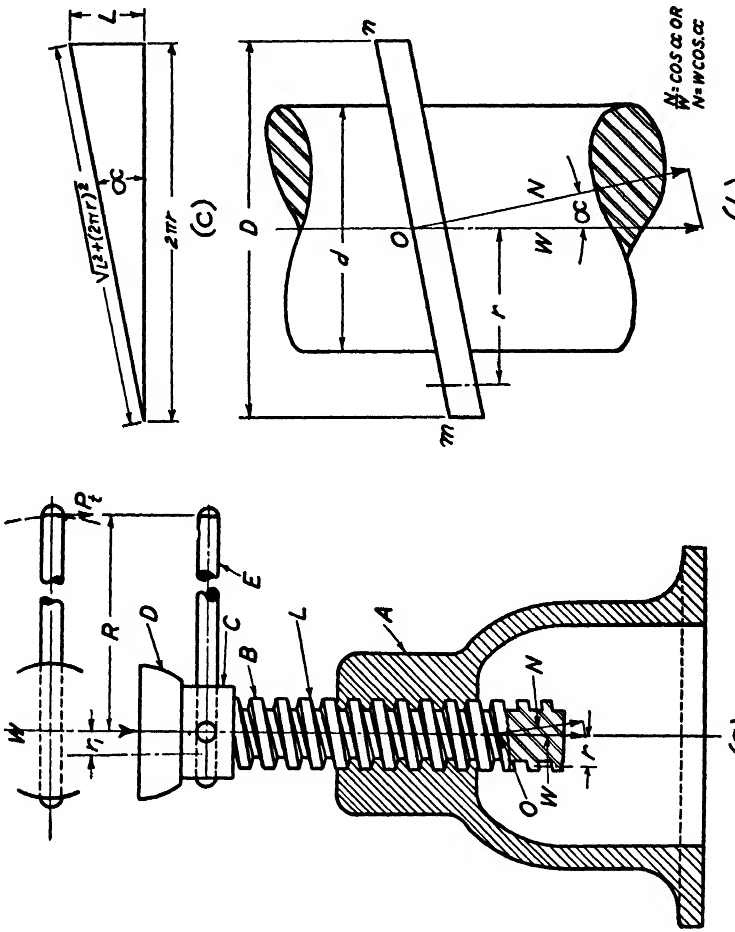


Fig 51. Screw Jack (*a*) Front Elevation; (*b*) Relation between *W* and *N*; (*c*) Development of Mean Helix of Thread

of a force and the distance through which it acts, the work of friction is the product of the frictional resistance and the distance through which the parts affected by this frictional resistance move relative to each other. Therefore frictional work done in a machine is done at the expense of the energy input and hence this energy supplied to the machine must be equal to the energy used in doing the useful

work plus the energy consumed in any other way such as in overcoming friction. The efficiency (mechanical) of a machine is a factor that tells how much of the energy input is used in doing the useful work of the machine. It is the ratio of the useful work done to the total energy supplied in a given length of time.

In determining the efficiency of this square-threaded transmission screw we shall use the following notations. All forces or loads are in pounds, all distances in inches.

W = load to be transmitted (in this case, to be raised)
P_t = the tangential force applied at the end of lever, E
R = radial moment arm through which P_t acts
r = mean radius of the thread
r_1 = mean radius of bearing surface between links C and D
N = normal pressure on thread
L = lead of thread
α = the angle of the thread $= \tan^{-1}\dfrac{L}{2\pi r}$
ϕ = the angle of friction $= \tan^{-1} \mu$, (by definition)
μ = coefficient of friction between screw and bolt
μ_1 = coefficient of friction between links, C and D
e = the efficiency of the transmission screw
n = the number of threads in contact between nut and screw

The useful work done in one revolution of the screw $= W \times L$ inch-pounds, for any screw will advance in its stationary nut a distance equal to its lead in one revolution. The total energy supplied per revolution is the work done by the tangential force, P_t, while moving through the circumference of its path which is $2\pi R$ inches. Therefore, total energy supplied per revolution $= P_t \times 2\pi R$ in.-lb. Since the efficiency as already stated is equal to the ratio of the useful work done to the total energy supplied, we have

$$e = \frac{W \times L}{P_t \times 2\pi R} \tag{67}$$

In dealing with friction it must now be remembered from Mechanics that the friction between two bodies in contact is equal to the normal pressure between them multiplied by the coefficient of friction. The normal pressure at the thrust bearing between links C and D, is equal to the load W for the bearing surface is perpendicu-

lar to the axis of the screw while the load W, acts along or parallel to this axis. Therefore if μ_1 is the coefficient of friction, the frictional resistance between C and $D = \mu_1 \times W$ pounds. In one revolution, this force acts through a distance equal to $2\pi r_1$, it being assumed that the frictional force is concentrated at a point located at a distance from the axis equal to the mean radius. Therefore the work of friction at this thrust bearing $= \mu_1 \times W \times 2\pi r_1$ in.-lb.

The main rubbing surface in a transmission screw is that surface of contact between the screw and its nut. The friction here is a function of the normal pressure, N, which is due to the load, W. These are shown in Fig. 51 (a), and in Fig. 51 (b). The latter is an enlarged view of a part of the former and shows the load, W, acting along the axis of the screw at point, O, on thread, mn. The vector, N, represents the normal pressure due to the load, W, and hence is perpendicular to mn at O. The angle between W and N is the angle of the helix of the thread or simply the angle of the thread, α (alpha). This angle is shown graphically in Fig. 51 (c), which is a development of the mean helix of the thread, through one complete turn. The figure shows that the angle, α, is the angle whose tangent is $\frac{L}{2\pi r}$ and that the length of the thread is the hypotenuse of the right triangle and is equal to $\sqrt{L^2+(2\pi r)^2}$. In Fig. 51 (b), $\cos\alpha = \frac{N}{W}$, therefore $N = W \times \cos\alpha$. The frictional resistance or force between the thread and its nut, as any frictional force, is equal to the normal pressure times the coefficient of friction which in this case we have called, μ (mu). Therefore the frictional resistance here $= \mu \times N = \mu \times W \cos\alpha$. In one turn, this frictional force, assumed to act along the mean element of the thread surface, works through a distance equal to the length of this mean element which is, from Fig. 51 (c), equal to $\sqrt{L^2+(2\pi r)^2}$, as previously stated. Therefore the work of friction at the thread becomes for one turn or revolution, $= \mu \times W \cos\alpha \times \sqrt{L^2+(2\pi r)^2}$ in.-lb. Therefore the total work done per revolution of the screw, which is the sum of the useful work and all the work of friction, $= W \times L + \mu_1 \times W \times 2\pi r_1 + \mu \times W \cos\alpha \times \sqrt{L^2+(2\pi r)^2}$.

If we now assume that r_1 is made equal to r and that μ is considered equal to μ_1, the total work done per revolution $= W \times L + \mu \times W \times 2\pi r + \mu \times W \cos\alpha \times \sqrt{L^2+(2\pi r)^2}$ in.-lb. But if this

screw is to operate, the energy that is supplied must equal the total work done.

Hence

$$P_t \times 2\pi R = W \times L + \mu \times W \times 2\pi r + \mu \times W \cos\alpha \times \sqrt{L^2 + (2\pi r)^2}$$

From Fig. 51 (*c*), it is seen that

$$\cos\alpha = \frac{adjacent\ side}{hypotenuse} = \frac{2\pi r}{\sqrt{L^2 + (2\pi r)^2}}$$

Substituting this value for $\cos\alpha$ in the preceding equation, we obtain,

$$P_t 2\pi R = W \times L + \mu \times W \times 2\pi r + \mu \times W \times \frac{2\pi r}{\sqrt{L^2 + (2\pi r)^2}} \times \sqrt{L^2 + (2\pi r)^2}$$

Simplifying,

$$\begin{aligned} P_t 2\pi R &= W \times L + \mu \times W \times 2\pi r + \mu \times W \times 2\pi r \\ &= W \times L + 2 \times \mu \times W \times 2\pi r \\ &= W \times L + \mu \times W \times 4\pi r \end{aligned}$$

Substitution of this value in formula (67) for its equal, $P_t 2\pi R$, gives the following:

$$e = \frac{W \times L}{W \times L + \mu \times W \times 4\pi r}$$

Dividing both numerator and denominator of the second member by W,

$$e = \frac{L}{L + \mu \times 4\pi r}$$

Now by dividing both numerator and denominator by $2\pi r$,

$$e = \frac{\dfrac{L}{2\pi r}}{\dfrac{L}{2\pi r} + \dfrac{\mu \times 4\pi r}{2\pi r}}$$

Since $\dfrac{L}{2\pi r} = \tan\alpha$, from Fig. 51 (*c*), we have for the efficiency of a square-threaded transmission screw,

$$e = \frac{\tan\alpha}{\tan\alpha + 2\mu} \qquad (68)$$

A consideration of this formula shows that if $\tan\alpha$ is less than the coefficient of friction, μ, or in other words, if the angle, α, is less than the angle of friction, ϕ (phi), the screw is self-locking. This means that the load transmitted can not cause rotation of the screw. Hence in the case of a screw jack, the tangential force can be removed

after the load has been raised to the desired position and the load will remain in that position. On the other hand, if $\tan \alpha$ is greater than μ, the load can cause rotation of the screw. The spiral screwdriver is an application of the latter condition. The formula also shows that the efficiency increases as the angle, α, increases. For this reason multiple threaded transmission screws are generally used. Such a thread may have a large lead, which provides a large angle, α, but a small pitch with its accompanying small thread and relatively large root diameter.

Design of Square-Threaded Transmission Screw. A transmission screw is generally short enough so that the load, W, which it takes along its axis places the screw under compression instead of buckling. The thread of the screw is not considered as giving any added strength in compression, so when formula (9), $P = AS_c$, is applied, the area, A, is based on the root diameter, d, becoming $\frac{\pi d^2}{4}$, and $P = W$, so that

$$W = \frac{\pi d^2}{4} \times S_c \qquad (69)$$

If the screw should be quite long, it may still be designed as if under compression using a lower value for the compressive stress and then checked for buckling.

This compressive load, W, is a shearing load on the threads of the screw as well as the nut. The area in shear of each thread on the screw can be taken as the cylindrical area, $\pi d \times \frac{p}{2}$, where p = the pitch of the thread. With n threads in use, the total area in shear will equal $n \times \pi d \times \frac{p}{2}$ square inches. Placing this value for A in formula (13),

$$P = AS_s$$

$$= n \times \pi d \times \frac{p}{2} \times S_s$$

Since

$$P = W$$

$$W = n \times \pi d \times \frac{p}{2} \times S_s \qquad (70)$$

Since the area of shear in the nut is the same as the area in shear of the screw except that it uses the outside diameter, D, instead of the inside diameter, d, formula (70) becomes for the nut,

$$W = n \times \pi D \times \frac{p}{2} \times S_s \qquad (71)$$

When a power-driven transmission screw is designed, the bearing area between the threads of the nut and screw becomes of major importance; for such a screw, designed safely in all other ways, often fails because of a too small bearing area creating between the threads an overly high unit bearing pressure for the conditions of operation. In hand-operated power screws, this need not as a rule be investigated. When considered, the bearing area per thread is taken as the projected area upon a plane perpendicular to the axis of the screw. It then becomes the area of a ring whose outside diameter is D and whose inside diameter is d. Hence the bearing area per thread $=\pi\frac{(D^2-d^2)}{4}$ sq. in. and for n threads in contact, the area,

$$A=n\times\pi\frac{(D^2-d^2)}{4} \text{ sq. in.}$$

The load per square inch or unit bearing pressure, p_n, in lb. per sq. in. is equal to the total load, W, divided by the above area, A.

Therefore,
$$p_n=\frac{W}{n\times\pi\frac{(D^2-d^2)}{4}}$$

or
$$p_n=\frac{4W}{n\pi(D^2-d^2)} \text{ lb. per sq. in.} \tag{72}$$

Authorities who have investigated the coefficient of friction, μ, have found it to be quite variable dependent on many things such as lubrication, condition of the contact surfaces, etc. It seems reasonable to suspect that in general, μ can be taken from 0.1 to 0.2.

Example. A square-threaded transmission screw used in a jack has a root diameter of 2 inches and a pitch of $\frac{1}{4}$ inch. It is double-threaded and designed to lift a load of 10 tons. The coefficient of friction is assumed to be 0.15. (*a*) Find the efficiency of this screw. (*b*) Is the screw self-locking? (*c*) Find the unit compressive stress in the screw in lb. per sq. in. (*d*) What is the unit shearing stress in lb. per sq. in. if there are 12 threads in the nut? (*e*) What is the bearing pressure in lb. per sq. in. of the projected area of the threads in contact?

Solution. (*a*) Since the screw is double-threaded and $p=\frac{1}{4}$ in., the lead, L, $= 2\times\frac{1}{4}$ in. $= \frac{1}{2}$ in.
From formula (**57**),

$$D = d + p$$
$$= 2 + \tfrac{1}{4} = 2\tfrac{1}{4} \text{ in.}$$

The mean diameter

$$= \frac{D+d}{2}$$

$$= \frac{2\frac{1}{4}+2}{2} = 2\tfrac{1}{8} \text{ in.}$$

The mean radius,

$$r = \frac{2\frac{1}{8}}{2} = 1\tfrac{1}{16} \text{ in.}$$

$$\tan \alpha = \frac{L}{2\pi r}$$

$$= \frac{\frac{1}{2}}{2 \times \pi \times 1\frac{1}{16}} = 0.075$$

From formula (68),

$$e = \frac{\tan \alpha}{\tan \alpha + 2\mu}$$

$$= \frac{0.075}{0.075 + 2 \times 0.15} = \frac{0.075}{0.375} = 0.20 \text{ or } 20\% \quad \textit{Ans.}$$

(*b*) Yes, because $\tan \alpha$ is less than μ. *Ans.*

(*c*) Using formula (69),

$$W = \frac{\pi d^2}{4} \times S_c$$

and substituting therein $W = 20{,}000$ lb. and $d = 2$ in., we have,

$$20{,}000 = \frac{\pi \times 2^2}{4} \times S_c$$

$$20{,}000 = \pi \times S_c$$

$$S_c = \frac{20{,}000}{\pi} = 6400- \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(*d*) The unit shearing stress in the screw will be greater than that in the nut because the formulas for the areas in shear in both screw and nut are the same with the exception that d is used in the screw whereas D is used in the nut.

Therefore, to find the larger unit shearing stress, we will use formula (70) in which $W = 20{,}000$ lb.; $n = 12$ threads; $d = 2$ in.; and $p = \frac{1}{4}$ in. Hence,

$$W=n\times\pi d\times\frac{p}{2}\times S_s$$

$$20{,}000=12\times\pi\times2\times\frac{\frac{1}{4}}{2}\times S_s$$

$$20{,}000=9.42S_s$$

$$S_s=\frac{20{,}000}{9.42}=2120+\text{ lb. per sq. in.}\quad \textit{Ans.}$$

(*e*) Applying formula (72), in which $W=20{,}000$ lb.; $n=12$ threads; $D=2\frac{1}{4}$ in.; and $d=2$ in.;

$$p_n=\frac{4W}{n\pi(D^2-d^2)}$$

$$=\frac{4\times20{,}000}{12\times\pi\times(2\frac{1}{4}^2-2^2)}=2000\text{ lb. per sq. in.}\quad \textit{Ans.}$$

PROBLEMS

1. Find the pitch and depth of thread of a 1-inch (outside diameter of 1 inch) screw that has 8 American (National) Standard threads per inch. *Ans.* 0.125 in.; 0.081+ in.

2. Find the pitch and the depth of thread of a 1-inch screw that has 8 V threads per inch. *Ans.* 0.125 in.; 0.108 in.

3. Find the pitch and the depth of thread of a 1-inch screw that has 8 square threads per inch. *Ans.* 0.125 in.; 0.063 in.

4. A $1\frac{1}{2}$-inch bolt carries 6 V threads per inch. It is required to find the following: (*a*) the pitch, (*b*) the depth of thread, (*c*) the root diameter, and (*d*) the root area.

Ans. (*a*) 0.167 in. (*b*) 0.144 in. (*c*) 1.211 in. (*d*) 1.152 sq. in.

5. According to Table VII, a 2-inch bolt carries $4\frac{1}{2}$ American (National) Standard coarse threads per inch. It is required to compute (*a*) the root diameter, (*b*) the root area, and (*c*) the body area of this bolt. *Ans.* (*a*) 1.711 in. (*b*) 2.30 sq. in. (*c*) 3.14 sq. in.

6. A $1\frac{1}{4}$-inch transmission screw has a square thread whose lead is $\frac{3}{4}$ inch. What are the pitch and root diameter if (*a*) the screw is single-threaded, (*b*) the screw is double-threaded, (*c*) the screw is triple-threaded?

Ans. (*a*) $\frac{3}{4}$ in.; $\frac{1}{2}$ in. (*b*) $\frac{3}{8}$ in.; $\frac{7}{8}$ in. (*c*) $\frac{1}{4}$ in.; 1 in.

7. How may the root diameter and hence the tensile strength of a square-threaded screw be increased without changing the lead of the screw? *Ans.* By the use of a multiple thread.

8. Is the height of a standard hexagonal nut for a given diameter of bolt more or less than the height of the corresponding standard hexagonal bolt-head? See Table VIII.

9. A bolt is loaded as in Fig. 48, with a tensile load of 10,000 pounds. This bolt is threaded with American standard coarse threads, as given in Table VII. The working tensile stress is 8000 pounds per square inch. (*a*) What is the theoretical root diameter? (*b*) What is the root diameter of the standard bolt

that must be selected? (*c*) What is the outside or nominal diameter of the bolt selected? (*d*) How many threads per inch does it carry? (*e*) What is its pitch? (*f*) What are its root and body areas? (*g*) If the ultimate tensile stress of the material of this bolt is 60,000 pounds per square inch, what factor of safety has been used in this design? (*h*) Is the bolt designed for dead, live, or shock loading? *Ans.* (*a*) 1.27 – in. (*b*) 1.284 in. (*c*) 1.5 in. (*d*) 6. (*e*) $\frac{1}{6}$ in. (*f*) 1.295 sq. in.; 1.767 sq. in. (*g*) 7.5.

10. A $1\frac{3}{8}$-inch bolt carries 6 V threads per inch. To what tensile load may it be subjected safely, if $S_t = 6000$ pounds per square inch? *Ans.* 5560 lb. nearly.

11. Two machine parts are fastened together tightly by means of a 2-inch American standard bolt. If the load tending to separate them is neglected, compute the unit working stress that is probably set up in this bolt by initial tightening. *Ans.* 13,900 lb. per sq. in.

12. A steam engine cylinder with a 12-inch diameter and a steam pressure of 125 lb. per sq. in. has its head secured thereto by 10 bolts. The allowable tensile stress in the bolts is taken as 3000 lb. per sq. in. (*a*) What is the total steam pressure exerted on the cylinder head? (*b*) What is the total steam pressure carried by each bolt? (*c*) Compute the root diameter of the bolts to be used. (*d*) What standard bolt should be selected from Table VII? *Ans.* (*a*) 14,150 lb. nearly. (*b*) 1415 lb. (*c*) 0.775 in. (*d*) 1 in.

13. What size of hole must be drilled in a standard $1\frac{3}{4}$-inch bolt to make the body area equal to the root area? Why is this sometimes done? *Ans.* 0.92 – in.

14. A standard $\frac{3}{4}$-inch bolt of nickel steel is subjected to a tensile load that sets up within it an induced tensile stress of 10,000 lb. per sq. in. (*a*) What tensile load is carried by the bolt? (*b*) What factor of safety is involved in tension if the ultimate tensile strength for its alloy steel is 100,000 lb. per sq. in.? (*c*) To what shearing load is the bolt-head subjected? (*d*) If a standard finished hexagonal bolt-head is used, what is the area in shear and the unit shearing stress induced therein? (*e*) What factor of safety is involved in shear of the bolt-head if the ultimate shearing strength is 75,000 lb. per sq. in.? *Ans.* (*a*) 3020 lb. (*b*) 10. (*c*) 3020 lb. (*d*) 1.325 sq. in.; 2280 lb. per sq. in. (*e*) 33 –.

15. What relation must exist between the tangent of the angle of a transmission screw thread and the coefficient of friction, (*a*) in order to have the screw self-locking, and (*b*) to have it possible for the axial load to drive the screw?

16. A square-threaded transmission screw, used in a jack, has a root diameter of $2\frac{1}{2}$ inches and a lead of $\frac{3}{4}$ inch. It is triple-threaded and is designed to lift a load of 12 tons. The coefficient of friction is assumed to be 0.15. (*a*) What is the pitch of this screw? (*b*) Compute its mean radius, *r*. (*c*) Find its efficiency, *e*. (*d*) Find the unit compressive stress in the screw (over the root area) in lb. per sq. in. (*e*) What is the unit shearing stress at the thread in lb. per sq. in. if there are 4 threads in the nut? (*f*) What is the bearing pressure in lb. per sq. in. of the projected area of the threads in contact? (*g*) Is the screw self-locking? *Ans.* (*a*) $\frac{1}{4}$ in. (*b*) $1\frac{5}{16}$ in. (*c*) 24% (*d*) 4900 lb. per sq. in. (*e*) 1750 lb. per sq. in. (*f*) 1680 lb. per sq. in. (*g*) Yes.

17. What horsepower can be safely transmitted by two $\frac{5}{8}$-inch set screws employed in a pulley installation if the rotative speed of the pulley is 250 revolutions per minute and the diameter of the shaft is $2\frac{1}{2}$ inches? *Ans.* 8.3 hp.

CHAPTER IV

CYLINDERS AND RIVETED JOINTS

Introduction. Pipes and cylindrical vessels in general are used in engineering practice to transmit and store liquids, vapors, and gases, which are as a rule under a considerable fluid pressure. Any fluid pressure is exerted equally in all directions and therefore plays upon the walls and ends of cylinders with equal intensity. If a fluid pressure within a cylinder is 50 pounds per square inch, every square inch of the inner surface of the cylinder is subjected to a force of 50 pounds, which is an internal loading that must be safely carried by the cylinder. The result is that this internal loading subjects certain sections of the cylinder wall to definite tensile loads. These sections in design then must be given areas of such magnitudes that they can safely resist these tensile loads. In other words, these involved areas must be large enough so that in setting up a tensile resistance equal to the total tensile load, each square inch of the section will be subjected to a safe working stress.

Extreme care in design must be exercised because rupture of a pressure vessel means an explosion with the accompanying possibility of loss of life and property.

Involved Sections of a Cylinder. The sections of the walls of a cylinder that are affected by an internal fluid pressure are two in number, namely,

1. The transverse section
2. The longitudinal section.

The transverse section of a cylinder is shown at M in Fig. 52 to be the section cut from a cylinder by a plane which is at right angles to the axis of the cylinder. Such a section is that which is generally called a right section. If we let

A_t = area of transverse section in square inches
D = inside diameter of the cylinder in inches
t = thickness of the cylinder wall in inches

then, $A_t = \pi D t$, approximately. **(73)**

The longitudinal section is shown at N in Fig. 53. This is a section cut from the cylinder by a plane that contains the axis. In

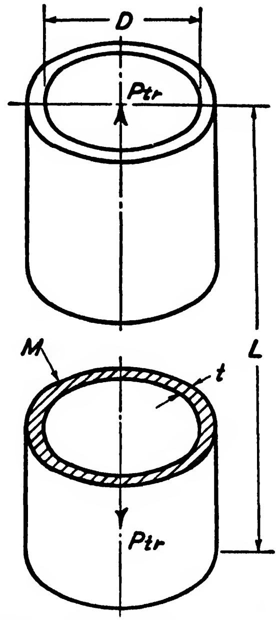


Fig. 52. Transverse Section of a Cylindrical Pressure Vessel

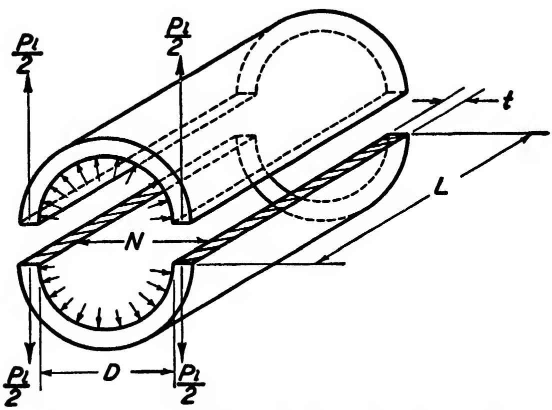


Fig. 53. Longitudinal Section of a Cylindrical Pressure Vessel

this text, we shall consider this plane to be a horizontal cutting plane as shown in Fig. 53. Then the section, N, is horizontally located and

a force acting perpendicular to it can be referred to as a vertical force. If we let

A_l = the area of the longitudinal section in square inches

L = the length of the cylinder in inches

with the other notation as used, we have,

$$A_l = 2tL \tag{74}$$

Stress in Transverse Section. It will be assumed that ends are affixed to the cylinder of Figs. 52 and 53, and that it is subjected from within to a fluid pressure of p lb. per sq. in. This pressure acting upon the end of the cylinder will produce a total load, P_{tr}, on the ends which is equal to the unit pressure p, multiplied by the area, $\frac{\pi D^2}{4}$, so that,

$$P_{tr} = p \times \frac{\pi D^2}{4} \tag{75}$$

This load, P_{tr}, is directed parallel to the axis of the cylinder and therefore perpendicular to all such sections like unto M, Fig. 52. Since the tendency is to elongate the cylinder parallel to its axis or elements, or rupture it at such an area as M, a tensile resistance, AS_t, is set up in the latter section equal to the load; for according to formula (5), $P = AS_t$

In this case $P = P_{tr}$, and

$A = A_t = \pi Dt$ (See formula 73).

Substituting these in the above, we have,

$$P_{tr} = A_t S_t = \pi Dt \times S_t$$

But from formula (75),

$$P_{tr} = p \times \frac{\pi D^2}{4},$$

$$p \times \frac{\pi D^2}{4} = \pi Dt \times S_t$$

Dividing both members of the equation by πDt

$$S_t = \frac{p \times \frac{\pi D^2}{4}}{\pi Dt} = \frac{p \times \pi D^2}{4\pi Dt}$$

Canceling like factors in the second member of the equation,

$$S_t = \frac{pD}{4t} \tag{76}$$

Stress in Longitudinal Section. Since the fluid pressure, p, acts

equally in all directions, the radial vectors of Fig. 53 show that there is a tendency to rupture the cylinder along the longitudinal section, shown cross-hatched in the figure. The tendency in this case is to lift one half of the cylinder off of the other half as pictured. The load, P_l, tending to produce such a rupture, acts perpendicular to the involved area, A_l, and hence is a tensile load. It is made up of the summation of the vertical parts or components of all the fluid pressure acting on one-half of the cylinder, and it can be proven that this summation gives

$$P_l = pLD \tag{77}$$

which is just as if a force of p lb. per sq. in. were working over the projected area of the cylinder.

(**Note.** A projected area of a cylinder will be mentioned again in this text. It is defined as the area of the projection of the cylinder upon a plane of projection taken parallel to the axis of the cylinder.)

Since the involved area in tension, A_l, is equal to $2tL$, (formula 74), one-half of which area is on each side of the cylinder, each half is subjected to the half-load, $\frac{P_l}{2}$, or the whole area is subjected to the whole load, P_l. Applying formula (5) to this work,

$$P_l = A_l\ S_t$$

Substituting the value of A_l taken from formula (74),

$$P_l = 2tLS_t$$

Now substituting the value of P given in formula (77), we have

$$pLD = 2tLS_t$$

Dividing both members of the above equation by the coefficient of S_t,

$$S_t = \frac{pLD}{2tL}$$

$$= \frac{pD}{2t} \tag{78}$$

Design of Thin Cylinders. From a comparison of formulas (76) and (78), several things may be brought out. The length of the cylinder is not involved in either formula and hence has no part to play in the determination of the stress in either the transverse or longitudinal sections. Furthermore, it is seen that in a given cylinder, the stress in the transverse section is only one-half as large as the stress in the longitudinal section. In other words, the transverse section is twice as strong as the longitudinal, so that the latter section

dictates the thickness of cylinder wall. To obtain a design formula for t, it will be necessary to solve formula (78) for t. This gives

$$t=\frac{pD}{2S_t} \tag{79}$$

In the derivation of the above formula, the stress was assumed to be uniformly distributed over the involved area. This is not the case. Instead, the stress is a maximum on the inside, and becomes progressively smaller toward the outside of the wall. However, if the wall is rather thin in comparison to the diameter of the cylinder, the assumption as made is consistent. For thick-walled cylinders as in the case of cylinders for hydraulic presses, another formula derived experimentally will be introduced later on.

The factor of safety to be used in the determination of the stress, S_t, for formula (79) depends upon a good many considerations such as the type of loading, the possibility of water hammer which creates a sudden increase in pressure, stresses set up in handling or transporting, thickness of wall, code requirements, etc. A factor of safety of 4 is probably as low as should ever be used. In the design of engine cylinders, it is well to add around $\frac{1}{4}$ to $\frac{1}{2}$ inch to the theoretical value obtained with formula (79) to permit reboring after wear has taken place.

In constructing large pressure vessels like steam boilers, several steel plates may be used which necessitates the use of riveted joints or welded joints in joining together the ends of the plates. When riveted joints are used, formula (79) is modified to include the efficiency, e, of the joint and becomes

$$t=\frac{pD}{2S_te} \tag{80}$$

The efficiency of a joint is the ratio of the strength of the joint to the strength of the plate. By including the efficiency as shown, formula (80) provides a plate that is enough heavier to allow the joint to possess the strength of a plate thickness provided by formula (79). In designing steam boilers, the result obtained by using formula (80) should be compared with the minimum plate thicknesses as set forth in the A. S. M. E. (American Society of Mechanical Engineers) Boiler Code. Should the calculated result be less than the code requirement, the latter should be taken in its place; otherwise use the calculated value. The plate thicknesses of the Boiler Code are given in Table X.

TABLE X

Boiler Diameter	Minimum Value of t
36 in. or less	$\frac{1}{4}$ in.
37 to 54 in.	$\frac{5}{16}$ in.
55 to 72 in.	$\frac{3}{8}$ in.
Over 72 in.	$\frac{1}{2}$ in.

This code also states that the factor of safety shall be at least 5 and the steel of the plates and rivets used shall have as a minimum the following ultimate stresses:

$$U_t = 55{,}000 \text{ lb. per sq. in.}$$
$$U_c = 95{,}000 \text{ lb. per sq. in.}$$
$$U_s = 44{,}000 \text{ lb. per sq. in.}$$

It is well that the student keep in mind that it is necessary for the designer to be aware of existent codes and standards and to see that his design is always consistent therewith.

Example. A cast iron steam engine cylinder is 12 inches in diameter. The maximum steam pressure occurring in the cylinder is 150 lb. per sq. in. Assuming a varying load, what thickness of cylinder wall should be used?

Solution. From Table I, $U_t = 20{,}000$ lb. per sq. in. and from Table II, let F be selected $= 8$.

Then since $$S_t = \frac{U_t}{8}$$

$$S_t = \frac{20{,}000}{8} = 2500 \text{ lb. per sq. in.}$$

Using this value for S_t, and with the statement of the example giving $p = 150$ lb. per sq. in. and $D = 12$ in., we have from formula (79),

$$t = \frac{pD}{2S_t}$$

$$= \frac{150 \times 12}{2 \times 2500} = 0.36 \text{ in., say } \tfrac{3}{8} \text{ in.}$$

Adding to this theoretical value, an amount equal to $\frac{3}{8}$ in. for reboring, counterboring, etc., the thickness becomes $\frac{3}{8}$ in. $+ \frac{3}{8}$ in. $=$ $\frac{3}{4}$ in. *Ans.*

Example. A steam boiler, 48 inches in diameter, generates

steam at a gauge pressure of 100 lb. per sq. in. Find the thickness of shell, letting $U_t = 55{,}000$ lb. per sq. in. and $F = 5$. Assume the efficiency of the riveted joints used as 75 per cent.

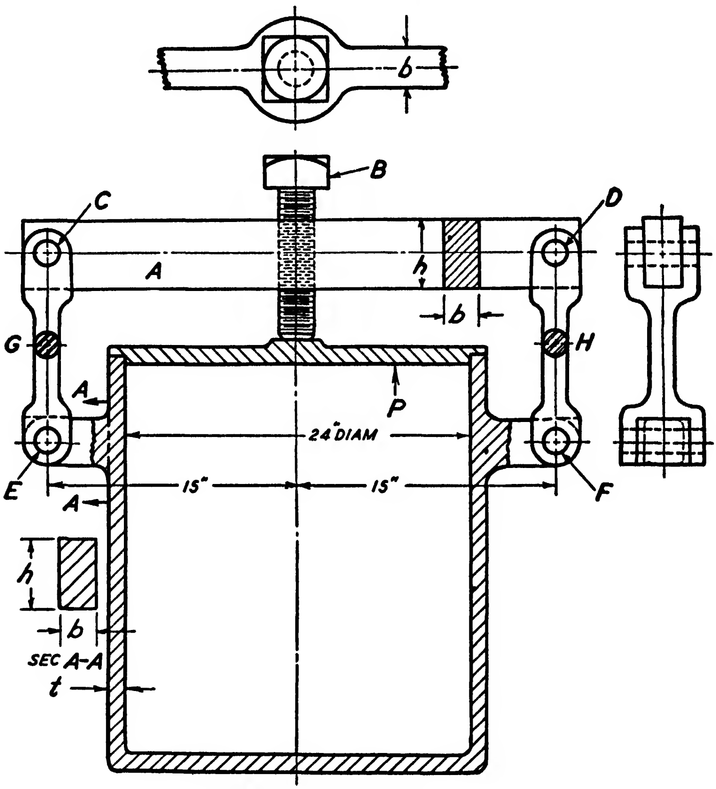


Fig. 54. Pressure Vessel

Solution. Here $D = 48$ in., $p = 100$ lb. per sq. in., $e = 0.75$, and from formula (6), $S_t = \dfrac{55{,}000}{5} = 11{,}000$ lb. per sq. in.

Substituting these values in formula (80),

$$t = \frac{pD}{2S_t e}$$

$$= \frac{100 \times 48}{2 \times 11{,}000 \times 0.75} = 0.291 \text{ in., say } \tfrac{5}{16} \text{ in.} \quad \textit{Ans.}$$

Comparing this answer with that recommended in Table X proves it to be acceptable.

Example. A pressure vessel as shown in Fig. 54 is used as a

digester in a chemical process. It is designed to withstand a fluid pressure of 30 lb. per sq. in. The vessel and its cover are made of cast iron. All other links are of steel. The cover is held tightly against the vessel by means of a screw, *B*, which is turned down through the tapped hole in the beam. *A*, so that the end of the screw presses firmly against the cover. The beam, *A*, has a rectangular section in which *h*, perpendicular to the neutral axis, is equal to $2b$. The rectangular section is opened up at the center to take the tapped hole, as shown in the figure. The beam is attached by pins, *C* and *D*, to the links, *G* and *H*, which are secured by pins, *E* and *F*, to the extensions cast onto the vessel. If S_t for cast iron$=2500$ lb. per sq. in., S_t for steel$=7500$ lb. per sq. in., S_c for steel$=7500$ lb. per sq. in., S_s for steel$=6000$ lb. per sq. in., it is required to find: (*a*) the thickness, *t*, of cylinder wall; (*b*) the total pressure, *P*, on the cover; (*c*) the diameter of the screw, *B*; (*d*) the solid rectangular section of the beam, *A*; (*e*) the section of the beam at the tapped hole; (*f*) the diameters of the pins *C* and *D*; (*g*) the diameters of the pins *E* and *F*; (*h*) the diameters of the links *G* and *H*; (*i*) the dimensions at the wall of the supports of pins *E* and *F*, assuming a rectangular section with $h=2b$.

Solution. (*a*) Here $S_t=2500$ lb. per sq. in., $D=24$ in., and $p=30$ lb. per sq. in.
Using formula (79),

$$t=\frac{pD}{2S_t}$$

$$=\frac{30\times 24}{2\times 2500}=0.144 \text{ in., say } \tfrac{1}{4} \text{ in. at least.} \quad \textit{Ans.}$$

(*b*) The area of the cover$=\dfrac{\pi D^2}{4}$

$$=\frac{\pi\times 24^2}{4}=452.4 \text{ sq. in.}$$

The total force on cover, $P=452.4\times p$

$$=452.4\times 30=13{,}570+ \text{ lb.} \quad \textit{Ans.}$$

(*c*) The screw, *B*, is under compression due to the load, *P*. Let d=root diameter of the screw. From the example, we have $P=13{,}570$ lb. and $S_c=7500$ lb. per sq. in. Using formula (9), $P=AS_c$, and evaluating therein,

$$13{,}570=\frac{\pi d^2}{4}\times 7500$$

Multiplying by 4 and dividing by 7500π,

$$d^2=\frac{13{,}570\times 4}{7500\times\pi}=2.3$$

$$d=\sqrt{2.3}=1.52 \text{ in.}$$

Selecting the next higher standard screw from Table VII, we obtain a root diameter = 1.7113 in. and an outside diameter = 2 in. *Ans.*

(*d*) This beam, *A*, is a simple beam with a single concentrated load at its center. From the symmetry of loading, the reaction at *C*, R_c, = the reaction at *D*, R_d; thus $R_c=R_d=\frac{P}{2}=\frac{13{,}570}{2}=6785$ lb.

The dangerous section and the maximum bending moment, *M*, are at the mid-point of the beam and

$$M=\frac{P}{2}\times 15=\frac{13{,}570}{2}\times 15=101{,}700 \text{ in.-lb.}$$

It is given that the section is rectangular with $h=2b$. From Table III, $Z=\frac{bh^2}{6}=\frac{b\times 4b^2}{6}=\frac{2b^3}{3}$

Applying formula (27), with the working stress = 7500 lb. per sq. in., $M=SZ$

$$101{,}700=7500\times\frac{2b^3}{3}$$

$$b^3=\frac{101{,}700\times 3}{2\times 7500}=20.3$$

$$b=\sqrt[3]{20.3}=2.72 \text{ in., say } 2\tfrac{3}{4} \text{ in.} \quad \textit{Ans.}$$

$$\therefore\ h=2b=2\times 2\tfrac{3}{4}=5\tfrac{1}{2} \text{ in.} \quad \textit{Ans.}$$

(**Note.** The tapped hole dictates the number of threads in shear on the screw. Since the depth of hole is greater than the height of a standard nut for this diameter of screw, the latter is very safe in shearing of its threads.)

(*e*) Since the axis of the tapped hole is parallel to the *h* dimension of the section, the solid rectangular section of the beam can be split into two halves at the hole or opened up for the hole, so that the metallic area in a section through the hole is equal to area of the solid section obtained in part *d*. The factor of safety will remain

unchanged. As the diameter of the hole is 2 in., this section will then have dimensions of $(2\frac{3}{4}+2)$ in. by $5\frac{1}{2}$ in. or $4\frac{3}{4}$ in. by $5\frac{1}{2}$ in. *Ans.*

(*f*) The pins, C and D, resist equal loads in shear, since their loads are the reactions of the beam, and $R_c = R_d$. The forked ends of the links, G and H, indicate that the pins are in double shear. If we let d represent the diameter of these pins, then A, the area of each pin in shear, $=\frac{2\times\pi d^2}{4}=\frac{\pi d^2}{2}$ sq. in.

Since the load is equal to R_c or R_d, it has a value of 6785 lb. $S_s = 6000$ lb. per sq. in. Substituting these values in formula (13), we have,

$$6785=\frac{\pi d^2}{2}\times 6000$$

$$d^2=\frac{6785\times 2}{6000\times\pi}=0.719$$

$$d=\sqrt{0.719}=0.8478 \text{ in., say } \tfrac{27}{32} \text{ in. or } \tfrac{7}{8} \text{ in.} \quad \textit{Ans.}$$

(*g*) The reactions of the beam, A, place links G and H under tension. Therefore pins E and F are subjected to the same shearing loads as pins C and D. Hence the diameter of E and F will equal the diameter of C and $D=\frac{27}{32}$ in. or $\frac{7}{8}$ in. *Ans.*

(*h*) The links G and H, in tension, are shown to have circular cross sections. If we now let d represent these diameters, the area of each $=\frac{\pi d^2}{4}$, the tensile load on each link $= R_c$ or $R_d = 6785$ lb. and $S_t = 7500$ lb. per sq. in. Applying formula (5),

$$6785=\frac{\pi d^2}{4}\times 7500$$

$$d^2=\frac{6785\times 4}{7500\times\pi}=1.150$$

$$d=\sqrt{1.15}=1.08 \text{ in., say } 1\tfrac{1}{8} \text{ in.} \quad \textit{Ans.}$$

(*i*) The extensions or brackets on the vessel are part of that casting, and both act as cantilevers. They are of the same length and sustain equal end loads. Hence their sections at their juncture with the vessel become the dangerous sections (for with all cantilevers the dangerous section is at the support) and their dimensions will be identical.

Length of cantilever from center line of pin to wall $=15-(12+\frac{1}{4})$

$=2\frac{3}{4}$ in. End load of cantilever is equal to the shearing load on the pin $=6785$ lb.
Therefore from formula (23),

$$M=6785\times2\tfrac{3}{4}=18{,}700-\text{ in.-lb.}$$

From Table III, $Z=\frac{bh^2}{6}$, Since $h=2b$,

$$Z=\frac{b\times4b^2}{6}=\frac{2b^3}{3}$$

Substituting our known values in the beam design formula (27), along with the bending stress for cast iron $=2500$ lb. per sq. in., we have,

$$18{,}700=2500\times\frac{2b^3}{3}$$

$$b^3=\frac{18{,}700\times3}{2500\times2}=11.2$$

$$b=\sqrt[3]{11.2}=2.23\text{ in., say }2\tfrac{1}{4}\text{ in.}\quad \textit{Ans.}$$

$$\therefore\ h=2b=2\times2\tfrac{1}{4}=4\tfrac{1}{2}\text{ in.}\quad \textit{Ans.}$$

Example. A standard 10-inch wrought-iron pipe has an inside diameter of 10.192 inches and an outside diameter of 10.750 inches. What unit tensile stress is created by an internal pressure of 200 pounds per square inch?

Solution. The thickness of pipe wall,

$$t=\frac{10.750-10.192}{2}=0.279\text{ in.}$$

Here $p=200$ lb. per sq. in. and $D=10.192$ in.
Substituting in formula (78),

$$S_t=\frac{pD}{2t}$$

$$=\frac{200\times10.192}{2\times0.279}=3660\text{ lb. per sq. in.}\quad \textit{Ans.}$$

Design of Thick Cylinders. When a cylinder is subjected to a very high internal fluid pressure, the walls of the cylinder must be extremely heavy or thick. In such a case the stress, as previously stated, cannot be assumed to be uniformly distributed over the involved area. This makes it impossible to use the rational formulas already derived for thin-walled cylinders. Bach, among others, has experimented along this line and from his experiments deduced the following empirical formula:

$$t=\frac{D}{2}\left(\sqrt{\frac{S_t+0.4p}{S_t-1.3p}}-1\right) \tag{81}$$

in which t = thickness of wall in inches

D = inside diameter in inches

p = internal fluid pressure in lb. per sq. in.

and S_t = safe tensile stress in lb. per sq. in.

Example. The pressure within the cylinder of a hydraulic press is 1000 pounds per square inch. The diameter of the cylinder is 15 inches. Determine the thickness of the cylinder, allowing a tensile stress of 2500 pounds per square inch.

Solution. Here $D=15$ in., $p=1000$ lb. per sq. in., $S_t=2500$ lb. per sq. in.

Substituting in formula (81), we have,

$$t=\frac{15}{2}\left(\sqrt{\frac{2500+0.4\times1000}{2500-1.3\times1000}}-1\right)$$

$$t=\frac{15}{2}\left(\sqrt{\frac{2900}{1200}}-1\right)$$

$$=7.5\times0.56=4.2 \text{ in., say } 4\tfrac{1}{4} \text{ in.} \quad \textit{Ans.}$$

Size of Pipe for Quantity of Fluid Carried. The design of a pipe not only involves the selection of a correct thickness of wall to stand up under the internal fluid pressure, it is also necessary to inquire into and determine the inside diameter of the pipe that must be used to deliver or convey a certain quantity of fluid per unit of time. Obviously this determination of the inside diameter must come first so that it may be used in finding the thickness or in checking the thickness of an assumed grade of pipe for its unit tensile stress.

Let A = inside area of the pipe in square inches

D = inside diameter of the pipe in inches

V = velocity of flow in feet per minute

Q = quantity of fluid carried in cubic feet per minute.

Now the volume or quantity of fluid, Q, passing any transverse section of the pipe in one minute would flow beyond that particular section and fill the pipe for a distance equal to the magnitude of its velocity. Hence in one minute, it would fill a length equal to V feet. Therefore the volume of the portion of the pipe that is filled by Q cubic feet per minute would be Q cubic feet and would also be the volume of a cylinder V feet in length, having a base area equal to

the inside area of a transverse section of the pipe, $\frac{\pi D^2}{4\times 144}$ square feet. Since in general the volume of any cylinder is equal to the product of its base and altitude, the preceding statement gives us the following,

$$Q=\frac{\pi D^2}{4\times 144}\times V$$

Multiplying both members of the above equation by 4×144 and dividing by $\pi\times V$, we have,

$$D^2=\frac{4\times 144\times Q}{\pi\times V}$$

Solving the above for D and then for Q,

$$D^2=183.35\times\frac{Q}{V}$$

Extracting the square root of each member,

$$D=\sqrt{183.35}\times\sqrt{\frac{Q}{V}}$$

$$=13.54\sqrt{\frac{Q}{V}} \tag{82}$$

and

$$Q=\frac{D^2V}{183.35} \tag{83}$$

Commercial Piping. Commercial pipe met with in design is generally made of wrought iron, steel, cast iron, or brass. Wrought iron and steel are used chiefly for conveying steam, air, and oil. Cast-iron pipe finds its use mainly in water and sewage systems. Brass pipe is not liable to corrosion. It is made up and threaded to the same standards as wrought iron and steel pipe. In small sizes it finds use in pressure lubrication systems on prime movers (heat engines).

Wrought-iron and steel pipes are obtainable in three different grades: (1) Standard, (2) Extra Strong, and (3) Double Extra Strong. Tables XI and XII give the dimensions in inches of the standard and extra strong pipes respectively, showing the so-called nominal sizes from $\frac{1}{8}$ inch to 12 inches. Notice in these tables that the outside diameters of standard and extra strong pipes are the same for any given size but that the inside diameters are different. This holds true for the double extra strong pipe as well. Above 12 inches, the size of pipes is based on the outside diameter. Such pipe is known as

TABLE XI
Dimensions of Standard Wrought-Iron and Steel Pipe

Size	Diameters		Thickness	Areas		Weight per Foot	Threads per Inch
	Exterior	Interior		Exterior	Interior		
.125	.405	.269	.068	.129	.057	.244	27
.25	.540	.364	.088	.229	.104	.424	18
.375	.675	.493	.091	.358	.191	.567	18
.5	.840	.622	.109	.554	.304	.850	14
.75	1.050	.824	.113	.866	.533	1.130	14
1.	1.315	1.049	.133	1.358	.864	1.678	11.5
1.25	1.660	1.380	.140	2.164	1.495	2.272	11.5
1.5	1.900	1.610	.145	2.835	2.036	2.717	11.5
2.	2.375	2 067	.154	4.430	3.355	3 652	11.5
2.5	2.875	2.469	.203	6.492	4.788	5.793	8
3.	3.500	3.068	.216	9.261	7 393	7 575	8
3.5	4.000	3.548	.226	12.566	9 886	9 109	8
4.	4.500	4.026	.237	15.904	12.730	10 790	8
5.	5.563	5.047	.258	24 306	20 006	14 617	8
6.	6.625	6.065	.280	34.472	28.891	18.974	8
8.	8 625	8 071	.277	58.426	51 161	24.696	8
8.	8.625	7.981	.322	58 426	50.027	28.554	8
10.	10.750	10.192	.279	90.763	81.585	31 201	8
10.	10 750	10 136	.307	90 763	80 691	34 240	8
10.	10.750	10.020	.365	90.763	78.855	40 483	8
12.	12 750	12 090	.330	127 676	114 800	43 773	8
12.	12 750	12 000	.375	127.676	113.097	49.562	8

Courtesy of Crane Company

TABLE XII
Dimensions of Extra Strong Wrought-Iron and Steel Pipe

Size	Diameters		Thickness	Areas		Weight per Foot	Threads per Inch
	Exterior	Interior		Exterior	Interior		
.125	.405	.215	.095	.129	.036	.314	27
.25	.540	302	.119	.229	.072	.535	18
.375	.675	.423	.126	.358	.141	.738	18
.5	.840	.546	.147	.554	.234	1.087	14
.75	1.050	.742	.154	.866	.433	1.473	14
1.	1.315	.957	.179	1.358	.719	2.171	11.5
1.25	1.660	1.278	.191	2 164	1 283	2.996	11 5
1.5	1.900	1.500	.200	2 835	1.767	3 631	11.5
2	2.375	1 939	.218	4 430	2 953	5 022	11.5
2.5	2 875	2.323	.276	6 492	4.238	7.661	8
3.	3 500	2.900	.300	9 621	6.605	10 252	8
3.5	4.000	3.364	.318	12 566	8 888	12.505	8
4.	4 500	3.826	.337	15 904	11 497	14.983	8
5.	5 563	4 813	.375	24.306	18 194	20.778	8
6.	6.625	5 761	.432	34.472	26.067	28.573	8
8.	8.625	7 625	.500	58 426	45 663	43.388	8
10.	10.750	9.750	.500	90 763	74.662	54.735	8
12.	12.750	11.750	.500	127.676	108.434	65.415	8

Courtesy of Crane Company

O.D. pipe and in specifying such, the outside diameter and thickness must be given.

Piping comes as a rule in lengths from 16 to 20 feet. It may or may not be threaded. The standard system of threads in use with pipes is the Briggs system.

Example. Find the diameter of standard steam pipe that is required to carry 700 cubic feet of steam per minute at a velocity of 6000 feet per minute.

Solution. Here $Q=700$ cu. ft. per min. and $V=6000$ f.p.m.

Using formula (82),

$$D=13.54\sqrt{\frac{Q}{V}}$$

$$=13.54\times\sqrt{\frac{700}{6000}}=13.54\sqrt{0.117}$$

$$=13.54\times0.342=4.64 \text{ in.}$$

Consulting Table XI, the commercial size that must be used is a 5-in. pipe. *Ans.*

Example. How many cubic feet of steam can be delivered by a standard 4-inch pipe if the velocity is 9000 feet per minute?

Solution. From Table XI, the inside diameter, D, of a 4-inch pipe$=4.026$ in. and it is given that $V=9000$ f.p.m.

Using formula (83),

$$Q=\frac{D^2V}{183.35}$$

$$=\frac{4.026^2\times9000}{183.35}=800 \text{ cu. ft. nearly.} \quad \textit{Ans.}$$

The Riveted Joint. When two pieces of metal are to be permanently fastened together, the riveted joint may be used. Its use is common in the construction of boilers, and other pressure vessels, steel chimneys (stacks), tanks, and structural work. While its field of usefulness has been rightfully encroached upon by welding, it still remains for many cases the best and simplest type of permanent fastening.

Rivets. Rivets are formed from round iron or mild steel bars, the latter being more generally used today. Each rivet consists of a head and a cylindrical shank. The head may be one of many shapes, several of which are shown and dimensioned in Fig. 55. The Button and Steeple heads are the ones more generally used unless a flush surface is desired, in which event the Countersunk head is employed. The shank of a rivet is the part below the head and is slightly tapered at the end. The Point of a rivet is formed from this tapered end.

When two plates are to be fastened together by a rivet as in Fig. 56, the holes in the plates through which the rivet is to pass may

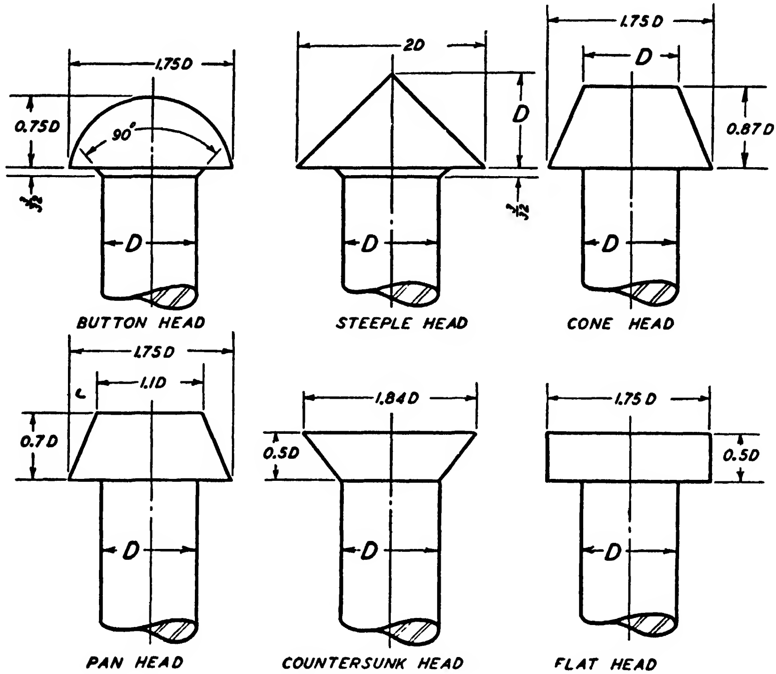


Fig. 55. Rivet Heads

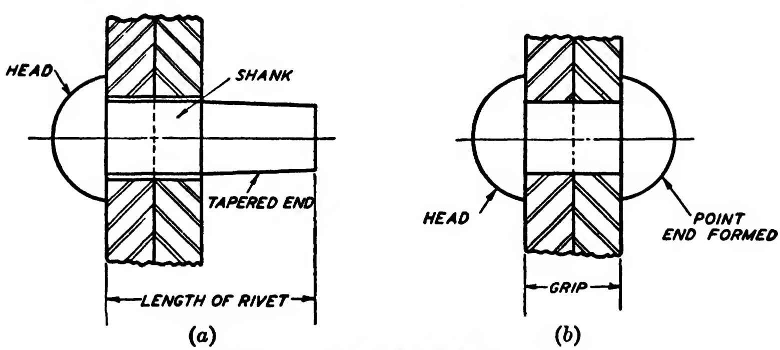


Fig. 56. Rivet before and after Being Driven

be punched, punched and reamed, or drilled. Punching is the cheapest method and is used as a rule for relatively thin plates and in structural work. Punching, however, injures the material around the hole and

hence drilling, although a more expensive operation, is used in most pressure-vessel work. The hole in Fig. 56 (*a*) is shown drilled to a diameter $\frac{1}{32}$ inch to $\frac{1}{16}$ inch larger than the rivet diameter. The plates are drilled while in position and then separated to remove any burrs or chips resulting from the drilling operation so as to have a tight flush joint between the plates. The red-hot rivet is introduced into the hole and the point is then formed. This is done sometimes by hand but more often by a riveting machine which either hammers or presses the point onto the rivet, and at the same time causes the shank to expand thus filling the hole. As the rivet cools, it tends to

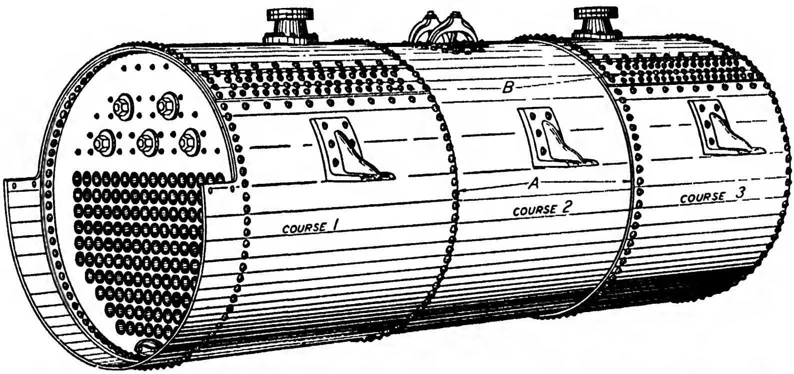


Fig. 57. Multitubular Boiler Showing the Use of Lap and Butt Joints in Its Three Courses

contract. This introduces into the rivet a force acting along its axis, which force holds the plates firmly together.

Rivets are manufactured in sizes which increase by $\frac{1}{16}$ inch in diameter from the $\frac{3}{8}$-inch rivet to the 1-inch rivet and by $\frac{1}{8}$ inch in diameter from the 1-inch rivet to the $1\frac{1}{2}$-inch rivet.

Types of Riveted Joints. The types of riveted joints to be discussed in this text are those commonly referred to as boiler joints. The fundamental principles and analyses of joints for all usages are similar, and consequently the material presented here becomes quite general.

Boiler joints can be classified under two general headings; namely, Lap joints and Butt joints. The former type is formed when two plates are lapped over each other and secured together by one or more rows of rivets passing through each plate. When one row of rivets is used, the joint is said to be a Single-Riveted Lap Joint; with

two rows of rivets it becomes a Double-Riveted Lap Joint; with three rows, a Triple-Riveted Lap Joint. Lap joints are employed on boilers of considerable length to provide circumferential or girth seams at the junctions of the several different plates or courses of the boiler, as at *A*, Fig. 57. A transverse section of the boiler through this girth

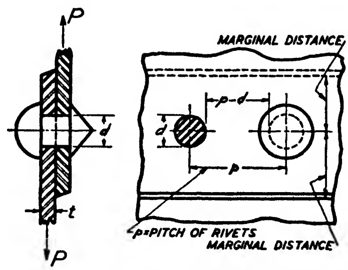


Fig 58 Single-Riveted Lap Joint

seam is shown in Fig. 58. Double-riveted lap joints are shown in Figs. 59 and 60. If in either a lap or butt joint as in the lap joint of Fig. 59, the rivets in the various rows are opposite each other, the riveting is called Chain Riveting. On the other hand if the riveting is staggered as in the lap joint of Fig. 60, it is called Zigzag Riveting.

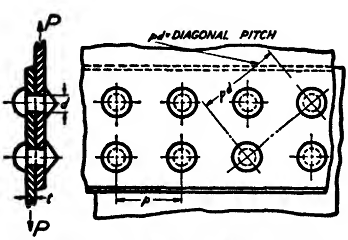


Fig. 59. Double-Riveted Lap Joint, Chain Riveting

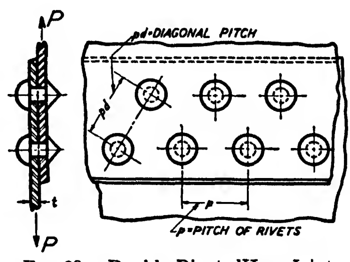


Fig. 60. Double-Riveted Lap Joint, Zigzag Riveting

The plates of a given course are bent into a cylindrical form and their ends are brought together so that they butt against each other. Strips of metal called butt straps or cover plates are placed on each side of them and the former are riveted to each other and to the latter. Such a joint is called a Butt Joint. The joint is said to be Single-Riveted, Double-Riveted, Triple-Riveted, or Quadruple-Riveted depending on whether there are one, two, three, or four rows of rivets respectively going through each plate. A double-riveted butt joint

is shown in Fig. 61. A triple-riveted butt joint forms the longitudinal seams at *B*, Fig. 57. Such a triple-riveted butt joint with zigzag riveting is shown in section in Fig. 62.

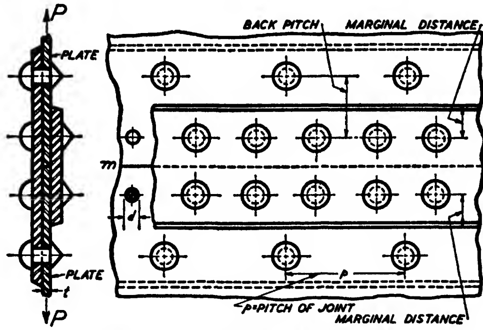


Fig. 61. Double-Riveted Butt Joint

Calking. To secure a leakless or fluid-tight joint, a process known as Calking is employed. In this process, the edge of one plate is

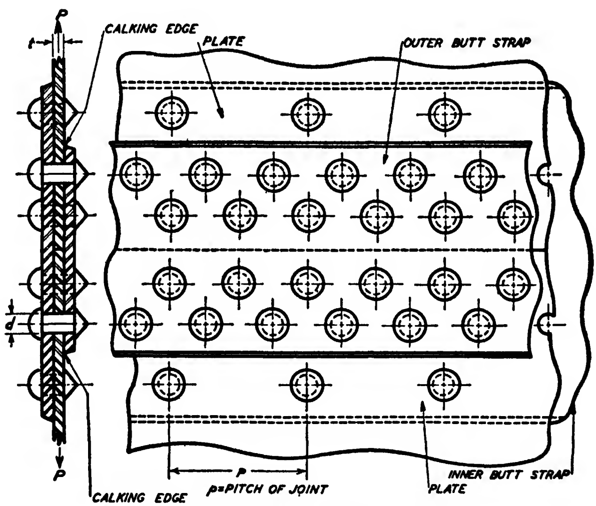


Fig. 62. Triple-Riveted Butt Joint

pressed tightly against the plate upon which it rests by means of a round nose tool or calking chisel as shown in Fig. 63 (also see Fig. 62). The outer edge (or edges) to be calked is beveled to an angle of not less than 70 degrees. The calking is done either by hand or by the

use of a pneumatic or hydraulic machine. Great care must be exercised to prevent injury to the plate with the accompanying weakening that would take place. The A.S.M.E. Boiler Code permits the use of fusion welding in the place of calking in unfired pressure vessels.

Size of Rivet. Practice has shown and dictated that the diameter of a rivet to be used with a given plate should depend on the thickness of the plate. Since, when the riveted joint is formed, the upset rivet fills the hole, the diameter of the rivet is taken equal to the diameter of the hole. This will be noted during the discussion of the strength of riveted joints later on in this chapter.

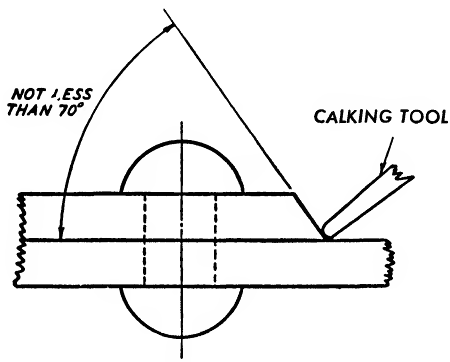


Fig 63 Calking

The diameter of the hole (or rivet) may be obtained from Table XIII where it is given together with the strap thickness for various thicknesses of plate, or it may be obtained by using the following formula:

$$d = 1.3\sqrt{t} \tag{84}$$

where d = the diameter of hole or upset rivet in inches
and t = the thickness of plate in inches.

Example. Find the diameter of rivet hole to be used in a riveted joint if the plate thickness is $\frac{3}{8}$ inch.

Solution. Substituting $\frac{3}{8}$ in. in formula (84) for t, we have,

$$d = 1.3\sqrt{t}$$

$$= 1.3\sqrt{\tfrac{3}{8}} = 1.3\sqrt{0.375} = 0.80- \text{ in., say } \tfrac{13}{16} \text{ in.} \quad \textit{Ans.}$$

(**Note.** It will be seen in this case that the value of d corresponds exactly with that recommended by the Hartford Steam Boiler Inspec-

TABLE XIII

Plate Thickness	Strap Thickness	Hole
1/4	1/4	11/16
9/32		
5/16	9/32	13/16
11/32		
3/8	5/16	
13/32		
7/16	3/8	15/16
15/32		
1/2	7/16	
17/32		
9/16		1 1/16
19/32	1/2	
5/8		
21/32		
11/16		1 3/16
23/32		
3/4		

Plate Thickness	Strap Thickness	Hole
25/32	9/16	1 5/16
13/16		
27/32		
7/8	5/8	
29/32		
15/16	11/16	
31/32		
1	3/4	1 7/16
1 1/32		
1 1/16		
1 3/32		
1 1/8		
1 5/32		
1 3/16	13/16	
1 7/32		
1 1/4	7/8	

(Hartford Steam Boiler Inspection and Insurance Company)

tion and Insurance Company as given in Table XIII. If, as previously stated in this chapter, a clearance of $\frac{1}{16}$ inch is allowed, the actual diameter of rivet will equal $\frac{13}{16}$ inch minus $\frac{1}{16}$ inch or $\frac{3}{4}$ inch, which is in agreement with standard practice.)

The Pitch of the Joint and the Unit Strip. The pitch of a row of rivets is the distance from the center line of one rivet to the center line of the next adjacent rivet of the same row. In lap joints, (see Figs. 59 and 60) it is generally the same in all rows, while in butt joints (see Figs. 61 and 62) the pitch of the rivets in the outside rows is generally twice that in the inside rows. The Pitch of the Joint, p, (see Figs. 58 to 62 inclusive) is the pitch of the rivets in that row where the pitch is a maximum; hence the pitch of the joint is the distance between the center lines of adjacent rivets in the outermost row.

A Unit Strip of a riveted joint is a strip whose length is equal to the pitch of the joint. Its length then is given by the dimension, p, in all of the figures illustrating riveted joints in this chapter. It is evident that a riveted joint is composed of several such recurring identical strips and that the strength of the entire joint bears the same relationship or ratio to the strength of the entire solid plate, that the strength of a unit strip of the joint bears to that of a unit

strip of the plate (that is, a strip of the plate without holes and with a length equal to p). This ratio has already been defined as the Efficiency of the Joint. Therefore in determining the latter, a unit strip of the joint need only be considered.

Diagonal Pitch, Back Pitch, and Marginal Distance. The Diagonal Pitch, p_d, (see Figs. 59 and 60) of a riveted joint is the distance from the center of one rivet to that of the nearest rivet to it, diagonally, in the next row. The Back Pitch (see Fig. 61) is the distance between the center lines of adjacent rows of rivets. The Marginal Distance (see Figs. 58 and 61) is the distance from the edge of the plate or strap to the center line of the adjacent row of rivets.

Possibilities of Failure of Riveted Joints. As we have seen, the internal fluid pressure of such a pressure vessel as the boiler of Fig. 57 tends to elongate and at the same time expand the vessel. The former is a tendency to pull one course away from the other in a direction along the axis of the boiler. This is resisted by the lap joint at the circumferential or girth seam. The latter is a tendency to separate the ends of a plate. This is resisted by the butt joint at the longitudinal seam. The loads thus created by the internal fluid pressure upon the various joints bring about the following possibilities of failure in them:

1. Tension, or tearing of the plate along the center line of a row of rivets as in Fig. 64 (*a*).

2. Shearing of the rivets as in Fig. 64 (*b*), which is "single shear" since only one area per rivet is in shear; or shearing of the rivets as in Fig. 64 (*c*), which is "double shear" for two areas of the same rivet are in shear.

3. Crushing of the plate in front of the rivet, or crushing of the rivet as in Fig. 64 (*d*).

4. Tearing of the plate in front of the rivet as in Fig. 64 (*e*).

5. Shearing of the plate in front of the rivet as in Fig. 64 (*f*).

It will be noticed that in each of the above, one or another of three simple stresses, tension, compression, or shear, is involved. Hence formula (1), $P=AS$, the general formula for these stresses, is used and it remains necessary for the joint to be so designed that it can set up a safe resistance, AS, in either tension, compression, or shear, that is equal to the maximum load, P, carried by the joint. Since such a resistance depends upon an area, A, in every case, and

the areas in methods *4* and *5* depend upon the marginal distance, possibility of failure in these two ways can be eliminated by making

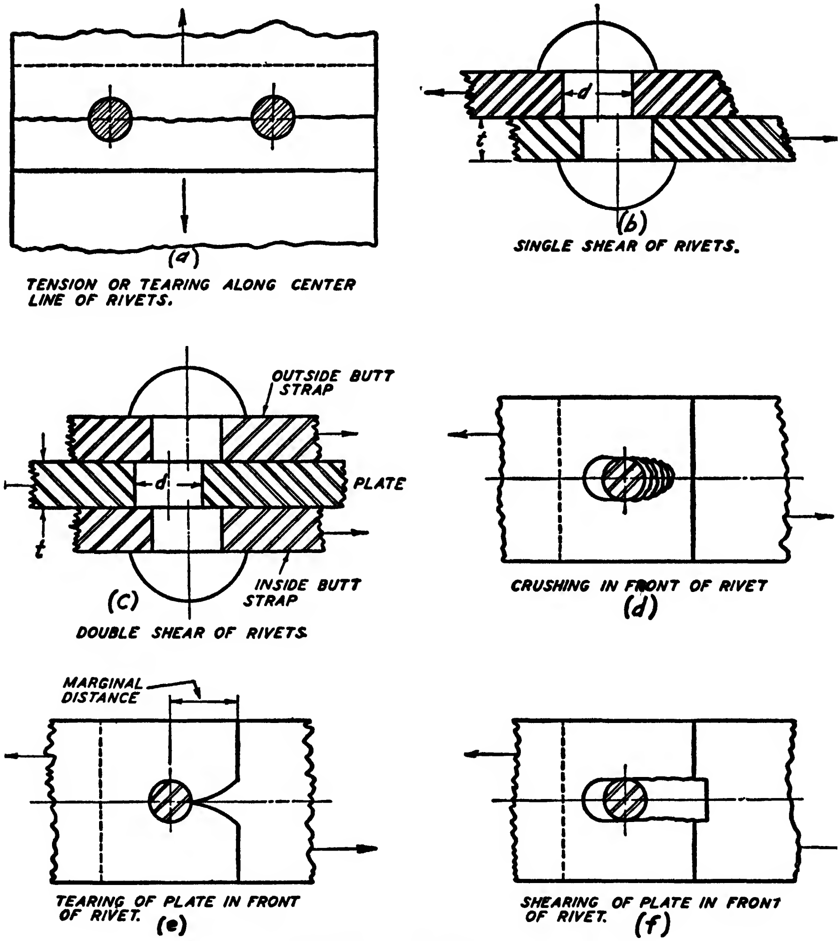


Fig. 64. Possibilities of Failure of a Riveted Joint

the marginal distance [see Figs. 58, 61, and 64 (*e*)] equal to $1\frac{1}{2}$ to $1\frac{3}{4}$ times the diameter, *d*, of the rivet hole. This leaves only the first three methods of failure for the simple types of joints. In the more complex types of joints, failure may occur by these same three methods or by a combination of them. In determining the resistance

a unit strip of the joint is considered and the following notation used:

p=pitch of the joint in inches

d=diameter of rivet hole in inches

t=thickness of plate in inches

t'=thickness of strap in inches

S_t=safe tensile stress of plate in lb. per sq. in.

S_c=safe crushing stress of plate or rivet in lb. per sq. in.

S_s=safe shearing stress of rivet in lb. per sq. in.

R=safe tensile resistance of a unit strip of the solid plate in pounds

R_t=safe tensile resistance of a unit strip of the joint in pounds

R_s=safe shearing resistance of a unit strip of the joint in pounds

R_c=safe crushing resistance of a unit strip of the joint in pounds

R_{ts}=safe combination tensile and shearing resistance of the joint in pounds

R_{tc}=safe combination tensile and compressive resistance of the joint in pounds

e=the efficiency of the joint

$e=\dfrac{R_t \text{ or } R_s \text{ or } R_c \text{ or } R_{ts} \text{ or } R_{tc}}{R}$ (whichever is the smallest value) $\times$ 100% (85)

The strengths or resistances of several different types of riveted joints will now be considered. The student should obtain from these a general idea of analysis which is applicable to all forms of riveted joints. He should not attempt to memorize the formulas but should see that by using a very few fundamentals of the subject of Machine Design, an analysis of any joint can be made. There are, in fact, but few formulas throughout the subject of Machine Design that need be memorized. They are the basic or fundamental formulas. All others are merely special cases of them, produced when an analysis of a specific problem calls for the application of the fundamental formula.

Single-Riveted Lap Joint. In this joint, shown in Fig. 58, the plates are secured to each other by a single row of rivets passing through both plates. The pitch of the joint, p, is the pitch of the rivets in this single row, and is the length of a unit strip of the joint and plate. It is evident that within this unit strip there is $\frac{1}{2}$ rivet$+\frac{1}{2}$ rivet or 1 rivet to resist the force, P, which is tending to slip one plate over the other. A unit strip of the solid plate will resist such a

force, P, by setting up a tensile resistance since the force tends to stretch the plate. Since in tension the involved area is perpendicular to the force, the tensile area of the plate within a unit strip will be a rectangular section whose length is p, the length of the unit strip, and whose thickness is t, the thickness of the plate. The safe tensile resistance, R, that the plate then is capable of having set up within it, is (see formulas 1 or 5) AS_t. Therefore,

$$\begin{aligned} R &= AS_t \\ &= p \times t \times S_t = ptS_t \end{aligned} \tag{86}$$

The above is the safe resistance per unit strip that the plate would have if in reality no joint were used. It is now necessary to find the resistances afforded by the joint so that the least or smallest of the latter can be compared with R of formula (86) in order to obtain the efficiency of the joint.

It is evident from Fig. 58, that along the center line of rivets the plate has been weakened by the drilling of holes for the rivets. It is also evident that the joint would fail, that is, the plates would separate, if tearing of the plate occurred along this center line of rivets. The tensile area here involved has a length, $p-d$, and a thickness, t, hence the safe resistance of the joint to tearing (tension) along the center line of rivets is

$$\begin{aligned} R_t &= AS_t \\ &= (p-d) \times t \times S_t = (p-d)tS_t \end{aligned} \tag{87}$$

The joint may fail if the rivets are sheared as in Fig. 64 (*b*), for this alone would allow the plates to separate. In a unit strip, there is one rivet in single shear and its involved area (parallel to the load or force as is always the case in shear) is $\frac{\pi d^2}{4}$. Therefore the safe resistance to shear,

$$\begin{aligned} R_s &= AS_s \\ &= \frac{\pi d^2}{4} S_s \end{aligned} \tag{88}$$

The above rivet, if not sheared, might be crushed or the plate in front of it might be crushed causing failure of the joint. (See Fig. 64, *d*.) The tendency toward failure in this manner would be the same if plate and rivet were of the same material. The area involved in this case is the bearing area between the rivet and the plate. While this is a semi-cylindrical area (with its elements perpendicular

to the direction of the load as must be the case in compression), it is always assumed in such a case where cylindrical bearing surfaces are under compression that the stress created by the load is uniformly distributed over the projected area of the surface. The projected area of a cylinder is its projection on a plane parallel to its axis and so becomes a rectangle whose length is the altitude of the cylinder and whose breadth is the diameter of the cylinder. Therefore the projected area, or the area in compression, is here equal to $d \times t$. (See Fig. 76.) Since one rivet is involved in compression per unit strip, the safe resistance which it can afford is given by the formula,

$$\begin{aligned} R_c &= AS_c \\ &= dtS_c \end{aligned} \tag{89}$$

In this joint, there are no other ways in which failure might take place; therefore no other resistances are afforded. It remains only to find in a given case the numerical values of each of these resistances of the joint. The ratio of the smallest of the latter values with respect to the resistance of the plate will then be the efficiency of the joint. For just as a chain is only as strong as its weakest link, so a riveted joint is only as strong as its least resistance. Although in the above we have used the safe resistances, AS, the ultimate resistances, AU, could just as well have been taken and would have yielded the same efficiency, since in the efficiency ratio the factors of safety will cancel. Failure of any joint, if it occurs, will naturally take place where the resistance is a minimum.

Example. Determine the safe resistances, the efficiency, and the method of failure of a single-riveted lap joint, in which the plate thickness is $\frac{1}{4}$ inch, the pitch of the joint is $1\frac{3}{4}$ inches, and the diameter of rivet hole is $\frac{11}{16}$ inch. Let $U_t = 55{,}000$ pounds per square inch, $U_c = 95{,}000$ pounds per square inch, $U_s = 44{,}000$ pounds per square inch, and $F = 5$.

Solution. From formula (4),

$$S = \frac{U}{F}$$

$$S_t = \frac{55{,}000}{5} = 11{,}000 \text{ lb. per sq. in.}$$

$$S_s = \frac{44{,}000}{5} = 8800 \text{ lb. per sq. in.}$$

$$S_c = \frac{95{,}000}{5} = 19{,}000 \text{ lb. per sq. in.}$$

It is also given that $t=\frac{1}{4}$ in.; $p=1\frac{3}{4}$ in.; $d=\frac{11}{16}$ in.

Evaluating in formula (86)

$$R = ptS_t$$

$$R = 1\tfrac{3}{4}\times\tfrac{1}{4}\times 11{,}000 = 4810 \text{ lb.} \quad \textit{Ans.}$$

Evaluating in formula (87),

$$R_t = (p-d)tS_t$$

$$R_t = (1\tfrac{3}{4} - \tfrac{11}{16})\times\tfrac{1}{4}\times 11{,}000$$

$$= 1\tfrac{1}{16}\times\tfrac{1}{4}\times 11{,}000 = 2920 \text{ lb.} \quad \textit{Ans.}$$

Evaluating in formula (88),

$$R_s = \frac{\pi d^2}{4} S_s$$

$$R_s = \frac{\pi\times(\frac{11}{16})^2}{4}\times 8800$$

$$= \pi\times 0.118\times 8800 = 3260 \text{ lb.} \quad \textit{Ans.}$$

Evaluating in formula (89),

$$R_c = dtS_c$$

$$R_c = \tfrac{11}{16}\times\tfrac{1}{4}\times 19{,}000 = 3270 \text{ lb.} \quad \textit{Ans.}$$

Comparing the three resistances of the joint, the least resistance is found to be R_t which is 2920 lb. Therefore from formula (85),

$$e = \frac{R_t}{R}\times 100\%$$

$$= \frac{2920}{4810}\times 100 = 61-\% \quad \textit{Ans.}$$

Since the least resistance is R_t, the method of failure in this case is by tension or tearing along the center line of rivets. *Ans.*

Double-Riveted Lap Joint. This type of joint is illustrated in Figs. 59 and 60, the only difference in the two illustrations being that in the former, there is chain riveting while in the latter there is zigzag riveting. A unit strip in either is of course equal in width to the pitch, p. There are two rows of rivets and two rivets in each unit strip. In other words since failure of the joint in shear or compression does not occur until the rivets in both rows are involved, the unit strip must be taken clear through the joint. This is in contrast to the butt-joint in which the unit strip stops at the place when the two

plates to be joined butt against each other, for the rivets that go through one plate do not go through the other as in the lap joint.

As in the single-riveted lap joint, the resistance of the plate,

$$R = ptS_t \tag{90}$$

If the joint tears along the center line of either row of rivets, failure will take place. The safe resistance set up against such a failure is then the same as for a single-riveted lap joint. Hence,

$$\begin{aligned} R_t &= AS_t \\ &= (p-d)tS_t \end{aligned} \tag{91}$$

In each unit strip of this joint, two rivets are in single shear. The plan view of either figure shows the number of rivets involved while the end view of the joint shown cross-hatched will always tell whether the rivets are in single or double shear. This is true for all riveted joints. Compare the end view of Fig. 59 with Fig. 64, (*b*).

The formula for the safe shearing resistance then becomes

$$\begin{aligned} R_s &= AS_s \\ &= 2 \times \frac{\pi d^2}{4} \times S_s = 2\frac{\pi d^2}{4} S_s \end{aligned} \tag{92}$$

Since there are two rivets in shear, it follows that there must be two rivets in crushing, so that

$$\begin{aligned} R_c &= AS_c \\ &= 2 \times dt \times S_c = 2dtS_c \end{aligned} \tag{93}$$

Example. Find the resistances, the efficiency, and the method of failure of a double-riveted lap joint, in which the pitch of the joint is 3 inches, the thickness of plate is $\frac{1}{2}$ inch, and the diameter of rivet hole is $\frac{15}{16}$ inch. Use the same ultimate stresses and factor of safety as in the preceding example.

Solution. From formula (4),

$$S = \frac{U}{F}$$

$$S_t = \frac{55{,}000}{5} = 11{,}000 \text{ lb. per sq. in.}$$

$$S_s = \frac{44{,}000}{5} = 8800 \text{ lb. per sq. in.}$$

$$S_c = \frac{95{,}000}{5} = 19{,}000 \text{ lb. per sq. in.}$$

We have also that $t=\frac{1}{2}$ in., $d=\frac{15}{16}$ in., and $p=3$ in. Using formula (**90**) to obtain the safe resistance of a unit strip of the solid plate,

$$R=ptS_t$$
$$R=3\times\frac{1}{2}\times 11{,}000=16{,}500 \text{ lb.} \quad \textit{Ans.}$$

Evaluating in formula (**91**),

$$R_t=(p-d)tS_t$$
$$R_t=(3-\tfrac{15}{16})\times\tfrac{1}{2}\times 11{,}000=11{,}300 \text{ lb.} \quad \textit{Ans.}$$

Evaluating in formula (**92**),

$$R_s=2\frac{\pi d^2}{4}S_s$$
$$R_s=2\times\frac{\pi\times(\frac{15}{16})^2}{4}\times 8800$$
$$=2\times 0.69\times 8800=12{,}200 \text{ lb.} \quad \textit{Ans.}$$

Evaluating in formula (**93**),

$$R_c=2dtS_c$$
$$R_c=2\times\tfrac{15}{16}\times\tfrac{1}{2}\times 19{,}000=17{,}800 \text{ lb.} \quad \textit{Ans.}$$

Since the lowest of the above resistances of the joint is R_t we have from formula (**85**),

$$e=\frac{R_t}{R}\times 100\%$$
$$=\frac{11{,}300}{16{,}500}\times 100=69-\% \quad \textit{Ans.}$$

Due to the fact that R_t is the lowest resistance of the joint, the method of failure is by tearing along the center line of rivets. *Ans.*

Double-Riveted Butt Joint. In any butt joint, there are two identical parts with exactly the same arrangement of rivets. Each part consists of one of the plates of the joint together with the rivets and those portions of the straps that are riveted to the individual plate. Each part is subjected to the same load; and failure of either part, so that one plate is released from the joint, results in failure of the joint. Therefore a unit strip of any butt joint, from a consideration of the strength of the joint, stops at the line where the plates butt against each other as for instance the line, *mn*, of Fig. 61.

In the double-riveted butt joint of Fig. 61, it should be noted that the length of the unit strip is twice the pitch of rivets in the inner row. Considering first the resistance of a unit strip of the solid

plate, we obtain for its safe resistance in tension or to tearing,

$$R = ptS_t \tag{94}$$

In the case of the resistance offered by the joint to tearing along the center line of rivets, it would appear at first glance that the tearing would occur along the inner row; for the length of this involved area in tension has been reduced by two rivet hole diameters, while along the outer row there is a reduction in the length of only one rivet hole diameter. Thus the former area and its corresponding resistance to tearing along the center line of rivets would be the weaker of the two. This is a correct conclusion from the standpoint of tearing alone. But it should be noted that the joint cannot fail by tearing along the inner row of rivets until one rivet in single shear or in crushing in the outer row has failed. Thus two resistances are here working together to give us a combination resistance of tearing and shear, R_{ts}, or a combination resistance of tearing and crushing, R_{tc}. Hence the only place that a straight tearing resistance is offered to failure of the joint is along the center line of the outer row, where it becomes

$$R_t = (p-d)tS_t \tag{95}$$

To obtain the combined resistance of tearing (tension) in the inner row and shear in the outer row, the former must be added to the latter. The former is equal to AS_t, where $A = (p-2d) \times t$, which gives $(p-2d)tS_t$ while the latter is equal to AS_s, where $A = \frac{\pi d^2}{4}$, which gives $\frac{\pi d^2}{4} S_s$.

Adding the above, we have for the combination resistance,

$$R_{ts} = (p-2d)tS_t + \frac{\pi d^2}{4} S_s \tag{96a}$$

To obtain the combined resistance to tearing (tension) in the inner row and crushing (compression) in the outer row, the former resistance must be added to the latter resistance. The resistance to tearing as given above is equal to $(p-2d)tS_t$. The resistance offered by the single rivet per unit strip in the outer row to crushing is equal to AS_c. Since this rivet has a bearing or crushing area, $d \times t$, with the plate, and a bearing area, $d \times t'$, with the strap (when the rivets in the outer row go through both straps, the bearing area of each rivet with the straps is $d \times 2t'$) the minimum area is $d \times t'$, for

the strap is always less in thickness than the plate. Hence the resistance to crushing becomes $dt'S_c$. Adding these resistances to tearing and crushing, we have for the combination resistance,

$$R_{tc}=(p-2d)tS_t+dt'S_c \tag{96, b}$$

By looking at the plan view of Fig. 61, we can see that there are two rivets in the inner row, and one rivet (two half-rivets) in the outer row per unit strip. The end view shows that the two rivets in the inner row are in double shear while the one rivet in the outer row is in single shear as noted above. Hence the shearing resistance of the joint is equal to the shearing resistance of two rivets in double shear plus the shearing resistance of one rivet in single shear, or

$$R_s=2\times2\times\frac{\pi d^2}{4}\times S_s+1\times\frac{\pi d^2}{4}\times S_s$$

Factoring the second member of the above equation,

$$R_s=(4+1)\frac{\pi d^2}{4}S_s$$

$$R_s=5\frac{\pi d^2}{4}S_s \tag{97}$$

Since there are three rivets involved in shear, there will likewise be three rivets per unit strip involved in compression or crushing. The minimum area in crushing for each rivet in the inner row will be $d\times t$ for the combined thickness of the two straps is greater than the thickness, t, of the plate. The minimum area in crushing for each rivet in the outer row as previously given is $d\times t'$. Therefore the formula for the safe crushing resistance becomes

$$\begin{aligned}R_c&=2dtS_c+dt'S_c\\&=(2dt+dt')S_c\end{aligned} \tag{98, a}$$

If, in a double-riveted butt joint of the type of Fig. 61, the thickness of the strap is equal to the thickness of the plate, it is evident from the preceding discussion that the resistance to crushing is given by the formula:

$$R_c=3dtS_c \tag{98, b}$$

The value of R_c as obtained by using formula (98, b) is a close approximation to the value obtained from formula (98, a).

The above material demonstrates that there are five resistances of the joint in this case that must be investigated in order to determine the minimum resistance that will permit the computation of the

efficiency of this joint. Some designs give immediate evidence of the fact that the resistance, R_{tc}, is greater than R_{ts}. When such is the case, obviously R_{tc} may be neglected.

Example. In a double-riveted butt joint of the type shown in Fig. 61, the thickness of plate is $\frac{3}{8}$ inch, the thickness of the strap is $\frac{5}{16}$ in., the pitch of the joint is $4\frac{3}{4}$ inches, and the diameter of rivet hole is $\frac{13}{16}$ inch.

Let

$$U_t = 55{,}000 \text{ lb. per sq. in.}$$
$$U_c = 95{,}000 \text{ lb. per sq. in.}$$
$$U_s = 44{,}000 \text{ lb. per sq. in.}$$
$$F = 5$$

Find (*a*) the safe resistance of the plate, (*b*) the safe resistances of the joint, (*c*) the efficiency of the joint, (*d*) the method of failure, (*e*) the total force or load per unit strip to produce failure.

Solution. (*a*) Here $p = 4\frac{3}{4}$ in., $t = \frac{3}{8}$ in., and $S_t = \frac{U_t}{F} = \frac{55{,}000}{5} =$ 11,000 lb. per sq. in.

Substituting these values in formula (**94**),

$$R = ptS_t$$
$$= 4\tfrac{3}{4} \times \tfrac{3}{8} \times 11{,}000 = 19{,}600 \text{ lb.} \quad \textit{Ans.}$$

(*b*) To secure the safe tensile resistance to tearing along the center line of rivets, we shall use formula (**95**) with $d = \frac{13}{16}$ in., and the other data as in part (*a*). Thus

$$R_t = (p - d)tS_t$$
$$= (4\tfrac{3}{4} - \tfrac{13}{16}) \times \tfrac{3}{8} \times 11{,}000$$
$$= \tfrac{63}{16} \times \tfrac{3}{8} \times 11{,}000 = 16{,}250 \text{ lb.} \quad \textit{Ans.}$$

To obtain R_{ts}, we have the values of p, d, t, and S_t already used

and $$S_s = \frac{U_s}{F} = \frac{44{,}000}{5} = 8800 \text{ lb. per sq. in.}$$

Evaluating in formula (**96, a**),

$$R_{ts} = (p - 2d)tS_t + \frac{\pi d^2}{4} S_s$$
$$= (4\tfrac{3}{4} - \tfrac{13}{8}) \times \tfrac{3}{8} \times 11{,}000 + \frac{\pi \times (\frac{13}{16})^2}{4} \times 8800$$
$$= 12{,}900 + 4550 = 17{,}450 \text{ lb.} \quad \textit{Ans.}$$

To obtain R_{tc}, we have

$$S_c=\frac{U_c}{F}=\frac{95{,}000}{5}=19{,}000 \text{ lb. per sq. in.}$$

Using formula (**96, b**)

$$R_{tc}=(p-2d)tS_t+dt'S_c$$

with $t'=\frac{5}{16}$ in.

$$R_{tc}=(4\tfrac{3}{4}-\tfrac{13}{8})\times\tfrac{3}{8}\times 11{,}000+\tfrac{13}{16}\times\tfrac{5}{16}\times 19{,}000$$
$$=12{,}900+4{,}830=17{,}730 \text{ lb.}$$

Substituting our known values in formula (**97**),

$$R_s=5\frac{\pi d^2}{4}S_s$$

$$=5\times\frac{\pi\times(\frac{13}{16})^2}{4}\times 8800=22{,}800 \text{ lb.} \quad \textit{Ans.}$$

Using formula (**98, a**),

$$R_c=(2dt+dt')S_c$$
$$=(2\times\tfrac{13}{16}\times\tfrac{3}{8}+\tfrac{13}{16}\times\tfrac{5}{16})19{,}000=16{,}400 \text{ lb.} \quad \textit{Ans.}$$

(*c*) Comparing the above resistances of the joint, R_t is found to be the smallest. Therefore from formula (**85**),

$$e=\frac{R_t}{R}\times 100\%$$

$$=\frac{16{,}250}{19{,}600}\times 100=83-\% \quad \textit{Ans.}$$

(*d*) In the event that this joint should fail, it will do so by tearing along the center line of the outer row of rivets. This is due to the fact that R_t is the smallest safe resistance. *Ans.*

(*e*) The load which causes failure of the joint is its lowest ultimate load. Since the ultimate load is equal to the ultimate resistance as shown by formula (**2**), and the ultimate resistance is equal to the safe resistance times the factor of safety, here

$$\text{the ultimate or breaking load per unit strip}=F\times R_t$$
$$=5\times 16{,}250$$
$$=81{,}250 \text{ lb.} \quad \textit{Ans.}$$

Triple-Riveted Butt Joint. The analysis of the triple-riveted butt joint is to a great extent similar to that of the double-riveted butt joint. The student should become thoroughly acquainted with Fig. 62 for which the following analysis is presented, and should attempt

to see the reasonableness of each step of this procedure. The analysis presents the following formulas:

The safe tensile resistance of a unit strip of the plate,

$$R = ptS_t \tag{99}$$

The safe tensile resistance of a unit strip of the joint (tearing along the outer row of rivets),

$$R_t = (p-d)tS_t \tag{100}$$

The combined safe resistance of a unit strip of the joint, to tearing along the middle row of rivets and to shear of the rivets in the outer row (which must accompany the former),

$$R_{ts} = (p-2d)tS_t + \frac{\pi d^2}{4}S_s \tag{101, a}$$

[**Note.** The resistance to tearing along the inner row of rivets is accompanied by double shear of two rivets in the middle row and single shear of one rivet in the outer row. Hence this resistance need not be considered for it obviously is greater than the preceding resistance given by formula (**101, a**)]

The combined safe resistance of a unit strip of the joint to tearing along the middle row of rivets and to crushing of the rivets in the outer row,

$$R_{tc} = (p-2d)tS_t + dt'S_c \tag{101, b}$$

The safe shearing resistance,

$$R_s = 4\times 2\times\frac{\pi d^2}{4}\times S_s + 1\times\frac{\pi d^2}{4}\times S_s$$

$$= 9\frac{\pi d^2}{4}S_s \tag{102}$$

The safe crushing resistance,

$$R_c = (4dt + dt')S_c \tag{103}$$

Design of Riveted Joints. In the preceding problems, the resistances of the various types of riveted joints were seen to vary considerably. Since the actual strength of a joint is measured by but one of its resistances, namely the least resistance, it is of no particular value to the riveted joint to have several relatively high resistances accompanied by a much lower one. The design of a riveted joint then attempts as far as possible to equalize these resistances and, at the same time, to provide a minimum resistance that approximates the strength of the plate closely enough to permit the efficiency to be

TABLE XIV — Approximate Efficiencies of Riveted Joints for Pressure Vessels

Single-riveted lap joint	50 to 60%	Single-riveted butt joint	55 to 65%
Double-riveted lap joint	60 to 75%	Double-riveted butt joint	65 to 80%
Triple-riveted lap joint	65 to 80%	Triple-riveted butt joint	80 to 88%
		Quadruple-riveted butt joint	90 to 95%

within that range of values which experience has dictated. These ranges of values for the different types of joints are set forth in Table XIV. In reality the design of a joint is commenced by selecting the particular type of joint that should be used. This permits one to anticipate the efficiency so that the assumed value of the latter can be used in finding the thickness of plate, t, by formula (80). Having obtained the value of t, the diameter of rivet consistent with this thickness of plate is selected either from Table XIII or by using formula (84). It now remains to find a value for the pitch of the joint, based on the dimensions already established. This is generally done by equating (and hence equalizing) the two resistances, R_t and R_s, the pitch, p, entering the equation in the formula for R_t. Having found the pitch, the resistances and efficiency can be determined. The latter is then checked with the value initially assumed. If the calculated value is equal to or more than the initial assumption, the design is all right in so far as this is concerned. Should the calculated efficiency be too low, the design must be revised. In the case of the design of lap joints, the equating of R_t to R_s generally proves effective. In the case of butt joints a preliminary trial by the use of this equation often shows the necessity of recalculating for the pitch by the equating of R_t to R_c. The pitch finally selected should also be practically acceptable from the standpoint of riveting and calking. To be sure of this, the inexperienced designer should consult known and proven satisfactory designs as set forth by such companies as the Hartford Steam Boiler Inspection and Insurance Company. The A. S. M. E. Boiler Code should likewise be freely consulted and strictly adhered to.

Example. It is required to find the pitch, the resistances, and the efficiency of a double-riveted lap joint (as shown in Fig. 60) used to connect two $\frac{3}{8}$-inch plates. Assume the ultimate strengths and factor of safety of the preceding problem.

Solution. It will be necessary to find first the diameter of rivet hole to be used when $t=\frac{3}{8}$ in.

From formula (84)

$$d=1.3\sqrt{t}$$

$$=1.3\sqrt{\tfrac{3}{8}}=1.3\times 0.613=0.80-, \text{ say } \tfrac{13}{16} \text{ in.}$$

Equating R_t to R_s [formulas (91) and (92)]

$$(p-d)tS_t=2\frac{\pi d^2}{4}S_s$$

Here $t=\frac{3}{8}$ in., $d=\frac{13}{16}$ in., $S_t=\dfrac{55{,}000}{5}$ lb. per sq. in., $S_s=\dfrac{44{,}000}{5}$ lb. per sq. in.

Substituting these values in the above formula,

$$(p-\tfrac{13}{16})\times\tfrac{3}{8}\times\frac{55{,}000}{5}=2\times\frac{\pi(\frac{13}{16})^2}{4}\times\frac{44{,}000}{5}$$

$$4125p-3350=9120$$

$$4125p=9120+3350$$

$$p=\frac{12{,}470}{4125}=3.02, \text{ say } 3\tfrac{1}{16} \text{ in.} \quad \textit{Ans.}$$

From formula (90),

$$R=ptS_t$$

$$=3\tfrac{1}{16}\times\tfrac{3}{8}\times\frac{55{,}000}{5}=12{,}650 \text{ lb.} \quad \textit{Ans.}$$

From formula (91)

$$R_t=(p-d)tS_t$$

$$=(3\tfrac{1}{16}-\tfrac{13}{16})\times\tfrac{3}{8}\times\frac{55{,}000}{5}=9280 \text{ lb.} \quad \textit{Ans.}$$

From formula (92)

$$R_s=2\frac{\pi d^2}{4}S_s$$

$$=2\times\frac{\pi(\frac{13}{16})^2}{4}\times\frac{44{,}000}{5}=9120 \text{ lb.}$$

From formula (93)

$$R_c=2dtS_c$$

$$=2\times\tfrac{13}{16}\times\tfrac{3}{8}\times\frac{95{,}000}{5}=11{,}600 \text{ lb.}$$

Since R_s is the least resistance of the joint, from formula (85)

$$e=\frac{R_s}{R}\times 100\%$$

$$=\frac{9120}{12{,}650}\times 100=72\% \quad \textit{Ans.}$$

Comparing this efficiency with Table XIV, it is found to be toward the upper limit and hence very satisfactory.

Example. Find the pitch, the resistances, and the efficiency of a double-riveted butt joint as illustrated in Fig. 61, in which

$t=\frac{15}{32}$ inch,
$t'=\frac{3}{8}$ inch,
$S_t=11{,}000$ lb. per sq. in.
$S_c=19{,}000$ lb. per sq. in.
$S_s=8800$ lb. per sq. in.

Solution. Applying formula (84) to find the diameter of rivet hole, d,

$$d=1.3\sqrt{t}$$

$$=1.3\sqrt{\tfrac{15}{32}}=0.89 \text{ in., say } \tfrac{15}{16} \text{ in.}$$

Equating R_t and R_s, [formulas (95) and (97)], to find the pitch of the joint, p,

$$(p-d)tS_t=5\frac{\pi d^2}{4}S_s$$

Evaluating in the above,

$$(p-\tfrac{15}{16})\times\tfrac{15}{32}\times 11{,}000=5\times\frac{\pi\times(\frac{15}{16})^2}{4}\times 8800$$

$$5150p-4820=30{,}400$$

$$5150p=30{,}400+4820$$

$$p=\frac{35{,}220}{5150}=6.83 \text{ in., say } 6\tfrac{7}{8} \text{ in.}$$

It is now in order to find the resistances and efficiency based upon the above pitch of the joint. The results obtained will determine whether the above pitch is to be used, or whether it would be advisable to solve for another. Applying formula (94),

$$R=ptS_t$$

and evaluating,

$$R=6\tfrac{7}{8}\times\tfrac{15}{32}\times 11{,}000=35{,}400 \text{ lb.}$$

Evaluating in formula (95),

$$R_t=(p-d)tS_t$$
$$=(6\tfrac{7}{8}-\tfrac{15}{16})\times\tfrac{15}{32}\times 11{,}000=30{,}600 \text{ lb.}$$

Evaluating in formula (96, a),

$$R_{ts}=(p-2d)tS_t+\frac{\pi d^2}{4}S_s$$
$$=(6\tfrac{7}{8}-\tfrac{15}{8})\times\tfrac{15}{32}\times 11{,}000+\frac{\pi\times(\frac{15}{16})^2}{4}\times 8800$$
$$=25{,}800+6070=31{,}870 \text{ lb.}$$

Evaluating in formula (96, b)

$$R_{tc}=(p-2d)tS_t+dt'S_c$$
$$=(6\tfrac{7}{8}-\tfrac{15}{8})\times\tfrac{15}{32}\times 11{,}000+\tfrac{15}{16}\times\tfrac{3}{8}\times 19{,}000$$
$$=32{,}500 \text{ lb.}$$

Evaluating in formula (97),

$$R_s=5\frac{\pi d^2}{4}S_s$$
$$=5\times\frac{\pi(\frac{15}{16})^2}{4}\times 8800=30{,}400 \text{ lb.}$$

Evaluating in formula (98),

$$R_c=(2dt+dt')S_c$$
$$=(2\times\tfrac{15}{16}\times\tfrac{15}{32}+\tfrac{15}{16}\times\tfrac{3}{8})19{,}000=23{,}400 \text{ lb.}$$

Since the least resistance of the joint is R_c, applying formula (85), we have

$$e=\frac{R_c}{R}\times 100\%$$
$$=\frac{23{,}400}{35{,}400}\times 100=66\%$$

The above efficiency is found from Table XIV to be rather low. It is therefore probably advisable to redesign this joint so as to bring the lowest resistance, R_c, of the previous calculation more closely into an equality with the other resistances. This can be accomplished by equating R_t and R_c. It will be noticed that in so doing, the pitch of the joint will be lowered. That this is consistent can be proven by investigating the known designs of authorities in this field.

Equating R_t and R_c, formulas (95) and (98, a),

$$(p-d)t\,S_t=(2dt+dt')S_c$$

Evaluating,

$$(p-\tfrac{15}{16})\times\tfrac{15}{32}\times 11{,}000=(2\times\tfrac{15}{16}\times\tfrac{15}{32}+\tfrac{15}{16}\times\tfrac{3}{8})19{,}000$$
$$5150p-4820=23{,}400$$
$$5150p=23{,}400+4820$$
$$p=\frac{28{,}220}{5150}=5.5-\text{in., say } 5\tfrac{1}{2}\text{ in.}\quad \textit{Ans.}$$

Using formula (94) to obtain the resistance of the solid plate,

$$R=ptS_t$$
$$=5\tfrac{1}{2}\times\tfrac{15}{32}\times 11{,}000=28{,}300\text{ lb.}\quad \textit{Ans.}$$

Evaluating in formula (95),

$$R_t=(p-d)tS_t$$
$$=(5\tfrac{1}{2}-\tfrac{15}{16})\times\tfrac{15}{32}\times 11{,}000=23{,}600\text{ lb.}\quad \textit{Ans.}$$

Evaluating in formula (96, a)

$$R_{ts}=(p-2d)tS_t+\frac{\pi d^2}{4}S_s$$
$$=(5\tfrac{1}{2}-\tfrac{15}{8})\times\tfrac{15}{32}\times 11{,}000+\frac{\pi\times(\frac{15}{16})^2}{4}\times 8800$$
$$=18{,}700+6070=24{,}770\text{ lb.}\quad \textit{Ans.}$$

Evaluating in formula (96, b)

$$R_{tc}=(p-2d)tS_t+dt'S_c$$
$$=(5\tfrac{1}{2}-\tfrac{15}{8})\times\tfrac{15}{32}\times 11{,}000+\tfrac{15}{16}\times\tfrac{3}{8}\times 19{,}000$$
$$=24{,}400\text{ lb.}\quad \textit{Ans.}$$

Evaluating in formulas (97) and (98) will give, as in the first calculation,

$$R_s=30{,}400\text{ lb. and } R_c=23{,}400\text{ lb.}\quad \textit{Ans.}$$

Investigation of these final resistances based on the smaller pitch shows that R_c is the least resistance. And from formula (85)

$$e=\frac{R_c}{R}\times 100\%$$
$$=\frac{23{,}400}{28{,}300}\times 100\%=83\%\quad \textit{Ans.}$$

(**Note.** If, in the preceding example, the numerical values of R_s and R_c had been found before equating R_t to R_s, it would have been made evident at the start that R_c is less than R_s. Hence the equating of R_t to R_s could have been neglected, and R_t equated to R_c only. Thus the solution of the example would have been shortened to quite an extent.)

PROBLEMS

1. A plane cuts a transverse section from a cylinder. How is the plane located with respect to the axis of the cylinder?

2. A plane cuts a longitudinal section from a cylinder. How is the plane located with respect to the axis of the cylinder?

3. A closed cylinder is subjected to an internal fluid pressure of p pounds per square inch. How does the stress that the force sets up in the longitudinal section of the cylinder wall compare with that which is set up in the transverse section of the wall?

4. A factor of safety of 8 is used in designing the wall of a cylinder. This is equivalent to what factor of safety in the transverse section of the cylinder wall? *Ans.* 16.

5. A cast-iron steam engine cylinder is 10 inches in diameter. The maximum steam pressure occurring in the cylinder is 200 lb. per sq. in. Assuming a varying load and allowing $\frac{7}{16}$ inch for counterboring and reboring, find the thickness of cylinder wall to be used. *Ans.* $\frac{7}{8}$ in.

6. A steam boiler, 50 inches in diameter, generates steam at a gauge pressure of 250 pounds per square inch. If $U_t = 55{,}000$ pounds per square inch, $F = 5$, and $e = 85$ per cent, what thickness of shell need be used? *Ans.* $\frac{11}{16}$ in.

7. If a pressure vessel like that of the example accompanying Fig. 54 is subjected to an internal fluid pressure of 50 pounds per square inch, find (*a*) the thickness of the cylinder wall, (*b*) the crushing load on the screw B, (*c*) the shearing load on the threads of B, (*d*) the loads on pins C, D, E, and F. *Ans.* (*a*) 0.240 in. say $\frac{1}{4}$ to $\frac{3}{8}$ in. (*b*) 22,610 lb. (*c*) 22,610 lb. (*d*) 11,305 lb.

8. A standard 10-inch wrought-iron pipe has an inside diameter of 10.192 inches and an outside diameter of 10.750 inches. What unit tensile stress is created by an internal pressure of 145 pounds per square inch? *Ans.* 2650 lb. per sq. in.

9. How many cubic feet of steam per minute can be carried by the pipe of the preceding problem if the velocity of the flow of steam is 6000 feet per minute? *Ans.* 3400 cu. ft. nearly.

10. The pressure within the cylinder of a hydraulic press is 1200 pounds per square inch. The inside diameter of the cylinder is 10 inches. Determine the thickness of the cylinder wall if a tensile stress of 2500 pounds per square inch is allowable. *Ans.* 4 in. nearly.

11. Name the two major types of riveted joints.

12. State the reason for calking a riveted joint.

13. In what kind of pressure vessels may fusion welding replace calking?

14. Define (*a*) marginal distance, (*b*) the pitch of the joint, (*c*) point of a rivet, (*d*) grip of a rivet, (*e*) back pitch of a joint, (*f*) unit strip of a joint, (*g*) efficiency of a joint.

15. Find the diameter of rivet hole to be used in a riveted joint if the plate thickness is $\frac{3}{4}$ inch. *Ans.* $1\frac{3}{16}$ in. From Table XIII.

16. Name the five methods of failure of a riveted joint.

(**Note.** In all problems dealing with a riveted joint design, let $U_t = 55{,}000$ lbs. per sq. in.; $U_c = 95{,}000$ lbs. per sq. in.; $U_s = 44{,}000$ lbs. per sq. in.; $F = 5$. Remember that most results in this text are slide rule results, since the accuracy of the slide rule is in general sufficient for this work.)

17. A single-riveted lap joint is used to connect two $\frac{5}{16}$-inch plates. The pitch of the joint is 2 inches and the diameter of rivet hole is $\frac{13}{16}$ inch. Find (*a*) the safe tensile resistance of a unit strip of the solid plate, (*b*) the safe resistance of a unit strip of the joint to tearing (tension) along the center line of rivets, (*c*) the safe resistance of a unit strip of the joint to shearing of the rivets, (*d*) the safe resistance of a unit strip of the joint to crushing of the rivets, (*e*) the efficiency of the joint. *Ans.* (*a*) 6870 lb. (*b*) 4080 lb. (*c*) 4560 lb. (*d*) 4820 lb. (*e*) 59.5%.

18. If the joint of the preceding problem should fail, by what method would failure take place?

19. A double-riveted lap joint (See Figs. 59 and 60) is used to connect two $\frac{1}{4}$-inch plates. The pitch of the joint is $2\frac{1}{2}$ inches and the diameter of rivet hole is $\frac{11}{16}$ inch. Find (*a*) the safe tensile resistance of a unit strip of the solid plate, (*b*) the safe resistances of a unit strip of the joint, (*c*) the efficiency of the joint. *Ans.* (*a*) 6880 lb. (*b*) $R_t = 4990$ lb., $R_s = 6530$ lb., $R_c = 6530$ lb. (*c*) 72.5%.

20. In a double-riveted butt joint of the type illustrated in Fig. 61, the thickness of plate is $\frac{5}{16}$ inch, the thickness of strap is $\frac{9}{32}$ inch, the pitch of the joint is $4\frac{1}{2}$ inches, and the diameter of the rivet hole is $\frac{13}{16}$ inch. It is required to find (*a*) the safe resistance of the plate, (*b*) the safe resistances of the joint, (*c*) the efficiency of the joint. *Ans.* (*a*) 15,500 lb. (*b*) $R_t = 12{,}700$ lb., $R_{ts} = 14{,}440$ lb., $R_{tc} = 14{,}200$ lb., $R_s = 22{,}800$ lb., $R_c = 14{,}000$ lb. (*c*) 82%.

21. It is required to find the rivet or hole diameter, the pitch, the safe resistances and the efficiency of a double-riveted lap joint (Fig. 60) used to connect two $\frac{1}{2}$-inch plates.

Ans. rivet diameter $= \frac{15}{16}$ in.
$p = 3\frac{3}{16}$ in.
$R = 17{,}500$ lb.
$R_t = 12{,}400-$ lb.
$R_s = 12{,}300+$ lb.
$R_c = 17{,}800$ lb.
$e = 70+\%$

22. What loads per unit strip would cause failure of the joints of Problems 17 and 19? *Ans.* 20,400 lb.; 24,950 lb.

23. If the thickness of strap in the triple-riveted butt joint of Fig. 62 were equal to the thickness of plate, what would be the formula for the safe resistance to crushing, R_c? Compare with formula (103). *Ans.* 5dt S_c.

LINE SHAFT ASSEMBLY, SHOWING HOW EASILY AND QUICKLY SPLIT LINE SHAFT BEARINGS CAN BE INSTALLED

Courtesy of Hyatt Bearings Division, General Motors Corporation, Harrison, New Jersey

CHAPTER V

SHAFTING AND KEYS

Introduction. A rotating bar or machine element which transmits power is called a Shaft. Power is delivered to it through the action of some tangential force and the resultant torque or torsional moment set up within the shaft permits the power to be distributed to various machines linked up to the shaft.

Another machine element called an Axle is shaft-like in appearance, but transmits no torque. It is generally a supporting or connecting link for a pair of wheels which may rotate with it or upon it. The result is that an axle is subjected to bending instead of twisting.

Since the introduction of power into a shaft generally requires the use of various links such as gears and pulleys, the latter together with the forces exerted upon them, subject the shaft to bending. Hence a shaft like an axle may be subjected to bending. In the case of a shaft, however, the bending if present is always accompanied by twisting.

The material used for ordinary commercial shafting is mild steel. In machines where the weight must be kept low even though the service is heavy, alloys such as vanadium and nickel steels are employed.

Shafting is generally turned or cold-rolled. The former is first hot-rolled to a diameter which is $\frac{1}{16}$ inch greater than the finished diameter. It is next turned down in a lathe to the final diameter and the surface is polished. Cold-rolled shafting is also first hot-rolled but the reduction in diameter is accomplished by rolling cold under great pressure. This produces a fairly uniform diameter as well as a surface of finished appearance. Large shafts of 5 inches in diameter or more are usually forged and turned to size in a lathe.

TABLE XV — Sizes of Transmission Shafting

Diameter in Inches

$\frac{15}{16}$	$1\frac{3}{16}$	$1\frac{7}{16}$	$1\frac{11}{16}$	$1\frac{15}{16}$	$2\frac{3}{16}$	$2\frac{7}{16}$
$2\frac{15}{16}$	$3\frac{7}{16}$	$3\frac{15}{16}$	$4\frac{7}{16}$	$4\frac{15}{16}$	$5\frac{7}{16}$	$5\frac{15}{16}$
$6\frac{1}{2}$	7	$7\frac{1}{2}$	8			

Classification of Shafting. Shafting can be classified according to the use to which it is put, as follows:

1. For prime movers
   (a) Engine shafts
   (b) Generator shafts
   (c) Turbine shafts
2. Machine shafts
3. For power transmission
   (a) Line shafts
   (b) Jackshafts
   (c) Countershafts

Line shafting is a term used to specify rather long and continuous lines of shafting employed in mills, shops, and factories for the distribution of power. A jackshaft is a shaft that is directly connected to the source of power and from which other shafts are driven. A countershaft is one placed between a line shaft and a machine. The power received by it from the line shaft is imparted to the drive-shaft of the machine.

Commercial Shaft Sizes. In the design of machine elements one must constantly keep in mind and be aware of the standard or commercial sizes that can be procured, for such parts are less expensive and more easily obtained. So a designer consults freely the trade catalogs of the concerns he may patronize so that the theoretical values obtained by him in his design may be increased to the stock sizes that are just higher. An immediate decision as to this must be made by him so that the design of subsequent interrelated parts will be based on the actual rather than the theoretical sizes or values.

Transmission shafting that is generally available comes in the diameters as given in Table XV.

Machine shafts vary as shown in Table XVI.

Stock lengths of line shafting are as a rule as follows: 12, 16, 20, and 24 feet.

TABLE XVI — Variation in Sizes of Machinery Shafting

$\frac{1}{16}$ inch increments for shafts from	$\frac{1}{2}$	to $2\frac{1}{2}$	inches in diameter
$\frac{1}{8}$ inch increments for shafts from	$2\frac{1}{2}$	to 4	inches in diameter
$\frac{1}{4}$ inch increments for shafts from	4	to 6	inches in diameter

The Design of Shafting. In Chapter II where twisting, or torsion, was brought out as one of the compound stresses, it became necessary to involve shafting in the discussion due to the fact that a shaft is the chief machine part to be subjected to a torsional moment. Therefore, at that time a good deal of information relative to shafts was introduced which should be reviewed by the student at this time. It remains now to consider the design of a shaft so as to obtain the diameter that it must have to safely resist the action of the involved forces. Consideration will be given to the following cases:

1. Shafts subjected to twisting only
2. Shafts subjected simultaneously to both bending and twisting
3. The design of shafting in which torsional rigidity dictates the selection of a diameter.

Shafts Subjected to Twisting Only. This includes the design of those shafts which are subjected to a torsional moment only, as well as those in which the bending moment is so small, in relation to the twisting moment, that the former can either be neglected entirely or considered indirectly through the selection of a lower working stress, the design proceeding on a basis of twisting only.

In this case, the torque in inch-pounds must first be obtained. If the horsepower, H, of the shaft and the r.p.m., N, at which it is transmitted are known, the torque can be found by applying formula (34), $T=\frac{12\times 33{,}000H}{2\pi N}$. If a tangential force, P_t, and the moment arm R, through which it is acting are known (See Fig. 8), the torque is found by applying formula (29), $T=P_tR$. With a knowledge of the material to be used, the safe shearing stress, S_s, is next obtained from formula (14) by consulting Table I for the ultimate shearing stress, U_s, and Table II for the factor of safety, F. The load on any shaft is generally a live or varying load so that the factor of safety for steel would be about 8. Occasionally a shock load is to be considered, in which case the factor will increase materially as is shown by the table. The type of cross section to be used permits the selection of the polar section modulus, Z_p, from Table IV. The section modulus is always given in terms of the dimensions of the section and so contains the diameter, d, for which the solution is to be made. The data thus obtained is substituted in the design formula for

twisting, formula (**31**),

$$T=S_sZ_p$$

which is then solved for the unknown diameter.

Example. It is required to find the diameter of a solid steel shaft to transmit 25 horsepower at 200 revolutions per minute.

Solution. Here $H=25$ hp.; $N=200$ r.p.m.; Z_p, from Table IV, $=\frac{\pi d^3}{16}$. Substituting the values of H and N in formula (**34**),

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 25}{2\times\pi\times 200}=7880 \text{ in.-lb.}$$

From Table I, $U_s=50{,}000$ lb. per sq. in. and from Table II, $F=8$. Evaluating in formula (**14**)

$$S_s=\frac{U_s}{F}$$

$$=\frac{50{,}000}{8}=6250 \text{ lb. per sq. in.}$$

From Table IV,

$$Z_p=\frac{\pi d^3}{16}$$

Evaluating in formula (**31**),

$$T=S_sZ_p$$

$$7880=6250\times\frac{\pi d^3}{16}$$

Multiplying by 16 and dividing by $6250\times\pi$,

$$d^3=\frac{7880\times 16}{6250\times\pi}=6.42$$

$$d=\sqrt[3]{6.42}=1.85 \text{ in., say } 1\tfrac{7}{8} \text{ in. or } 1\tfrac{15}{16} \text{ in.} \quad \textit{Ans.}$$

Example. Find the inside and outside diameters of a hollow shaft to be used in the place of the solid shaft of the preceding example. Let d (the inside diameter) $=\frac{1}{2}D$.

Solution. From Table IV, $Z_p=\frac{\pi}{16}\,\frac{(D^4-d^4)}{D}$

Since $$d=\frac{D}{2},$$

$$Z_p=\frac{\pi}{16}\,\frac{\left[D^4-\left(\frac{D}{2}\right)^4\right]}{D}=\frac{\pi}{16}\,\frac{\left[D^4-\frac{D^4}{16}\right]}{D}$$

$$Z_p=\frac{\pi}{16}\,\frac{\frac{15D^4}{16}}{D}=\frac{\pi}{16}\times\frac{15D^4}{16}\times\frac{1}{D}$$

$$=\frac{15\pi D^3}{256}$$

There is no change in the given data, therefore $T=7880$ in.-lb. and $S_s=6250$ lb. per sq. in.

Evaluating in formula (31),

$$T=S_sZ_p$$

$$7880=6250\times\frac{15\pi D^3}{256}$$

Multiplying by 256 and dividing by $6250\times15\pi$,

$$D^3=\frac{7880\times256}{6250\times15\pi}=6.85$$

$$D=\sqrt[3]{6.85}=1.9-\text{ in., say } 1\tfrac{15}{16}\text{ in.}\quad \textit{Ans.}$$

$$d=\frac{D}{2}=\tfrac{31}{32}\text{ in.}\quad \textit{Ans.}$$

(**Note.** The result of the above example is evidence of the fact that a hollow shaft is much lighter than a solid shaft that will transmit the same torque.)

Example. What is the safe torsional moment of a 3-inch solid steel machine shaft, whose ultimate shearing stress is 50,000 lb. per sq. in. if the factor of safety is assumed to be 8?

Solution. The safe torsional moment, T, of any shaft is shown by formula (31) to be equal to S_sZ_p.

Here $S_s=\frac{50{,}000}{8}=6250$ lb. per sq. in. and from Table IV,

$$Z_p=\frac{\pi d^3}{16}=\frac{\pi\times3^3}{16}=5.3\text{ in.}^3$$

Therefore $T=6250\times5.3=33{,}125$ in.-lb. *Ans.*

Example. A 20-inch gear is keyed to the shaft of the preceding problem and the shaft delivers all its power through this gear to another shaft. What tangential force or tooth pressure will be set up at the pitch circle of the gear?

Solution. Here $R=\frac{20}{2}=10$ in. and $T=33{,}125$ in.-lb.
From formula (39),

$$P_t=\frac{T}{R}$$

$$=\frac{33{,}125}{10}=3312.5 \text{ lb.} \quad \textit{Ans.}$$

Example. A 3000-pound load is raised by a cable working over a drum which is 36 inches in diameter. Find the diameter of shaft to which the drum is keyed if $S_s=6250$ lb. per sq. in.

Solution. Here $P_t=3000$ lb., $R=\frac{36}{2}=18$ in.
From formula (29), $T=P_tR$

$$=3000\times 18=54{,}000 \text{ in.-lb.}$$

From Table IV, $Z_p=\frac{\pi d^3}{16}$, and it is given that $S_s=6250$ lb. per sq. in
Substituting in the design formula,

$$T=S_sZ_p$$

$$54{,}000=6250\times\frac{\pi d^3}{16}$$

$$d^3=\frac{54{,}000\times 16}{6250\times\pi}=44$$

$$d=\sqrt[3]{44}=3.51, \text{ say } 3\tfrac{5}{8} \text{ in.} \quad \textit{Ans.}$$

Shafts Subjected to Combined Bending and Twisting. A shaft is held in position by one or more supports called bearings, which permit freedom of rotation of the shaft about its axis. Power is introduced into the shaft and taken from it by links such as gears and pulleys which are keyed thereto. In the performance of their duties, these links are subjected to forces which together with their weights become loads acting on the shaft at the points where the links are located. These loads cause the shaft to act as a beam setting up reactions at the bearings and bending moments within the shaft. Thus such a shaft is subjected to some maximum bending moment as well as a twisting moment. The design of the shaft must give to it a cross-sectional area that can safely resist the combined effect of both bending and twisting. This makes it necessary first to find the twisting or torsional moment, T, of the shaft and then to analyze

the shaft exactly as a beam in order to find its maximum bending moment, M. T and M are then used in producing a so-called Ideal Torque, the effect of which is equal to the combined effects of the twisting and bending moments. This combination of T and M is effected or obtained by the following formula,

$$T_i = M + \sqrt{M^2 + T^2} \tag{104}$$

in which

T_i = the ideal torque or twisting moment in inch-pounds
M = the maximum bending moment in inch-pounds
T = the twisting moment or torque in inch-pounds.

A design formula for combined bending and twisting is then obtained by substituting or using T_i for T and S_t for S_s in formula (31), thus

$$T_i = S_t Z_p \tag{105}$$

Example. A solid steel shaft is subjected to a bending moment of 121,000 in.-lb. and a twisting moment of 56,000 in.-lb. If S_t is equal to 8000 lb. per sq. in., what diameter of shaft need be used?

Solution. Here $M = 121{,}000$ in.-lb., and $T = 56{,}000$ in.-lb. Evaluating in formula (104),

$$\begin{aligned} T_i &= M + \sqrt{M^2 + T^2} \\ &= 121{,}000 + \sqrt{121{,}000^2 + 56{,}000^2} \\ &= 121{,}000 + 133{,}000 = 254{,}000 \text{ in.-lb.} \end{aligned}$$

With $S_t = 8000$ lb. per sq. in., $Z_p = \frac{\pi d^3}{16}$, and the above value of T_i, we obtain from formula (105),

$$\begin{aligned} T_i &= S_t Z_p \\ 254{,}000 &= 8000 \times \frac{\pi d^3}{16} \\ d^3 &= \frac{254{,}000 \times 16}{8000 \times \pi} = 162 \\ d &= \sqrt[3]{162} = 5.44 \text{ in., say } 5\tfrac{7}{16} \text{ or } 5\tfrac{1}{2} \text{ in.} \quad \textit{Ans.} \end{aligned}$$

Example. A nickel-steel shaft carries two gears, C and D, located at distances of 10 inches and 16 inches respectively from the center lines of the left and right bearings as shown in Fig. 65. Gear C is 24 inches in diameter, gear D, 8 inches in diameter. The distance between the center lines of bearings is 8 feet. The shaft transmits 15 horsepower at 100 revolutions per minute. The power

is delivered to the shaft at gear C and taken out at gear D, in such a manner that the tooth pressures, P_{tc} and P_{td}, which become loads supported by the shaft, act vertically downward as shown in the end view of Fig. 65. Find the diameter of the shaft if $U_t = 90{,}000$ lb. per sq. in., $F=8$, and gears C and D weigh 200 pounds and 75 pounds respectively.

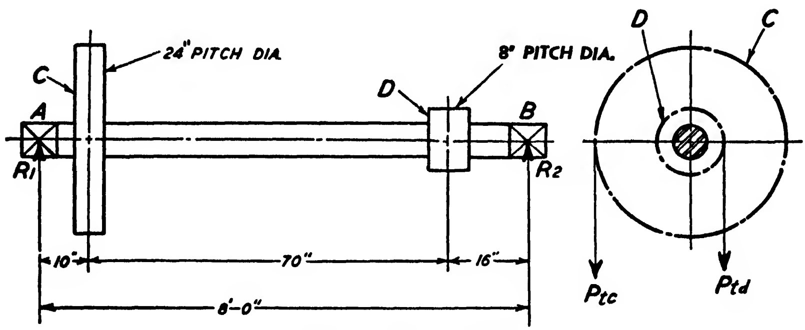


Fig 65. Shaft with Gears

Solution. Step 1. To find the torque in the shaft. With $H=$ 15 hp. and $N=100$ r.p.m., from formula (**34**),

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 15}{2\times\pi\times 100}=9450 \text{ in.-lb.}$$

Step 2. To find the maximum bending moment of the shaft acting as a simple beam.

With the torque as found in Step 1 and the radius of gear C as $\frac{24}{2}=12$ in., formula (**39**) will permit us to obtain the tangential force, P_{tc}, occurring at gear C, which together with the weight of C forms the total load on the shaft at that point. Thus

$$P_{tc}=\frac{T}{R}$$

$$=\frac{9450}{12}=788- \text{ lb.}$$

In a similar manner, since for D the radius of gear $=\frac{8}{2}=4$ in.. the tangential force at D,

$$P_{ta}=\frac{T}{R}$$

$$=\frac{9450}{4}=2363-\text{ lb.}$$

The total load at $C=788+200=988$ lb.

The total load at $D=2{,}363+75=2{,}438$ lb.

With these loads now known, we can proceed to find the reactions R_1 and R_2.

Taking the center of moments at B,

$$96R_1-988\times 86-2438\times 16=0$$

$$96R_1=85{,}000+39{,}000$$

$$96R_1=124{,}000$$

$$R_1=\frac{124{,}000}{96}=1291\text{ lb.}$$

Taking the center of moments at A,

$$-96R_2+2438\times 80+988\times 10=0$$

$$96R_2=195{,}040+9880$$

$$96R_2=204{,}920$$

$$R_2=\frac{204{,}920}{96}=2135\text{ lb.}$$

To check, $R_1+R_2=1291+2135=3426$ lb., the sum of the loads.

Neglecting the weight of the shaft, the maximum bending moment will be at either C or D.

$$M_c=1291\times 10=12{,}910\text{ in.-lb.}$$

$$M_d=-2135\times 16=-34{,}160\text{ in.-lb.}$$

$$\therefore\ M=+34{,}160\text{ in.-lb.}$$

Step 3. To find T_i. From formula **(104)**,

$$T_i=M+\sqrt{M^2+T^2}$$

$$=34{,}160+\sqrt{34{,}160^2+9450^2}$$

$$=34{,}160+35{,}400=69{,}560\text{ in.-lb.}$$

Step 4. To find the diameter.

Substituting our known values together with Z_p, found in Table IV as $\frac{\pi d^3}{16}$, in formula **(105)**,

$$T_i=S_tZ_p$$

$$69{,}560=\frac{90{,}000}{8}\times\frac{\pi d^3}{16}$$

$$d^3=\frac{69{,}560\times8\times16}{90{,}000\times\pi}=31.5$$

$$d=\sqrt[3]{31.5}=3.15-\text{ in., say }3\tfrac{1}{4}\text{ in.}\quad \textit{Ans.}$$

Torsional Rigidity of Shafting. When a tangential force acts upon a shaft to set up a torsional moment therein, it tends to deflect the elements of the shaft as shown in Fig. 66. Here, an element of the shaft, AB, has been deflected through the angle of twist, θ (theta), (shown exaggerated) to the position CB by the action of the tangential force, P_t.

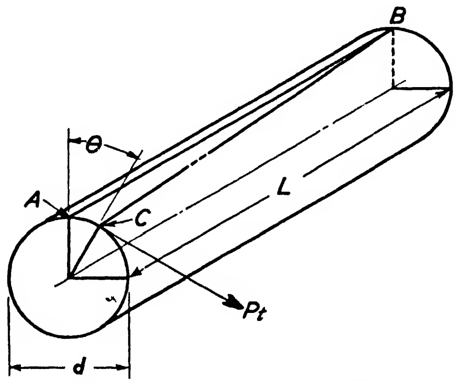


Fig 66 Torsional Deflection of a Shaft

This torsional deflection in a shaft when excessive is the cause of torsional vibration, and failure of the shaft may take place due to it. Hence the design of a shaft should take this torsional deflection into consideration so that the shaft will have a large enough diameter to give it stiffness or rigidity.

The diameter of a shaft which is suitable from a standpoint of torsional rigidity may be obtained from the following formula:

$$d=0.272\sqrt[4]{T} \tag{106}$$

where T is the torque in inch-pounds. This formula limits the angle of twist or the torsional deflection to 0.1 degree per foot of length. In its derivation, a value of 13,000,000 representing the modulus of elasticity in shear was used.

Example. Find the diameter of a steel shaft to transmit 30,000

in.-lb. of torque, allowing S_s to be 7000 lb. per sq. in. Will the resulting diameter be satisfactory from a standpoint of torsional rigidity?

Solution. Here $T=30{,}000$ in.-lb. $S_s=7{,}000$ lb. per sq. in. $Z_p=\frac{\pi d^3}{16}$. Substituting these values in formula (**31**),

$$T=S_sZ_p$$

$$30{,}000=7000\times\frac{\pi d^3}{16}$$

$$d^3=\frac{30{,}000\times16}{7000\times\pi}=21.8$$

$$d=\sqrt[3]{21.8}=2.78 \text{ in.} \quad \textit{Ans.}$$

Applying formula (**106**),

$$d=0.272\sqrt[4]{T}$$

$$=0.272\sqrt[4]{30{,}000}$$

$$=0.272\times13.2=3.59 \text{ in.} \quad \textit{Ans.}$$

The former diameter obtained on the basis of torsion is smaller than the diameter needed for torsional rigidity. Therefore the former is unsatisfactory and the latter, probably a $3\frac{5}{8}$-inch shaft, should be used. The larger value, whichever it is, should always be adopted.

Definition of a Key. Keys are wedge-like steel fastenings that are inserted within two machine or structural parts to prevent them from having relative motion with respect to each other. They cause the two members which they fasten together to act as a unit or a single part. For example when a gear is keyed to a shaft, the key prevents the gear from having any relative rotation with the shaft, that is, it keeps the gear from turning on the shaft. It makes the gear act like an enlargement of the shaft itself. A key used in the latter capacity generally prevents any lengthwise or axial motion as well. Most keys used in machinery have for their chief office the fastening of some machine part to a rotating shaft.

Forms of Keys. Among the many forms of keys that have made their appearance in the design of machinery, those shown in Figs. 67, 68, and 69 are probably the most commonly used.

In Fig. 67 (*a*), the Saddle key is illustrated. A keyway is provided only in the attached part, and the key is hollowed to fit the

shaft. (A slot cut in a shaft or hub to receive a key, is called a keyway or keyseat.) When the key is fitted into its keyway it presses down on the shaft, and thus tends to separate the contact surfaces of shaft

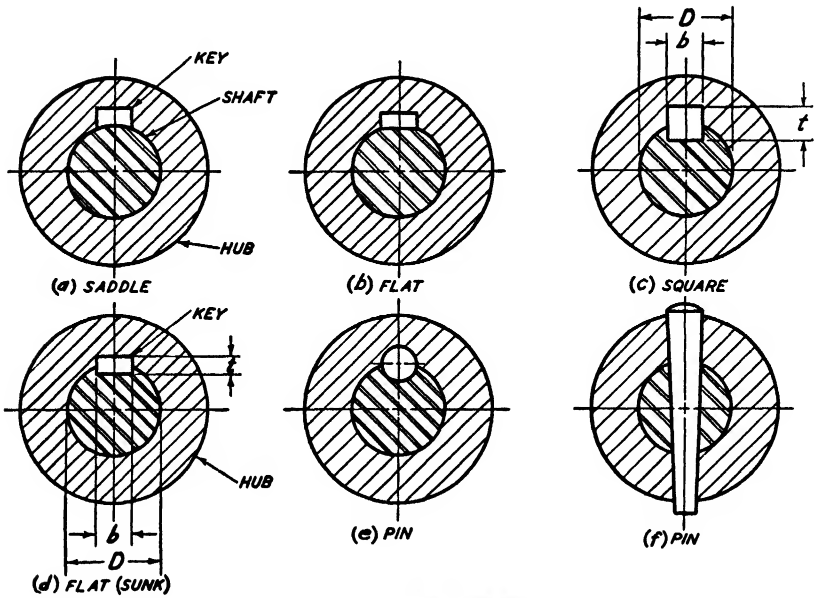


Fig. 67. Forms of Keys

and hub and to set up the frictional resistance which is its only holding power. Sometimes to secure a trifle more holding power, the surface of the saddle key is left flat and the shaft is planed off to

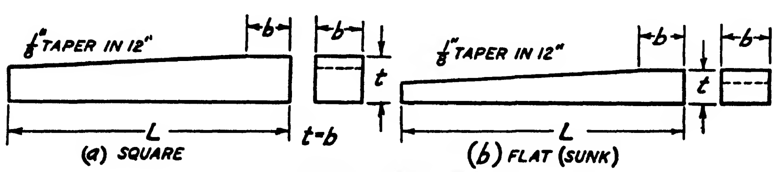


Fig. 68. Taper Keys

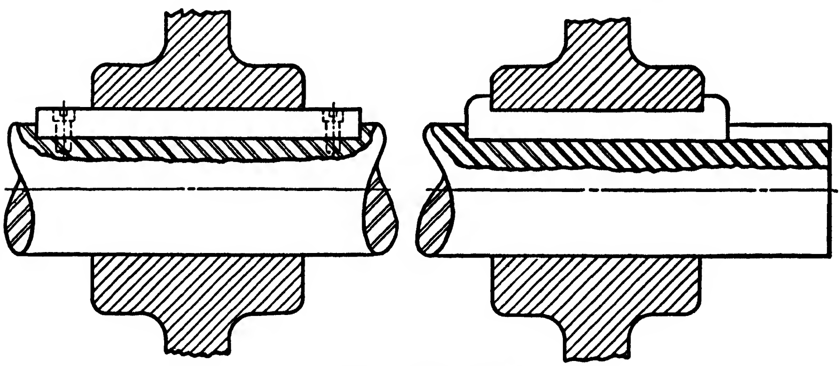
match. The key is then called a Flat key. See Fig. 67 (*b*). Both of these keys can only be used with relatively light loads.

The Square key and the Flat or Rectangular key of Figs. 67 (*c*) and 67 (*d*), are the two most common forms of keys in use today. The square key is square in section as the name implies, while the

TABLE XVII — Square and Flat (Rectangular) Keys (American Standard)

(To be cut from cold-finished stock and to be used without machining)

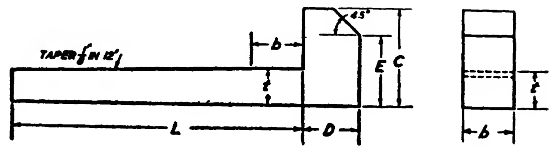
Shaft Diameter, in Inches	Square Stock Key	Flat Stock Key	
	Width, b, in Inches ($b = t$)	Width, b, in Inches	Thickness, t, in Inches
1/2 to 9/16	1/8	1/8	3/32
5/8 to 7/8	3/16	3/16	1/8
15/16 to 1 1/4	1/4	1/4	3/16
1 5/16 to 1 3/4	3/8	3/8	1/4
1 13/16 to 2 1/4	1/2	1/2	3/8
2 5/16 to 2 3/4	5/8	5/8	7/16
2 7/8 to 3 1/4	3/4	3/4	1/2
3 3/8 to 3 3/4	7/8	7/8	5/8
3 7/8 to 4 1/2	1	1	3/4
4 3/4 to 5 1/2	1 1/4	1 1/4	7/8
5 3/4 to 6	1 1/2	1 1/2	1



(a) Fig. 69. Feather Keys (b)

flat or rectangular key has a thickness which is a trifle smaller than the width. They are designed so that one half of the key is in the shaft while the other half is in the hub. Their holding power is due to the resistances which they offer in shear and compression. Either of these keys may be Straight with parallel sides or Tapered, as in Fig. 68, (a) and (b). Tapering is an aid to the insertion and removal of a key and occurs on the upper or hub side. Its use sets up a high pressure between the shaft and the hub, which produces a large frictional force that is helpful in the transmission of power. This high pressure however, is a bursting pressure on the hub. The result of the attempt by the A. S. M. E. to standardize the sizes and dimensions of these keys is given in Table XVII. Square and flat keys come also with a Gib-Head provided as shown in the figure which ac-

TABLE XVIII — Gib-Head Taper Stock Keys (American Standard)



Diameter of Shaft	Square Type					Flat Type					Stock Length of Key		
	Key		Gib-Head			Key		Gib-Head					
	b	t	C	D	E	b	t	C	D	E	L		
1/2 to 9/16	1/8	1/8	1/4	7/32	5/32	1/8	3/32	3/16	1/8	1/8	1/2 to 2	increment	1/4 in.
5/8 to 7/8	3/16	3/16	5/16	9/32	7/32	3/16	1/8	1/4	3/16	5/32	3/4 to 1 1/8	"	3/8 in.
15/16 to 1 1/4	1/4	1/4	7/16	11/32	11/32	1/4	3/16	5/16	1/4	3/16	1 to 4	"	1/2 in.
1 5/16 to 1 3/4	3/8	3/8	11/16	15/32	15/32	3/8	1/4	7/16	3/8	5/16	1 1/2 to 2 1/4	"	3/4 in.
1 13/16 to 2 1/4	1/2	1/2	7/8	19/32	5/8	1/2	3/8	5/8	1/2	7/16	2 to 8	"	1 in.
2 5/16 to 2 3/4	5/8	5/8	1 1/16	23/32	3/4	5/8	7/16	3/4	5/8	1/2	2 1/2 to 3 3/4	"	1 1/4 in.
2 7/8 to 3 1/4	3/4	3/4	1 1/4	7/8	7/8	3/4	1/2	7/8	3/4	5/8	3 to 12	"	1 1/2 in.
3 3/8 to 3 3/4	7/8	7/8	1 1/2	1	1	7/8	5/8	1 1/16	7/8	3/4	3 1/2 to 14	"	1 3/4 in.
3 7/8 to 4 1/2	1	1	1 3/4	1 3/16	1 3/16	1	3/4	1 1/4	1	13/16	4 to 16	"	2 in.
4 3/4 to 5 1/2	1 1/4	1 1/4	2	1 7/16	1 7/16	1 1/4	7/8	1 1/2	1 1/4	1	5 to 20	"	2 1/2 in.
5 3/4 to 6	1 1/2	1 1/2	2 1/2	1 3/4	1 3/4	1 1/2	1	1 3/4	1 1/2	1 1/4	6 to 24	"	3 in.

(All dimensions are in inches)

companies Table XVIII. This table provides standard dimensions of the gib-head and keys including the stock lengths. The purpose of the head is to permit easy removal of the key when one end of the keyseat is inaccessible.

Figs. 67 (*e*) and 67 (*f*) show two methods of using a round tapered key called a Pin key. Such keys are easy to use and are suitable for light loads. The use of several pin keys in the manner of Fig. 67 (*e*) in the same hub and shaft will permit the transmission of considerable power.

Feather keys, or Splines, are used to prevent a machine part from turning on a shaft, but, unlike other keys, permit the part to move lengthwise or along the axis of the shaft. Fig. 69 shows two

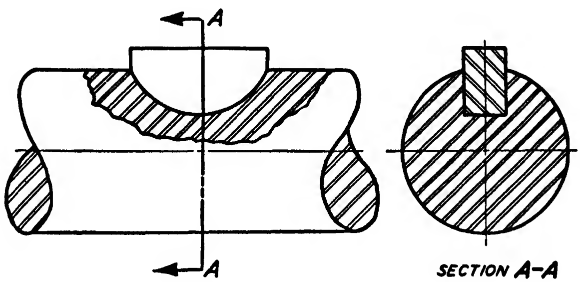


Fig 70. Woodruff Key

different installations of feather keys. In Fig. 69 (*a*) the key is made relatively long to give the desired latitude of axial movement and is held fixed to the shaft by two or more flat fillister-head machine screws. In Fig. 69 (*b*) the key is held to the hub of the wheel as shown, and moves freely with the hub along the keyseat of the shaft.

The Woodruff key of Fig. 70 is nearly semicircular in outline and rests in a keyseat, which is curved in conformity to the key. The semicircular shape permits this key to rock or rotate a little in the keyseat of the shaft until the upper edge of the key is fitting perfectly against the keyway of the hub. When considerable power is to be transmitted, several keyseats for these keys can be cut in a line in the shaft matching a common keyseat of the hub. This avoids cutting deeply into the shaft for the keyseat of a single larger key to transmit the same power.

The Strength of Rectangular (Square or Flat) Keys. An investi-

gation of the strength of these keys will now be made. This will enable one to design a key or to check a standard key for its working stresses and factors of safety under the given conditions.

Fig. 71 illustrates a shaft with its key in place. The hub is not shown but the student should visualize it over the upper half of the key. He is then more likely to realize how the key resists the tangential force that must be set up on the outside of the shaft when this latter transmits power that is delivered to it or taken from it at the location of the key. The area of the key in shear and an area of the

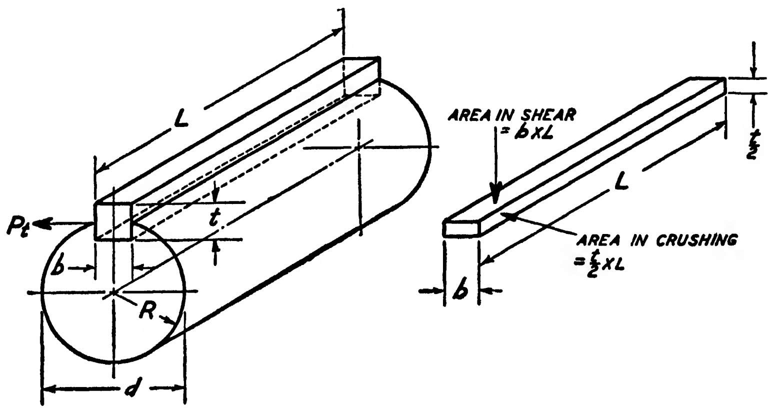


Fig. 71

key (or shaft) in crushing are shown in Fig. 71, having been projected over from their locations in the main view. We shall let

P_t = tangential force set up at the outside of shaft, in pounds
R = radius of shaft in inches
d = diameter of shaft in inches
L = length of key in inches
b = width of key in inches
t = thickness of key in inches
S_s = shearing stress of shaft or key (assuming same material) in pounds per square inch
S_c = crushing stress of shaft or key in pounds per square inch
T = twisting or torsional moment of shaft in inch-pounds.

From formula (39), the tangential force that is set up at the outside of the shaft by the transmission of a torque, T, is,

$$P_t = \frac{T}{R}$$

The key resists this load, P_t, placed upon it over a rectangular area, $b \times L$ throughout the middle of the key. This area is parallel to the direction of the load (or tangential force) and hence is an area in shear, and the tangential force, P_t, is a shearing load on the key. Hence from formula (13),

$$\begin{aligned} P_t &= AS_s \\ &= bLS_s \end{aligned} \tag{107}$$

Solving formula (107) for S_s,

$$S_s = \frac{P_t}{bL} \tag{108}$$

The key resists this same load, P_t, over another rectangular area, $\frac{t}{2} \times L$. This area is perpendicular to the direction of the load. Since the load tends to compress or crush this area of the key against the shaft, the area is involved in compression and the load, P_t, with respect to this area is a compressive or crushing load. From formula (9)

$$\begin{aligned} P_t &= AS_c \\ &= \frac{t}{2}LS_c \end{aligned} \tag{109}$$

Solving formula (109) for S_c,

$$S_c = \frac{P_t}{\frac{t}{2}L} = \frac{2P_t}{tL} \tag{110}$$

As we investigate formulas (107) and (109), we notice that the shearing resistance of the key, bLS_s, and the crushing resistance, $\frac{t}{2}LS_c$, are both equal to the tangential force, P_t. As things which are equal to the same thing are equal to each other, we have

$$bLS_s = \frac{t}{2}LS_c$$

$$bS_s = \frac{t}{2}S_c$$

Dividing both members of the above by $b \times S_c$,

$$\frac{S_s}{S_c} = \frac{t}{2b} \tag{111}$$

If formulas (**108**) and (**110**) are solved for the induced shearing and compressive stresses of a key, or if formula (**111**) is used to find the ratio of these stresses, it will appear that the factor of safety in compression is much lower than the factor of safety in shear. The computation of these factors of safety for a given case will prove this discrepancy to exist. However, it is known that when a key is fitted tightly in its keyseat, the actual compressive stress in the key is much less than the computed value. For this reason, the actual factor of safety in compression is larger than its computed value, and in all probability closely approaches the factor of safety in shear. It will be noted from formula (**111**) that the ratio, $\frac{S_s}{S_c}$, is equal to one-half when $t=b$, as in a square key, and that it is a trifle less than one-half for a flat (sunk) key.

From formula (**107**),

$$P_t=bLS_s$$

since

$$P_t=\frac{T}{R}$$

$$\frac{T}{R}=bLS_s$$

or

$$T=bLS_s\times R \tag{112}$$

The shearing resistance of the key, bLS_s, operates at the outside of the shaft. Its moment arm with respect to the axis of the shaft then is R. Therefore $bLS_s\times R$ is the moment of the shearing resistance of the key, and formula (**112**) states that it is equal to the torque transmitted. In a similar manner, it can be proved that

$$T=\frac{t}{2}LS_c\times R \tag{113}$$

or that the moment of the crushing resistance is also equal to the torque transmitted.

(**Note.** In many different cases of design, the moment of some safe resistance must equal the torque transmitted. This becomes the basic statement or underlying principle of many an analysis such as that used in the design of clutches, couplings, etc.)

A fair assumption for the width of a square or flat key is about one-fourth the shaft diameter or $\frac{d}{4}$. It is evident that the dimensions of a key must be consistent with the diameter of the shaft. The previous assumption will permit a keyseat that will not decrease

the strength of the shaft to too great an extent. A key is generally designed safely to transmit the entire torque of the shaft whether or not it is delivered entirely to the link with which the key is used. In such a case, the minimum length of key should not be less than $1\frac{1}{2}d$. Naturally the length of a hub must be at least equal to the theoretical length of the key.

Example. A solid steel machine shaft with a safe shearing stress of 7000 lb. per sq. in. transmits a torque of 10,500 in.-lb. (*a*) Find the diameter of the shaft. (*b*) A square key is used whose width is equal to $\frac{1}{4}$ the shaft diameter and whose length is equal to $1\frac{1}{2}$ times the shaft diameter. Find the dimensions of the key and check the key for its induced shearing and compressive stresses. (*c*) Obtain the factors of safety of the key in shear and crushing allowing $U_s=50{,}000$ pounds per square inch and $U_c=60{,}000$ pounds per square inch.

Solution. (*a*) Here $S_s=7000$ lb. per sq. in., $T=10{,}500$ in.-lb., and $Z_p=\frac{\pi d^3}{16}$.

Applying formula (**31**),

$$T=S_sZ_p$$

$$10{,}500=7000\times\frac{\pi d^3}{16}$$

$$d^3=\frac{10{,}500\times 16}{7000\times\pi}=7.63$$

$$d=\sqrt[3]{7.63}=1.96 \text{ in., say 2 in.} \quad \textit{Ans.}$$

(*b*) $$b=t=\frac{d}{4}=\frac{2}{4}=\frac{1}{2} \text{ in.} \quad \textit{Ans.}$$

$$L=1\tfrac{1}{2}\times 2=3 \text{ in.} \quad \textit{Ans.}$$

From formula (**39**)

$$P_t=\frac{T}{R}$$

$$P_t=\frac{10{,}500}{1}=10{,}500 \text{ lb.}$$

The shearing stress of the key is obtained from formula (**108**) as follows:

$$S_s=\frac{P_t}{bL}$$

$$=\frac{10{,}500}{\frac{1}{2}\times 3}=7000 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

The crushing stress of the key is obtained from formula (**110**) as follows:

$$S_c=\frac{2P_t}{tL}$$

$$=\frac{2\times10{,}500}{\frac{1}{2}\times3}=14{,}000 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(*c*) Here $U_s=50{,}000$ lb. per sq. in.
Applying formula (**3**),

$$F=\frac{U_s}{S_s}$$

$$=\frac{50{,}000}{7000}=7.1 \quad \textit{Ans.}$$

With $U_c=60{,}000$ lb. per sq. in. Applying formula (**3**)

$$F=\frac{U_c}{S_c}$$

$$=\frac{60{,}000}{14{,}000}=4.3 \quad \textit{Ans.}$$

Example. A medium-steel line shaft transmits a torque of 17,000 inch-pounds. (*a*) Find the diameter of the shaft assuming $F=8$. (*b*) Select a standard flat key from Table XVIII and check it for the induced unit shearing and crushing stresses. (*c*) Obtain the factors of safety of the key in shear and crushing. Assume the material of key and shaft to be the same.

Solution. (*a*) Here $T=17{,}000$ in.-lb., U_s from Table I $=50{,}000$ lb. per sq. in., and $F=8$. From formula (**14**)

$$S_s=\frac{U_s}{F}$$

$$=\frac{50{,}000}{8}=6{,}250 \text{ lb. per sq. in.}$$

Applying formula (**31**) with $Z_p=\frac{\pi d^3}{16}$

$$T=S_sZ_p$$

$$17{,}000=6250\times\frac{\pi d^3}{16}$$

$$d^3=\frac{17{,}000\times16}{6250\times\pi}=13.85$$

$$d=\sqrt[3]{13.85}=2.4 \text{ in., say (see Table XV) } 2\tfrac{7}{16} \text{ in.} \quad \textit{Ans.}$$

(*b*) From Table XVIII, for the above diameter of shaft, a $\frac{7}{16}\times\frac{5}{8}\times3\frac{3}{4}$ in. ($t\times b\times L$) flat key is selected. *Ans.*
Applying formula (39),

$$P_t=\frac{T}{R}$$

$$=\frac{17{,}000}{\frac{1}{2}\times2\frac{7}{16}}=14{,}000 \text{ lb. nearly.}$$

Applying formula (108) with $b=\frac{5}{8}$ in. and $L=3\frac{3}{4}$ in.

$$S_s=\frac{P_t}{bL}$$

$$=\frac{14{,}000}{\frac{5}{8}\times3\frac{3}{4}}=\frac{14{,}000}{\frac{5}{8}\times\frac{15}{4}}$$

$$=\frac{14{,}000}{\frac{75}{32}}=14{,}000\times\frac{32}{75}=6000- \text{ lb. per sq. in.} \quad \textit{Ans.}$$

Applying formula (110) with $t=\frac{7}{16}$ in. and $L=3\frac{3}{4}$ in.,

$$S_c=\frac{2P_t}{tL}$$

$$=\frac{2\times14{,}000}{\frac{7}{16}\times3\frac{3}{4}}=\frac{28{,}000}{\frac{7}{16}\times\frac{15}{4}}$$

$$=\frac{28{,}000}{\frac{105}{64}}=28{,}000\times\frac{64}{105}=17{,}000 \text{ lb. per sq. in.}$$

(*c*) From Table I, U_s=50,000 lb. per sq. in. and U_c=60,000 lb. per sq. in.
From formula (3), the factor of safety in shear is as follows:

$$F=\frac{U}{S}$$

$$=\frac{50{,}000}{6000}=8.3 \quad \textit{Ans.}$$

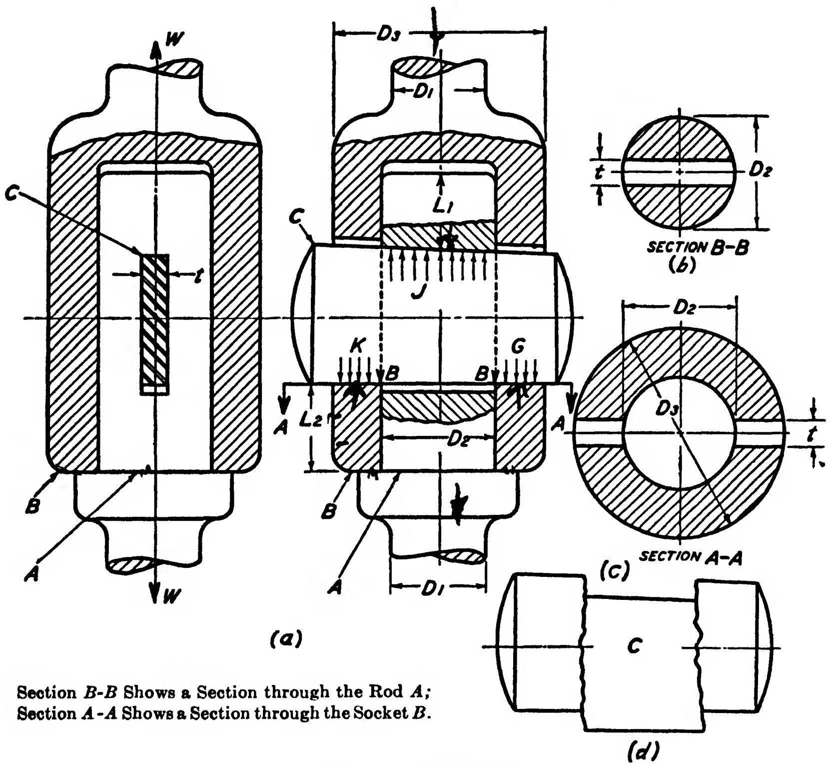
Likewise the factor of safety in crushing is

$$F=\frac{60{,}000}{17{,}000}=3.5 \quad \textit{Ans.}$$

The value of F in crushing will be materially higher than 3.5 due to the selection of a tight-fitting taper key from Table XVIII. This can be obtained with or without the gib-head.

Cotter Key. A Cotter is a form of key used to fasten together

two machine parts which tend to become separated through the action of the forces to which they are subjected. A so-called cottered joint employing such a cotter key is illustrated in Fig. 72, (*a*), (*b*), (*c*), and (*d*) where a rod, *A*, is acted upon by a force, *W*, which tends to pull the rod out of the socket, *B*, of another rod. The cotter, *C*,



Section *B-B* Shows a Section through the Rod *A*;
Section *A-A* Shows a Section through the Socket *B*.

Fig. 72. Cottered Joint

holds the two rods together by being driven through matched openings in the rods. The sections given in the two views of the joint, Fig. 72 (*a*), show the arrangement of the various parts. Fig. 72 (*b*) shows a section through the rod cut by the plane *B-B* showing the opening for the cotter. Fig. 72 (*c*) shows a section through the socket cut by the plane, *A-A*. Fig. 72 (*d*) shows the possibility of failure of the cotter by double shear.

In this discussion, all forces are in pounds, all dimensions in inches, and all stresses in pounds per square inch.

D_1 = the diameter of the rod

D_2 = the diameter of the rod where it is enlarged upon entering the socket.

D_3 = the outside diameter of the socket

t = the thickness of the cotter

h = the mean width of the cotter

L_1 = the distance from the cotter to the end of the rod.

L_2 = the distance from the cotter to the end of the socket.

S_t = the safe tensile stress of the rod and socket.

S_s = the safe shearing stress of the cotter or rod, whichever is involved.

S_c = the safe crushing stress of the cotter, rod or socket, whichever is the least.

W = the load acting on the rods.

The load W, is a tensile load on the rods, so that the diameter, D_1, can be found by applying formula (5), in which P will be replaced by the load, W. Thus

$$W = AS_t = \frac{\pi D_1^2}{4} \times S_t \tag{114}$$

The area in tension through the rod at the location of the cotter, see Fig. 72 (*b*) must be equal to the area $\frac{\pi D_1^2}{4}$. Therefore D_2 must be larger than D_1, for some of the circular area based on D_2 is eliminated by the opening for the cotter. The diameter, D_2, can be obtained by equating the areas, which gives

$$\frac{\pi D_1^2}{4} = \frac{\pi D_2^2}{4} - D_2 t \tag{115}$$

or by applying formula (5), which gives

$$W = AS_t = \left(\frac{\pi D_2^2}{4} - D_2 t\right) S_t \tag{116}$$

The thickness, t, of the cotter, can be taken equal to $0.3D_2$.

The outside diameter of the socket, D_3, is obtained in relation to the weakest section of the socket in tension. This is the section shown in Fig. 72 (*c*). Applying formula (5),

$$W = AS_t$$

$$= \left[\frac{\pi D_3^2}{4} - \frac{\pi D_2^2}{4} - t(D_3 - D_2)\right] S_t$$

$$=\left[\frac{\pi}{4}(D_3^2-D_2^2)-0.3D_2(D_3-D_2)\right]S_t \qquad (117)$$

While the value of D_3 obtained in solving formula (117) will be satisfactory from a standpoint of tension, it may be unsatisfactory from the standpoint of crushing between the cotter and the end of the socket. In other words, crushing between the cotter and socket may require an area that insists on a larger value of D_3 than that already obtained for the tensile area of Fig. 72 (*c*). The larger value would then be used, which of course would make the socket still safer in tension.

The cotter, itself, is in double shear. See Fig. 72 (*d*). Since the cotter is slightly tapered, the area in shear will be based on the mean width of the cotter, *h*, and is equal to $2ht$. From formula (13),

$$W=AS_s=2htS_s \qquad (118)$$

The cotter or rod may fail by crushing over the area, $D_2\times t$, indicated by the letter, *J*, in Fig. 72 (*a*). Since D_2 and t have already been determined, it remains to check this area for its unit crushing stress. If the result obtained in a given case is less than a safe crushing stress for the materials involved, the design is safe. If the result is more than the safe crushing stress, either D_2 or t must be increased, probably D_2. From formula (9)

$$W=AS_c=D_2tS_c$$

or

$$S_c=\frac{W}{D_2t} \qquad (119)$$

The cotter and socket must also be checked for crushing over the area at *K* and *G* of the figure. This area is $t(D_3-D_2)$. If this area as designed proves unsatisfactory in crushing, D_3 should be increased by using formula (120) as a design formula for D_3 instead of as a check formula for S_c. From formula (9),

$$W=AS_c=t(D_3-D_2)S_c$$

or

$$S_c=\frac{W}{t(D_3-D_2)} \qquad (120)$$

The design is not finished until the dimensions, L_1 and L_2 are established. To obtain L_1, it will be noted that due to the load, *W*, as before, the rod might be pulled through the cotter. This would result in shear over two areas, each of which would be equal to $D_2\times L_1$. From formula (13),

$$W = AS_s = 2D_2L_1S_s \tag{121}$$

The socket might on the other hand be pulled through the cotter which places an area, $2L_2(D_3 - D_2)$ in shear. Hence

$$W = AS_s = 2L_2(D_3 - D_2)S_s \tag{122}$$

The cotter should be tapered from $\frac{1}{2}$ inch to 1 inch per foot of length. When it is subjected to fairly live loads, provision should be made for locking it in place.

Example. Design a cottered joint like that of Fig. 72 safely to resist a load, W, of 12,000 pounds that acts along the coincident axes of the rods connected by the cotter. The material of the cotter and rods will permit the following safe stresses: $S_t = 8000$ lb. per sq. in.; $S_c = 12{,}000$ lb. per sq. in.; $S_s = 6000$ lb. per sq. in.

Solution. Applying formula (**114**) to find the diameter, D_1,

$$W = \frac{\pi D_1^2}{4} S_t$$

$$12{,}000 = \frac{\pi D_1^2}{4} \times 8000$$

$$D_1^2 = \frac{12{,}000 \times 4}{8000 \times \pi} = 1.91$$

$$D_1 = \sqrt{1.91} = 1.39 \text{ in., say } 1\tfrac{7}{16} \text{ in. Ans.}$$

Applying formula (**115**) with $t = 0.3D_2$, to find D_2,

$$\frac{\pi D_1^2}{4} = \frac{\pi D_2^2}{4} - D_2 t$$

$$\frac{\pi \times (1\frac{7}{16})^2}{4} = \frac{\pi D_2^2}{4} - D_2 \times 0.3D_2$$

(Note that the actual value of D_1, $1\frac{7}{16}$ in., is used in the preceding step rather than the theoretical value, 1.39 in.)

$$\frac{\pi \times 529}{1024} = \frac{\pi D_2^2}{4} - 0.3D_2^2$$

$$1.62 = D_2^2(0.7854 - 0.3) = 0.485D_2^2$$

$$D_2 = \sqrt{\frac{1.62}{0.485}} = \sqrt{3.34} = 1.83 \text{ in., say } 1\tfrac{7}{8} \text{ in. } \textit{Ans.}$$

Now that D_2 is known, t can be found. Thus

$$t = 0.3D_2 = 0.563 \text{ in., say } \tfrac{5}{8} \text{ in. } \textit{Ans.}$$

To find the value of D_3 from a consideration of tension, use formula (117).

$$W=\left[\frac{\pi}{4}(D_3^2-D_2^2)-t(D_3-D_2)\right]S_t$$

Evaluating in the above with $D_2=1\frac{7}{8}$ in., $t=\frac{5}{8}$ in., $S_t=8000$ lb. per sq. in., and $W=12{,}000$ lb., we have

$$12{,}000=\left[\frac{\pi}{4}(D_3^2-(1\tfrac{7}{8})^2)-\tfrac{5}{8}(D_3-1\tfrac{7}{8})\right]\times 8000$$

$$\frac{12{,}000}{8000}=\frac{\pi}{4}D_3^2-\frac{\pi}{4}\times 1.875^2-\tfrac{5}{8}D_3+\tfrac{5}{8}\times 1.875$$

$$1.5=\frac{\pi}{4}D_3^2-2.76-0.625\ D_3+1.17$$

Transposing all terms to the first member,

$$-\frac{\pi}{4}D_3^2+0.625\ D_3+1.5+2.76-1.17=0$$

Simplifying,

$$\frac{\pi}{4}D_3^2-0.625\ D_3-3.09=0$$

Dividing by $\frac{\pi}{4}$,

$$D_3^2-0.8\ D_3-3.93=0$$

Solving this affected quadratic equation by the algebraic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

in which $x=D_3$, $a=1$, $b=-0.8$, and $c=-3.93$, we have

$$D_3=\frac{-(-0.8)\pm\sqrt{(-0.8)^2-4\cdot 1\cdot(-3.93)}}{2\cdot 1}$$

$$=\frac{+0.8\pm\sqrt{0.64+15.72}}{2}$$

$$=\frac{+0.8\pm\sqrt{16.36}}{2}$$

$$=\frac{+0.8\pm 4.04}{2}=2.42 \text{ in., say } 2\tfrac{7}{16} \text{ in.} \quad \textit{Ans.}$$

Applying formula (**118**) to obtain h, the mean width of the cotter,

$$W = 2htS_s$$

$$12{,}000 = 2 \times h \times \tfrac{5}{8} \times 6000$$

$$h = \frac{12{,}000 \times 8}{2 \times 5 \times 6000} = 1.6 \text{ in., say } 1\tfrac{5}{8} \text{ in.} \quad \textit{Ans.}$$

Checking for crushing at J between the rod and cotter, Fig. 72 (a), by using formula (**119**),

$$S_c = \frac{W}{D_2 t}$$

$$= \frac{12{,}000}{1\frac{7}{8} \times \frac{5}{8}} = 10{,}400 \text{ lb. per sq. in., safe.} \quad \textit{Ans.}$$

The above value of S_c is less than the assumed safe crushing stress of 12,000 lb. per sq. in. Therefore the design is safely dimensioned for this area in crushing.

Checking for crushing between the socket and cotter (at K and G of Fig. 72, (a)), by using formula (**120**),

$$S_c = \frac{W}{t(D_3 - D_2)}$$

$$= \frac{12{,}000}{\frac{5}{8}(2\frac{7}{16} - 1\frac{7}{8})} = \frac{12{,}000}{\frac{45}{128}}$$

$$= 12{,}000 \times \tfrac{128}{45} = 34{,}100 \text{ lb. per sq. in., unsafe.} \quad \textit{Ans.}$$

Since the above answer is altogether too high, it is proved that this area in crushing is too small. To increase its size, we shall solve for a new value of D_3 on the basis of crushing by using formula (**120**) in the form,

$$W = t(D_3 - D_2)S_c$$

$$12{,}000 = \tfrac{5}{8}(D_3 - 1\tfrac{7}{8}) \times 12{,}000$$

Dividing by 12,000,

$$1 = \tfrac{5}{8}D_3 - \frac{75}{64}$$

$$0.625D_3 = 1 + 1.17$$

$$D_3 = \frac{2.17}{0.625} = 3.47 \text{ in., say } 3\tfrac{1}{2} \text{ in.} \quad \textit{Ans.}$$

This value of D_3 definitely replaces the lower value in the design of this joint.

To obtain L_1, by using formula (**121**),

$$W=2D_2L_1S_s$$

$$12{,}000=2\times1\tfrac{7}{8}\times L_1\times6000=\tfrac{15}{4}\times L_1\times6000$$

$$L_1=\frac{12{,}000\times4}{15\times6000}=0.53\text{ in., say }\tfrac{9}{16}\text{ in. to }\tfrac{5}{8}\text{ in.}\quad \textit{Ans.}$$

Solving for L_2 by using formula (**122**),

$$W=2L_2(D_3-D_2)S_s$$

$$12{,}000=2L_2(3\tfrac{1}{2}-1\tfrac{7}{8})\times6000$$

$$12{,}000=42{,}000L_2-22{,}500L_2$$

$$12{,}000=19{,}500L_2$$

$$L_2=\frac{12{,}000}{19{,}500}=0.62\text{ in., say }\tfrac{5}{8}\text{ in. to }\tfrac{3}{4}\text{ in.}\quad \textit{Ans.}$$

Tabulation of results:

$D_1=1\tfrac{7}{16}$ in.	$h=1\tfrac{5}{8}$ in.
$D_2=1\tfrac{7}{8}$ in.	$L_1=\tfrac{9}{16}$ in. to $\tfrac{5}{8}$ in.
$D_3=3\tfrac{1}{2}$ in.	$L_2=\tfrac{5}{8}$ in. to $\tfrac{3}{4}$ in.
$t=\tfrac{5}{8}$ in.	

PROBLEMS

1. Classify shafting.

2. State the difference between a line shaft and a countershaft.

3. Find the diameter of a solid steel shaft which transmits 20,000 inch-pounds of torque, allowing the safe shearing stress to be 7000 pounds per square inch. *Ans.* $2\tfrac{7}{16}$ in.

4. What torsional moment can a 4-inch shaft safely withstand if the allowable shearing stress is 12,000 lb. per sq. inch? *Ans.* 151,000 in.-lb.

5. A mild steel shaft transmits 40 horsepower at 150 revolutions per minute. If the ultimate shearing strength is 50,000 lb. per sq. in. and the factor of safety to be used is 8, find the diameter of the shaft. *Ans.* $2\tfrac{7}{16}$ in.

6. A mild steel line shaft transmits 150 horsepower at 100 r.p.m. With $U_s=50{,}000$ lb. per sq. in. and $F=6$, find (*a*) the diameter of a solid shaft to be used here, (*b*) the size of a hollow shaft whose inside diameter is one-half the outside diameter. *Ans.* (*a*) $3\tfrac{15}{16}$ in. (*b*) $1\tfrac{31}{32}$ in.; $3\tfrac{15}{16}$ in.

7. A 5000-pound load is raised by a cable working over a drum which is 40 inches in diameter. The allowable shearing stress in the shaft of the drum is 8000 lb. per sq. in. (*a*) Find the torque in the drum shaft. (*b*) How large a shaft need be used? *Ans.* (*a*) 100,000 in.-lb. (*b*) 4 in.

8. A solid steel shaft is subjected to a bending moment of 15,000 in.-lb. and a twisting moment of 10,000 in.-lb. If the safe tensile stress is taken as 10,000 lb. per sq. in., what diameter of shaft need be used? *Ans.* $2\tfrac{5}{8}$ in.

9. Find the diameter of a steel transmission shaft to transmit 14,000 in.-lb. of torque if the permissible working stress is taken as 7000 lb. per sq. in. Use formula (**31**). *Ans.* $2\tfrac{3}{16}$ in.

10. Find the diameter of a steel machine shaft to transmit 10,000 in.-lb. of torque if the angle of twist is limited to 0.1 degree per foot of length. (Use formula (**106**)). *Ans.* $2\frac{3}{4}$ in.

11. A steel machine shaft carries two gears, C and D, (See Fig. 65) located at distances of 7 inches and 12 inches, respectively, from the center lines of the left and right bearing. Gear C is 30 inches in diameter; gear D, 10 inches in diameter. The distance between center lines of bearings is 4 feet. The shaft transmits 20 horsepower at 80 revolutions per minute. The power is delivered to the shaft at gear C, and taken out at gear D, in such a manner that the tooth pressures P_{tc} and P_{td}, which become loads supported by the shaft, act vertically downward as shown in the end view of Fig. 65. $S_t = 10{,}000$ lb. per sq. in. and the weights of gears C and D are 250 pounds and 75 pounds, respectively. It is required to find: (*a*) the torque transmitted, (*b*) the maximum bending moment, (*c*) the ideal torque, (*d*) the diameter of a solid shaft to be used under these conditions. *Ans.* (*a*) 15,800 – in.-lb. (*b*) 31,400 in.-lb. (*c*) 66,200 in.-lb. (*d*) $3\frac{1}{4}$ in.

12. What is a key?

13. When is a feather key employed?

14. What resistances act as the holding power of keys?

15. What in general can be assumed as the ratio of the width of a square or flat (sunk) key to the diameter of the shaft? What is the minimum length in terms of the diameter of the shaft, that should be used for these keys?

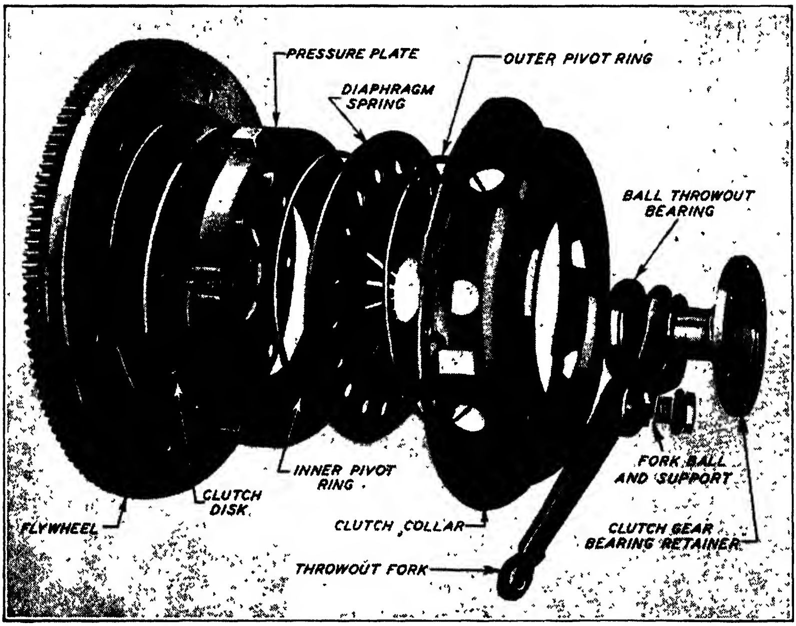
16. A standard square key is $\frac{3}{8}$ x $\frac{3}{8}$ x $2\frac{1}{4}$ inches. It is used with a $1\frac{7}{16}$-inch shaft whose allowable working stress in shear is 6000 lb. per sq. in. (*a*) Find the torque in the shaft using formula (**31**). (*b*) Find the tangential force, P_t, at the outside of the shaft, using formula (**39**). (*c*) What is the load in shear on the key? (*d*) What is the compressive load on the key? (*e*) Find the induced unit shearing stress, S_s, in the key. (*f*) Find the induced crushing stress, S_c, in the key. (*g*) Find the factor of safety in shear of the key if its ultimate shearing strength is 50,000 lb. per sq. in. (*h*) Find the factor of safety in crushing of the key if its ultimate compressive strength is 60,000 lb. per sq. in. *Ans.* (*a*) 3500 in.-lb. (*b*) 4900 in.-lb. (*c*) 4900 in.-lb. (*d*) 4900 in.-lb. (*e*) 5800 lb. per sq. in. (*f*) 11,600 lb. per sq. in. (*g*) 8.6 (*h*) 5.2.

17. Define a cotter key.

18. Design a cottered joint like that of Fig. 72 to safely resist a load, W, of 15,000 pounds which acts along the axes of the rods connected by the cotter. The material of the cotter and rods will permit the following safe stresses: $S_t = 8{,}000$ lb. per sq. in. $S_c = 12{,}000$ lb. per sq. in. $S_s = 6{,}000$ lb. per sq. in.

Ans. $D_1 = 1\frac{9}{16}$ in.	$h = 2$ in.
$D_2 = 2$ in.	$L_1 = \frac{5}{8}$ in.
$D_3 = 4$ in.	$L_2 = \frac{5}{8}$ in.
$t = \frac{5}{8}$ in.	

(**Note.** It will be helpful to the student in solving this problem to follow carefully step by step the solution of the cottered joint that is given in this chapter.)



1938 CHEVROLET CLUTCH SHOWING PARTS ASSEMBLED

Courtesy of Chevrolet Motor Co.

CHAPTER VI

COUPLINGS AND CLUTCHES

Introduction. When it becomes necessary to join or fasten together the ends of two shafts so that they may act as a single unit or so that power can be supplied directly from one to the other, shaft Couplings and Clutches are employed. The former are permanent fastenings to be disconnected only for repairs, etc., while the latter are such as will permit of instant connection or disconnection of the shafts at the will of the operator. Thus the driving shaft is free to rotate at all times, supplying its motion and power to the driven shaft as occasion demands.

Shaft couplings are used under the following conditions:

1. With shafts having collinear axes, that is, axes in the same straight line. Rigid or Flexible couplings of various forms are here used.

2. With shafts having intersecting axes, in which case a Universal Coupling is employed.

3. With shafts whose axes are parallel and at a relatively small distance apart. Here the double-slider crank principle of mechanism is employed.

Clutches are used with shafts whose axes are collinear. They may be classified as:

1. Friction clutches, which drive by virtue of a frictional force set up between plane, conical, or cylindrical surfaces.

2. Positive clutches, the outline or contour of whose contact surfaces give assurance of positive or compulsory driving without the action of friction.

Sleeve or Muff Coupling. This coupling as shown in Fig. 73 is simply a hollow cylinder which is fitted over the ends of the two shafts to be connected and is then keyed thereto by a sunk key. In this manner, the motion of one shaft is communicated to the sleeve and thence to the other shaft, the three links acting as a single rigid member. It is the simplest form of rigid coupling and is most often

constructed of cast iron. It has no projecting parts as is evidenced by its perfectly smooth exterior, which is a distinct advantage from the standpoint of safety. This is especially true in the case of machine parts that have a motion of rotation. Careful consideration should be given to this element of safety in operation in the design and selection of all machine parts. The sleeve coupling is however rather difficult to remove and requires an extra space of half its length on the shaft over which it may be slipped back.

It is evident that any coupling must be designed to transmit safely the full power of its shaft. Hence the sleeve and key of this coupling must safely resist the torsional moment, T, of the shaft.

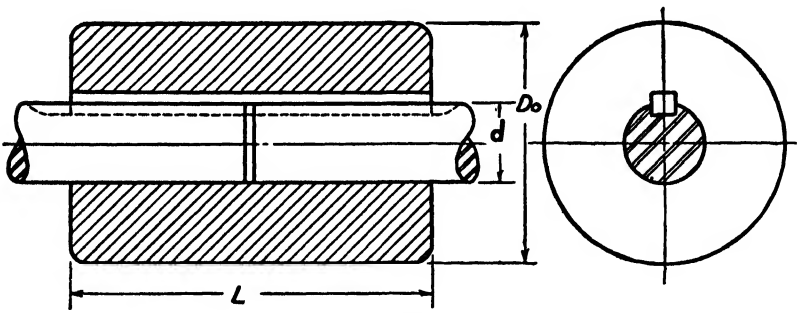


Fig 73 Sleeve Coupling

The method of procedure in the design of the key has been considered in the previous chapter, where it was shown that the moments of the shearing and crushing resistances of the key must each be equal to the torque of the shaft. The effective length of key to be used in the cross-sectional areas which provide the resistances of the key is only that part of the length of the coupling key that is in one of the two shafts connected. Often the coupling key is made in two parts, so that there are in reality two keys in line, one in each shaft. With the latter arrangement, the effective length of key is the length of one part, and with either arrangement becomes equal to approximately one-half the length of the coupling. In so far as the sleeve is concerned, Fig. 73 shows that the sleeve is in reality acting as a section of a hollow shaft in transmitting the torque through itself from one shaft to the other. Therefore formula (31) $T=S_sZ_p$ applies in the design of the sleeve. Applying the notation of Fig. 73 to the polar

section modulus for a hollow circular section (Table IV), we have

$$Z_p = \frac{\pi}{16}\left(\frac{D_o^4 - d^4}{D_o}\right)$$

Substituting this in the preceding formula,

$$T = S_s \times \frac{\pi}{16}\left(\frac{D_o^4 - d^4}{D_o}\right)$$

Dividing both members of the equation by $\frac{\pi}{16}\left(\frac{D_o^4 - d^4}{D_o}\right)$ in order to solve the equation for S_s,

$$S_s = \frac{T}{\frac{\pi}{16}\left(\frac{D_o^4 - d^4}{D_o}\right)} = \frac{T}{\frac{\pi(D_o^4 - d^4)}{16D_o}}$$

$$= \frac{16TD_o}{\pi(D_o^4 - d^4)} \qquad (123)$$

in which D_o = outside diameter of the sleeve in inches

d = inside diameter of the sleeve or the diameter of the shaft in inches

S_s = induced unit shearing stress in the sleeve in lb. per sq. in.

T = torque transmitted in in.-lb.

The form in which formula (**123**) is stated suggests that the mathematical computation will be simplified if the outside diameter of the sleeve for a known diameter of shaft is first assumed and the design checked for S_s. The factor of safety can then be determined which will dictate whether or not a new assumption should be made for the outside diameter. Fair assumptions for D_o and L in the case of a cast-iron sleeve coupling as well as in the cast-iron clamp or compression coupling that follows are:

$$D_o = 2d + \tfrac{1}{2} \text{ in.}$$
$$L = 3\tfrac{1}{2}d$$

It will be noted in the following example that the above formula for D_o yields a very low working stress and hence a high factor of safety especially when used in conjunction with relatively small shafts.

Example. A sleeve coupling is used to connect two 2-inch shafts whose safe shearing stress is 10,000 pounds per square inch. (*a*) What torque is transmitted by the shafting? (*b*) To what torsional moment

is the coupling subjected? (*c*) Find the outside diameter and length of the coupling on the basis of the assumptions as given in the text. (*d*) What will be the induced stress, S_s, in the sleeve if the diameter as found in part (*c*) is used? (*e*) If a cast-iron sleeve with an ultimate shearing stress of 20,000 lb. per sq. in. is used, what is the numerical value of the factor of safety? (*f*) The length of the key being approximately equal to one-half the length of the sleeve, find the unit shearing and crushing stresses of a $\frac{1}{2}$-inch square key to be used.

Solution. (*a*) Here $S_s=10{,}000$ lb. per sq. in. and Z_p, from Table IV, $=\frac{\pi d^3}{16}$

Applying formula (**31**)

$$T=S_sZ_p=S_s\times\frac{\pi d^3}{16}$$
$$=10{,}000\times\frac{\pi\times 2^3}{16}=15{,}700\text{ in.-lb.}\quad \textit{Ans.}$$

(*b*) The torsional moment of the coupling is equal to the torsional moment of the shaft $=15{,}700$ in.-lb. *Ans.*

(*c*)
$$D_o=2d+\tfrac{1}{2}$$
$$=2\times 2+\tfrac{1}{2}=4\tfrac{1}{2}\text{ in.}\quad \textit{Ans.}$$
$$L=3\tfrac{1}{2}d$$
$$=3\tfrac{1}{2}\times 2=7\text{ in.}\quad \textit{Ans.}$$

(*d*) Applying formula (**123**) in which $T=15{,}700$ in.-lb., $D_o=4\frac{1}{2}$ in., and $d=2$ in., we have

$$S_s=\frac{16TD_o}{\pi(D_o^4-d^4)}$$
$$=\frac{16\times 15{,}700\times 4\frac{1}{2}}{\pi(4.5^4-2^4)}=913\text{ lb. per sq. in.}\quad \textit{Ans.}$$

(*e*) Here $U_s=20{,}000$ lb. per sq. in., and $S_s=913$ lb. per sq. in.,

Evaluating in formula (3), $F=\frac{U}{S}=\frac{20{,}000}{913}=21.9$ *Ans.*

(*f*) Here the length of key $=3\frac{1}{2}$ in., the breadth of the key, b, $=\frac{1}{2}$ in., radius of shaft, R, $=\frac{1}{2}\times 2=1$ in., and $T=15{,}700$ in.-lb.

Applying formula (**112**), $T=bLS_sR$

$$15{,}700=\tfrac{1}{2}\times 3\tfrac{1}{2}\times S_s\times 1$$
$$15{,}700=1.75\ S_s$$
$$S_s=\frac{15{,}700}{1.75}=8970\text{ lb. per sq. in.}\quad \textit{Ans.}$$

Applying formula (113) with $t=b=\frac{1}{2}$ in.

$$T=\frac{t}{2}LS_cR$$

$$15{,}700=\tfrac{1}{4}\times3\tfrac{1}{2}\times S_c\times1$$

$$15{,}700=\tfrac{7}{8}S_c$$

$$S_c=\frac{15{,}700}{\frac{7}{8}}=\tfrac{8}{7}\times15{,}700=17{,}940 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

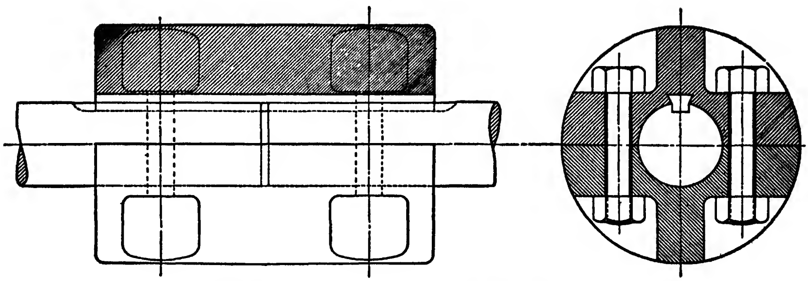


Fig. 74 (a). Clamp or Compression Coupling

Fig 74 (b) Clamp or Compression Coupling
Courtesy of Link-Belt Company, Chicago, Ill.

Clamp or Compression Coupling. This rigid coupling, illustrated in Fig. 74, (*a*) and (*b*), is a modification and improvement of the sleeve coupling. It is simply the latter split in halves which are recessed, Fig. 74 (*a*), for the through bolts which are used to hold the two parts together. The coupling is placed over the ends of the two shafts to be connected and is keyed thereto by a single key passing along its entire length. Its construction in two halves permits of its being readily assembled and removed which is a distinct advantage. The coupling is so bored that upon assembly the through bolts clamp the coupling tightly against the shaft. In this manner a considerable frictional holding power is set up between the shafts and coupling. The coupling when constructed as in Fig. 74 (*b*) is properly reinforced by ribs, and may be had with a smooth protecting casing

which provides an improved appearance and greater safety. The clamp coupling may be used for the transmission of rather large torques.

Flange Coupling. Referring to Fig. 75, it is seen that this rigid coupling consists of two hubs keyed to the two shafts. Sometimes they are forced into place on the shafts and then keyed. The hubs extend into flanges whose faces are brought and held together by a series of bolts arranged concentrically about the shaft so that their axes are parallel to the coincident or collinear axes of the shafts and

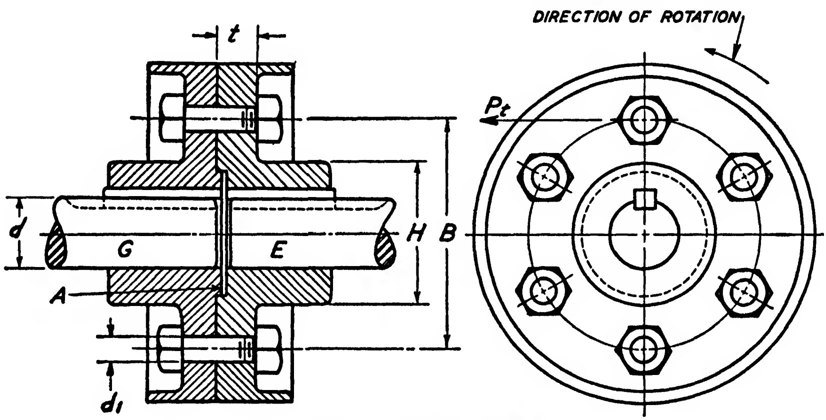


Fig 75. Flange Coupling

coupling. These bolts must be carefully fitted in reamed holes so that each one will take its proportionate part of the load. An additional flange at the outside of the coupling is a safety flange to guard the projecting bolt heads and nuts and add to the rigidity of the coupling. Alignment of the shafts is provided in two ways: 1. by a cylindrical projection on one flange that fits into a corresponding recess in the other as shown in Fig. 75 at A; 2. by extending one shaft through its flange and into the bore of the other. This coupling is generally constructed of cast iron but steel is sometimes used. It is adapted to heavy loads and hence is used almost exclusively on rather large shafting.

Accurate alignment of shafts is essential with all rigid couplings. The most satisfactory location for them is one which is very close to a bearing of the shaft.

The analysis of the design of this coupling is another analysis which includes the application of a number of the fundamentals of this subject. It is thus advantageous in developing the ability to analyze and in bringing the student to the point of view that a consistent logical sequence of properly selected and applied fundamentals yields the design of a machine part, rather than meaningless, mechanical, evaluations in some formulas that seem to give the desired results.

In this analysis, we will use the following notation in which all forces will be in pounds and dimensions in inches:

d = diameter of shaft
d_1 = nominal, or outside diameter of bolts
B = diameter of bolt circle
H = diameter of hub
t = thickness of flange
P_t = total tangential force occurring at bolt circle
n = number of bolts
S_s = allowable unit shearing stress of the steel bolts
S'_s = allowable unit shearing stress of the steel shaft
S_c = allowable unit crushing or compressive stress of the steel bolts or of the material of the flange, whichever is the smaller value.

A few initial assumptions which are consistent with the usual practice are:

$B=3d$
$H=1\frac{1}{2}d+1$ in.
$n=3$, for shaft diameters up to $1\frac{1}{2}$ in.
$n=4$, for shaft diameters up to 4 in.
$n=6$, for shaft diameters up to 7 in.

Referring to Fig. 75, let us assume that shaft E is the driver and that it transmits a torque, T, while driving shaft G. Due to this torque there will be set up at the bolt circle a tangential force, P_t; and from formula (39)

$$P_t=\frac{T}{R}=\frac{T}{\frac{B}{2}}$$

This tangential force is a shearing load on the bolts, for the tendency of the load is to cut through the bolts in a plane that coincides with

the faces of the flanges and to act parallel to the resisting area. The latter is then equal to the sum of the involved areas of the bolts, which gives a total resisting area in shear,

$$A = n \times \frac{\pi d_1^2}{4} = n\frac{\pi d_1^2}{4}$$

From formula (13), $P = AS_s$, it is seen that the load is equal to the shearing resistance so that here, the load,

$$P_t = n \times \frac{\pi d_1^2}{4} \times S_s = n\frac{\pi d_1^2}{4} S_s \qquad (124)$$

Dividing both members of the above equation by n,

$$\frac{P_t}{n} = \frac{\pi d_1^2}{4} S_s \qquad (125)$$

in which $\frac{P_t}{n}$ is the load per bolt. It will also be noted that d_1 is the nominal or outside diameter of the bolts since the threads do not extend along the bolts as far as the face of the flange.

The total shearing resistance, $n\frac{\pi d_1^2}{4} S_s$, see formula (124), is a tangential force that acts through the moment arm, $\frac{B}{2}$, to set up a torque in the driven shaft, G, which is equal to $n\frac{\pi d_1^2}{4} S_s \times \frac{B}{2}$ in.-lb. Since $P_t \times \frac{B}{2} = T$, the torque in the driving shaft, and $P_t = n\frac{\pi d_1^2}{4} S_s$, it is evident that the torque in the driven shaft is equal to the torque in the driving shaft, T. Therefore

$$T = n\frac{\pi d_1^2}{4} S_s \times \frac{B}{2} \qquad (126)$$

Applying formula (31) with S_s' used here as the allowable shearing stress of the shaft

$$T = S' Z_p$$

with

$$Z_p = \frac{\pi d^3}{16}$$

$$T = S_s' \times \frac{\pi d^3}{16}$$

Substituting this value of T in formula (126), we obtain

$$S_s' \times \frac{\pi d^3}{16} = n\frac{\pi d_1^2}{4} S_s \times \frac{B}{2} \qquad (127)$$

Either formula (126) or formula (127) is a statement that the moment of the shearing resistance of the bolts is equal to the torque trans-

mitted. In solving for the diameter of the bolts to be used, either formula (**125**) or (**127**) may be used.

The tangential force, P_t, is not only a shearing load on the bolts but a compressive load as well. As a compressive load, it acts over the bearing areas of the n bolts. The bearing area of each bolt is the projected area of the cylindrical bearing surface between the bolt and either flange. Such a projected area is obtained on a plane of projection which is taken parallel to the axis of the cylindrical surface as shown in Fig. 76. It is therefore equal to $d_1 \times t$ square inches, so that the total area of n bolts in compression is $n \times d_1 \times t$ square inches.

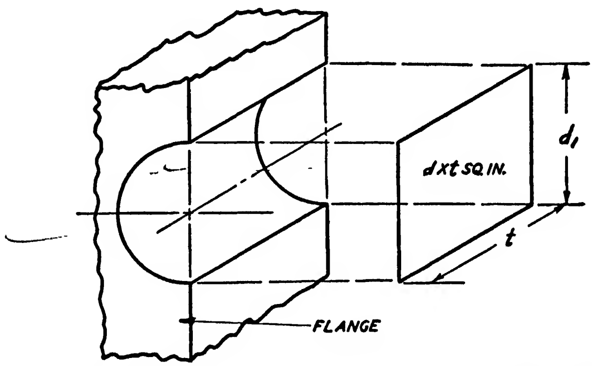


Fig 76 The Bearing or Projected Area of a Cylindrical Surface

This area necessarily yields a total compressive resistance of $nd_1t \times S_c$ pounds which from formula (9), $P = AS_c$, is equal to the load, P_t. Hence we have

$$P_t = nd_1t \times S_c \tag{128}$$

Dividing both members of the above equation by h

$$\frac{P_t}{n} = d_1t \times S_c \tag{129}$$

Since the diameter of the bolts is usually obtained on a basis of shear, formula (**129**) is used to obtain t, assuming the allowable or safe crushing stress, S_c, for the material of the bolt or flange, whichever is the lower. Often t is assumed so as to be consistent with the other dimensions of the coupling and in such a case, formula (**129**) is solved for S_c to find if it, the induced crushing stress, is a safe stress.

As in the case of the shearing resistance, the moment of the safe

crushing resistance must equal the torque transmitted by the shafts. Since the moment arm of the crushing resistance is the radius of the bolt circle, $\frac{B}{2}$, we have

$$T = nd_1tS_c \times \frac{B}{2} \tag{130}$$

Although either formula (**129**) or (**130**) may be used in the design on the basis of compression, the former is possibly more easily and directly applied, for it is merely an expression of a simple stress formula.

Example. It is required to connect two 4-inch shafts by means of a cast-iron flange coupling which employs 6 bolts. The allowable shearing stress of the bolts is 6000 lb. per sq. in. while that of the shafting is 8000 lb. per sq. in. (*a*) Find the diameter of bolts to be used. (*b*) Find the induced crushing stress, S_c, if the thickness of the flange is $\frac{5}{8}$ inch. Is it a safe stress?

Solution. 1st Method: Step 1. To find torque, T.
Applying formula (**31**) to find the torque; with S_s for shaft = 8000 lb. per sq. in., $d = 4$ in., and $Z_p = \frac{\pi d^3}{16}$, (Table IV)

$$T = S_s'Z_p = S_s' \times \frac{\pi d^3}{16}$$

$$= 8000 \times \pi \times \frac{4^3}{16} = 100{,}500 \text{ in.-lb.}$$

Step 2. To find the tangential force, P_t, occurring at the bolt circle.

$$B = 3d$$

$$= 3 \times 4 = 12 \text{ in.}$$

From formula (**39**)

$$P_t = \frac{T}{\frac{B}{2}}$$

$$= \frac{100{,}500}{\frac{12}{2}} = 16{,}750 \text{ lb.}$$

Step 3. To find the bolt diameter, d_1. Here $P_t = 16{,}750$ lb., $n = 6$, and $S_s = 6000$ lb. per sq. in.
Applying formula (**125**),

$$\frac{P_t}{n}=\frac{\pi d_1^2}{4}S_s$$

and evaluating therein

$$\frac{16{,}750}{6}=\frac{\pi\times d_1^2\times 6000}{4}=1500\times\pi\times d_1^2$$

Dividing by $1500\times\pi$,

$$d_1^2=\frac{16{,}750}{6\times 1500\times\pi}=0.59$$

$d_1=\sqrt{0.59}=0.77$ in., say $\frac{7}{8}$ in., which is the next higher standard diameter. *Ans.*

2nd Method. The basis of this method of finding the bolt diameter, d_1, is the statement that the moment of the shearing resistance is equal to the torque transmitted.

With $S_s'=8000$ lb. per sq. in. and with other values as given in the first solution, we shall use formula (**127**),

$$S_s'\times\frac{\pi d^3}{16}=n\frac{\pi d_1^2}{4}S_s\times\frac{B}{2}$$

Evaluating in the above,

$$8000\times\frac{\pi\times 4^3}{16}=6\times\frac{\pi\times d_1^2}{4}\times 6000\times\frac{12}{2}$$

Simplifying,

$$32{,}000\pi=54{,}000\times\pi d_1^2$$

Dividing by $54{,}000\times\pi$,

$$d_1^2=\frac{32{,}000\pi}{54{,}000\pi}=0.59$$

$$d_1=\sqrt{0.59}=0.77 \text{ in., say } \tfrac{7}{8} \text{ in.} \quad \textit{Ans.}$$

It is seen that the second solution is an exact check of the first solution.

(*b*) We shall solve for S_c by using formula (**129**),

$$\frac{P_t}{n}=d_1t\times S_c$$

Evaluating in the above formula with $t=\frac{5}{8}$ in. as given,

$$\frac{16{,}750}{6}=\tfrac{7}{8}\times\tfrac{5}{8}\times S_c$$

$$S_c=\frac{16{,}750\times 8\times 8}{6\times 7\times 5}=5100 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

This value of S_c is the induced unit crushing stress that is set

up by a crushing load of $\frac{16{,}750}{6}$ pounds on the projected area or bearing area of each bolt. If we assume from Table I that U_c, for cast iron, $=80{,}000$ lb. per sq. in. and U_c, for steel, $=60{,}000$ lb. per sq. in., it is evident that there will be a greater tendency to crush the steel than to crush the cast iron. Hence we shall determine the factor of safety for steel. Thus from formula (3)

$$F=\frac{U}{S}$$

Evaluating

$$F=\frac{60{,}000}{5100}=11.8,$$

which is proof that the induced stress, S_c, is also a safe stress. *Ans.*

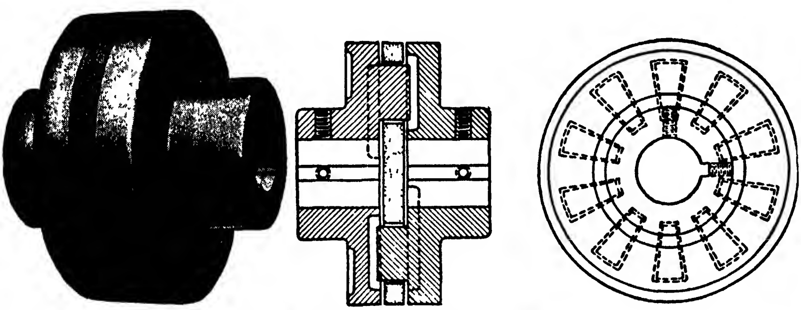


Fig 77 (a). Flexible Coupling
Courtesy of Link-Belt Company, Chicago, Ill.

Flexible Couplings. This type of coupling is used to permanently connect two shafts permitting at the same time some non-alignment to exist between them. Hence such a coupling is particularly advantageous in a direct drive in connecting the shaft of a machine to that of a prime mover, such as an electric motor. Another important characteristic of the flexible coupling is its ability to absorb shock which may be imparted to one of the shafts. Its use is also helpful in obtaining the proper contact between the shaft and its bearings, which is necessary if the latter are to function properly.

There are many different designs of flexible couplings in use today. One of these is illustrated in Fig. 77 (*a*). Here two cast-iron flanges are secured in place on the shafting by means of keys and set screws. These flanges are provided with lugs which are cast as

an integral part of them. These lugs fit into openings in a leather disk which is placed between the flanges and which serves as the flexible intermediate driving member of the assembly. The leather disk is composed of several plies which are cemented and stitched together.

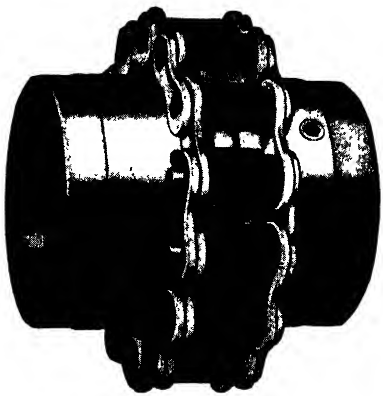


Fig. 77 (b). Flexible Coupling
Courtesy of Link-Belt Company, Chicago, Ill.

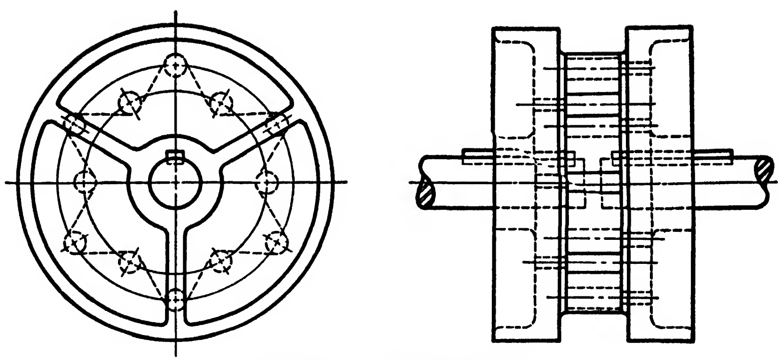


Fig. 77 (c). Flexible Coupling

It provides flexibility in all directions. This type of coupling is strong and long wearing and is especially intended for connecting motors directly to the machines they are to drive. The coupling works equally well at high and low speeds and insures noiseless operation.

Other flexible couplings are shown in Figs. 77 (*b*) and 77 (*c*). The former is made up of two coupling halves upon which are cut identical tooth sprocket wheels. Flexibility is obtained by connecting the coupling halves with a specially constructed roller chain, which

is applied over both sprockets. It may be procured with or without a protective casing. The Link-Belt Company of Chicago considers it good practice and advisable to enclose and lubricate this type of coupling and states that the housing and lubrication while always desirable are not definite requirements unless the couplings are used under one or all of the following conditions:

1. Speeds above 500 r.p.m.
2. Dusty atmosphere
3. Moisture-laden atmosphere, or subject to water splash.

The coupling of Fig. 77 (*c*) employs two flanges which are similar to those used in the ordinary flange coupling. In this type however the flanges are separated so as to admit of a leather driving and connecting belt, which passes alternately inside and outside the inner and outer pins which project from the faces of the flanges. The pins are set in two circles, the outer attached to one flange, the inner to the other. Either flange may drive, and in either direction. In driving, the driven pins will lag, settling back into loops of the flexible belt.

Universal Coupling. When two shafts are not in line but have intersecting axes, they may be connected by a Universal Coupling as shown in Fig. 78 (*a*) and Fig. 78 (*b*). The action peculiar to this joint is made possible by the use of the two pins, *c* and *d*, whose axes are at right angles to each other. Either yoke, *a* or *b*, can be turned about the axis of each pin, so that adjustment to the angle, θ, between the shaft axes can be made. While the two shafts make a complete revolution in the same length of time, their velocity ratio is not quite constant throughout the revolution. The smaller the angle, θ, the smaller will be this variation in angular velocities. This variation in the angular velocities of driving and driven shafts can be entirely eliminated by the use of an intermediate shaft coupled to each of them by means of a universal coupling. In such a case, the intermediate shaft is so placed between the two main shafts that it makes the same angle with each. When a single universal is employed, the angle, θ, should not exceed 15 degrees.

Oldham Coupling. This is a form of flexible coupling that is especially adapted for connecting two shafts which are parallel but not in line. Disks, *A* and *B*, with grooved faces are keyed on the ends of the shafts as shown in the sketch of Fig. 79 (*a*) and in the photograph of Fig. 79 (*b*). A third disk, *C*, with two tongues at right

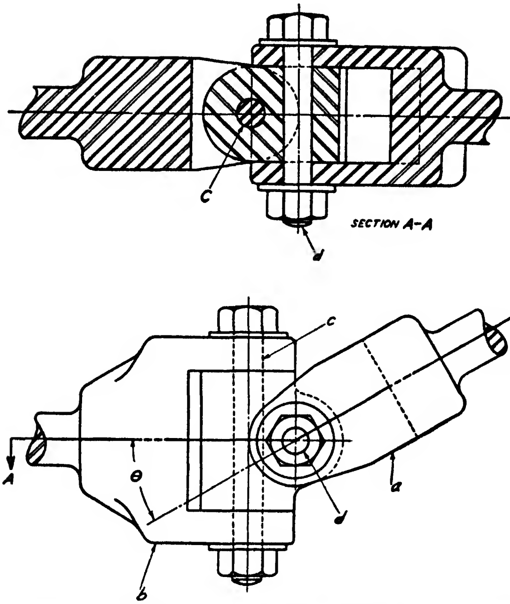


Fig 78 (a) Universal Coupling

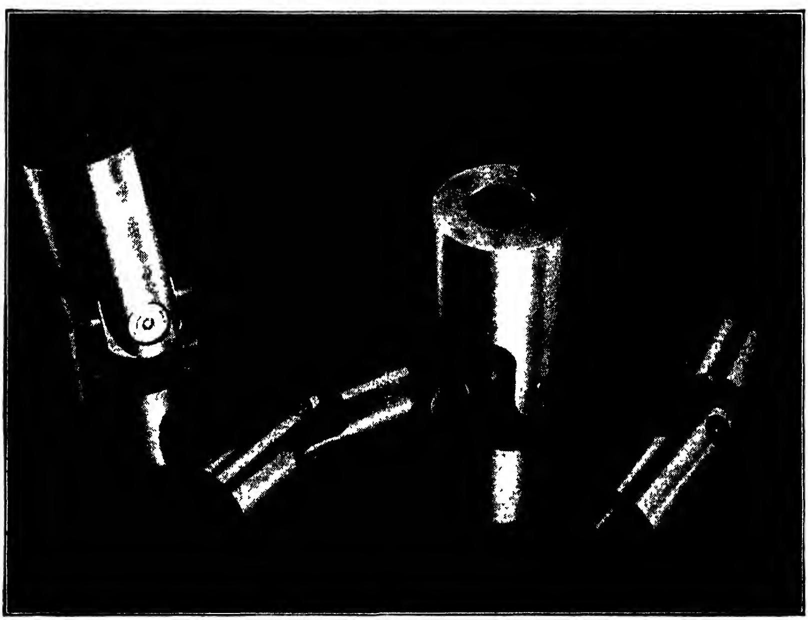


Fig. 78 (b). Universal Couplings
Courtesy of Boston Gear Works, Inc., North Quincy, Mass.

angles to each other and located on opposite sides of the disk, lies between the other two disks. The tongues on C fit into the grooves of A and B. With this type of coupling the connected shafts have the same angular velocity.

Cone Clutch. This friction clutch illustrated in Fig. 80 consists in the main of two links which are the driver or cup, D, and the follower or cone, E. The cup is keyed to the driving shaft, B, by a

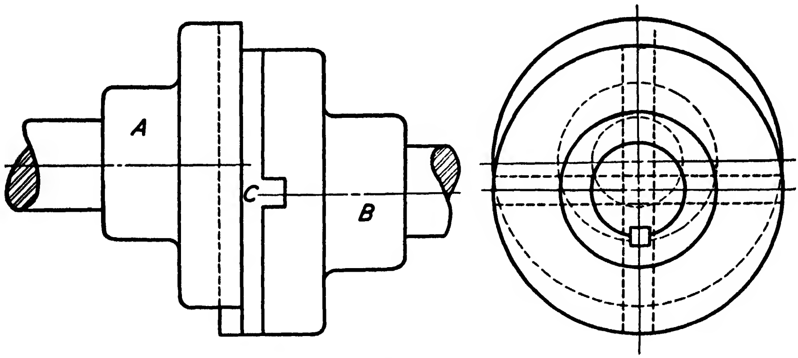


Fig. 79 (a) Oldham Coupling

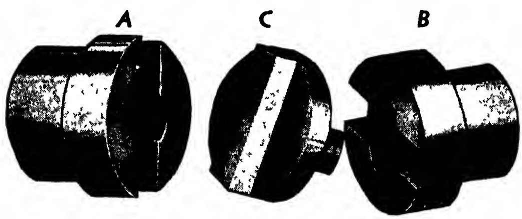


Fig 79 (b). Oldham Coupling
Courtesy of Foote Bros. Gear and Machine Corp., Chicago, Ill.

sunk key and has an inside conical surface or face which exactly fits the outside conical surface of the cone, or male member, E. The latter, riding on a feather key in shaft C, may be shifted along its shaft by the forked shifting lever, G, to engage the clutch by bringing the two conical surfaces into contact. The frictional resistance set up at this contact surface makes it possible to have the torque transmitted from one shaft to the other. In some cases, a spring placed around the driven shaft in contact with the hub of the cone, holds the clutch faces in contact and maintains the pressure between them. In the latter event, the forked lever is used only for disengagement

of the clutch. The contact surfaces of the clutch may present metal to metal contact but more often the male member is lined with some such material as wood, leather, cork, or an asbestos composition. Upon the material selected for the clutch faces and the conditions of operation will depend the allowable normal pressure and coefficient of friction that may obtain in the design and operation of the clutch. The cone clutch was formerly used in automotive design but has been superseded in that field by the single dry-plate disk clutch. However it still is of great use in industrial practice. It is adaptable in cases where loads are to be engaged and disengaged at frequent intervals.

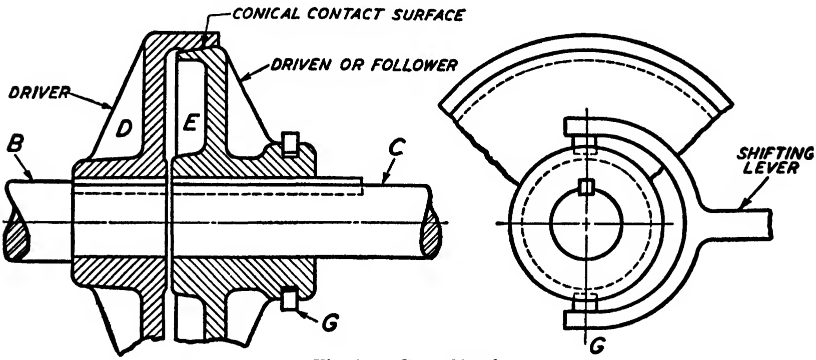


Fig. 80. Cone Clutch

The design of a cone clutch rests primarily in obtaining the mean radius, R, and the width of the contact surfaces, w. The latter is referred to as the face of the clutch. Since one of these dimensions is dependent on the other (or, in other words, one is a function of the other) either may be assumed and the other calculated on the basis of the assumption. In analyzing the forces which are concerned in this design, we shall refer to Fig. 81 and shall use the following notation in which the forces will all be given in pounds and the dimensions in inches.

T = torque in the shafts
R = the (mean) radius of the clutch
w = width of clutch or the clutch face
α = the angle of the clutch, to be assumed from 8° to 15°
P_t = the frictional resistance, a tangential force
P_n = the total normal pressure between contact surfaces

p_n = the allowable unit normal pressure between contact surfaces
P_a = the axial pressure
μ = the coefficient of friction
A = the area of contact surface of clutch.

The cone of the clutch is forced against the cup of the clutch by an axial force, P_a. This subjects the contact surfaces to a total normal pressure, P_n. Now P_n can be assumed to be concentrated at the two points N and N shown in Fig. 81, so that one-half of P_n, or $\frac{P_n}{2}$, acts at

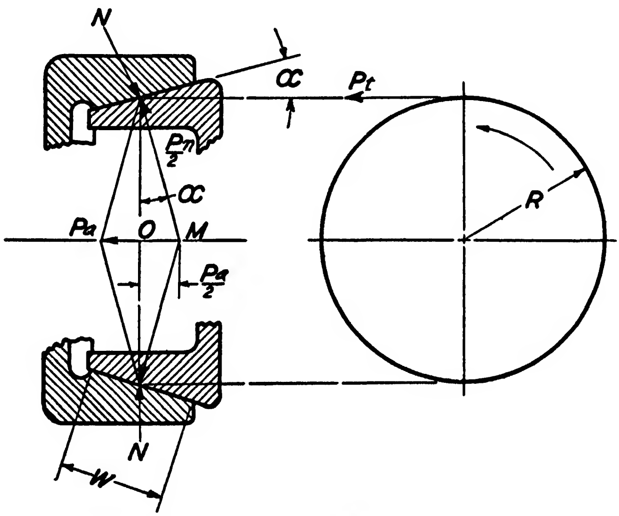


Fig. 81. Diagram of Forces of Cone Clutch

each point. This permits the construction of the parallelogram of forces as given in the figure. From the right triangle, MON, it is evident that

$$\sin \alpha = \frac{\frac{P_a}{2}}{\frac{P_n}{2}} = \frac{P_a}{P_n}$$

from which

$$P_a = P_n \sin \alpha \tag{131}$$

or

$$P_n = \frac{P_a}{\sin \alpha} \tag{132}$$

It must now be remembered that the frictional force that is set up between two contact surfaces is equal to the product of the normal pressure and the coefficient of friction, and that the frictional force

in the case of a clutch is the tangential force that introduces a torque into the driven shaft through a moment arm, R, which torque is equal to that in the driving shaft. Hence the frictional resistance set up between the cup and the cone must be equal to the tangential force that the torque to be transmitted would produce at a radial distance, R, from the axis of the shaft. From these statements, we obtain

$$P_t = \mu P_n$$

or

$$P_n = \frac{P_t}{\mu} \tag{133}$$

and by formula (39)

$$P_t = \frac{T}{R}$$

Substituting the value of P_n of formula (133) in formula (131),

$$P_a = \frac{P_t}{\mu} \times \sin \alpha = \frac{P_t \sin \alpha}{\mu} \tag{134}$$

Since the area of a conical frustum is equal to the product of its mean circumference and its slant height,

$$A = 2\pi R w \tag{135}$$

and since the total normal pressure divided by the allowable normal pressure per square inch is equal to the area,

$$A = \frac{P_n}{p_n} \tag{136}$$

The final solution of the cone clutch problem for w or R can now be effected by obtaining algebraic expressions for the area, A, from both formulas (135) and (136). Since these expressions are equal to the same thing, they are equal to each other. So the equating of these two equal expressions sets up an equation which may be solved for its unknown, w or R, as the case may be. The axial pressure which is required to hold the cone in the cup can next be found by substituting in either formula (131) or formula (134).

Example. It is required to find the width of clutch face and the axial pressure of a cast-iron cone clutch, using the following data:

$H = 25$ horsepower
$N = 200$ r.p.m.
$R = 9$ inches
$p_n = 45$ pounds per square inch
$\mu = 0.12$
$\alpha = 10$ degrees

Solution. Step 1. To find the torque, T. Applying formula (**34**)

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 25}{2\times\pi\times 200}=7880 \text{ in.-lb.}$$

Step 2. To find the tangential force or frictional resistance, P_t. Applying formula (**39**)

$$P_t=\frac{T}{R}$$

$$=\frac{7880}{9}=875 \text{ lb.}$$

Step 3. To find the total normal pressure, P_n. Applying formula (**133**)

$$P_n=\frac{P_t}{\mu}$$

$$=\frac{875}{0.12}=7300 \text{ lb.}$$

Step 4. To find the area, A, by applying formula (**135**).

$$A=2\pi Rw$$

$$=2\times\pi\times 9\times w=18\pi w \text{ sq. in.}$$

Step 5. To find the area, A, through formula (**136**).

$$A=\frac{P_n}{p_n}$$

$$=\frac{7300}{45}=162 \text{ sq. in.}$$

Step 6. Equating the values of A found in the two preceding steps,

$$18\pi w=162$$

$$w=\frac{162}{18\pi}=2.87 \text{ in., say } 2\tfrac{7}{8} \text{ in.} \quad \textit{Ans.}$$

Step 7. To find the axial pressure, P_a. Applying formula (**131**).

$$P_a=P_n \sin\alpha$$

$$=7300\times\sin 10°=7300\times 0.174=1270 \text{ lb.} \quad \textit{Ans.}$$

Example. It is required to find the radius and axial pressure of a

cast-iron cone clutch whose cone is faced with leather. The following data is given:

$H = 25$ horsepower
$N = 1500$ revolutions per minute
$p_n = 8$ pounds per square inch
$w = 2$ inches
$\mu = 0.20$
$\alpha = 15$ degrees

Solution. Step 1. To find the torque, T. Applying formula (**34**).

$$T = \frac{12 \times 33{,}000H}{2\pi N}$$

$$= \frac{12 \times 33{,}000 \times 25}{2 \times \pi \times 1500} = 1050 \text{ in.-lb.}$$

Step 2. To find the tangential force or frictional resistance, P_t. Applying formula (**39**).

$$P_t = \frac{T}{R}$$

$$= \frac{1050}{R} \text{ lb.}$$

Step 3. To find the total normal pressure, P_n. Applying formula (**133**), and substituting therein for P_t, its value from the preceding step,

$$P_n = \frac{P_t}{\mu}$$

$$= \frac{\frac{1050}{R}}{0.20} = \frac{1050}{0.20R} = \frac{5250}{R} \text{ lb.}$$

Step 4. To find the area, A, by using formula (**135**).

$$A = 2\pi Rw$$

$$= 2 \times \pi \times R \times 2 = 4\pi R \text{ sq. in.}$$

Step 5. To find the area, A, by using formula (**136**).

$$A = \frac{P_n}{p_n}$$

Here $P_n = \frac{5250}{R}$ and $p_n = 8$ lb. per sq. in. as given.

Evaluating
$$A = \frac{\frac{5250}{R}}{8} = \frac{5250}{8R} \text{ sq. in.}$$

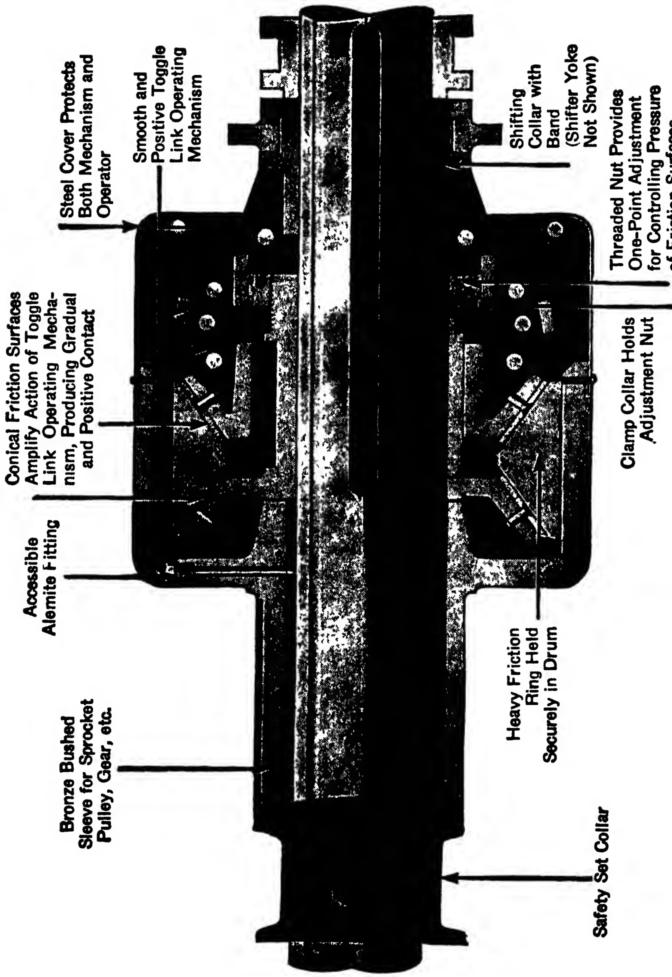


Fig. 82. "Twyncone" Clutch
Courtesy of Link-Belt Company, Chicago, Ill.

Step 6. Equating the two values (or expressions) of A of the two preceding steps,

$$4\pi R = \frac{5250}{8R}$$

$$4\pi R \times 8R = 5250$$

$$32\pi R^2 = 5250$$

$$R^2 = \frac{5250}{32\pi} = 52.2$$

$$R = \sqrt{52.2} = 7.23 \text{ in., say } 7\tfrac{1}{4} \text{ in.} \quad \textit{Ans.}$$

Step 7. Find the axial pressure, P_a, by using formula (**131**).

Here $\alpha = 15°$ and $P_n = \frac{5250}{R}$, (from Step 3).

$$P_a = P_n \sin \alpha$$

$$= \frac{5250}{R} \times \sin 15°$$

$$= \frac{5250}{7.25} \times 0.259 = 188 \text{ lb.} \quad \textit{Ans.}$$

Fig. 82 is an illustration of the Twyncone friction clutch manufactured by the Link-Belt Company of Chicago. It will be noted from the figure that this clutch employs two pairs of conical contact surfaces. It is made in sizes which rate from 27 horsepower to 120 horsepower at 100 r.p.m., and which are simple, easy to operate and maintain, and unusually small in diameter for their capacities.

Simple Disk Clutch. This form of friction clutch employs one pair of flat rings or disks to form a contact surface over which a frictional resistance is set up. The contact surface may be metal on metal, or metal on asbestos, leather, cork, or other suitable frictional material as in the case of the cone clutch. Fig. 83 shows the driven plate with a friction lining material inserted into or fastened to the face of the driven member of the clutch. Engagement of the clutch is generally effected through the use of a spring or a toggle linkage. Such a linkage is shown with the multiple-disk clutch of Fig. 84.

It will be noted in that which follows that the notation used is the same as for the cone clutch, with the exception that

R = the outside radius of the contact surface

r = the inside radius of the contact surface.

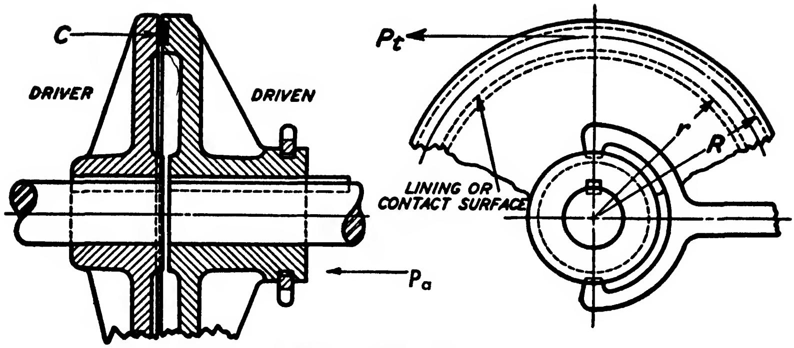
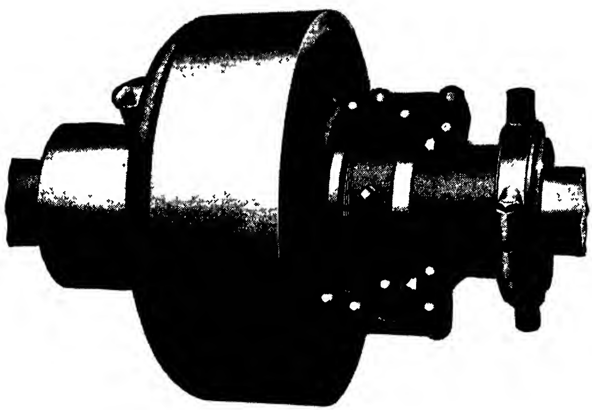


Fig. 83. Simple Disk Clutch



Courtesy of Dodge Manufacturing Corp., Mishawaka, Indiana

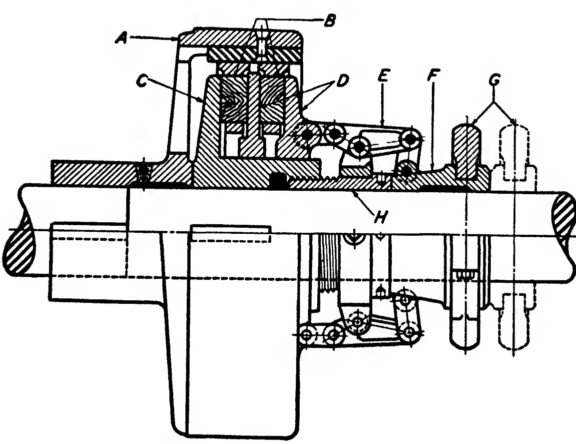


Fig. 84. Multiple-Disk Clutch

As in any type of clutch, the frictional resistance occurring at the contact surface is a tangential force, P_t, whose moment must equal the torque transmitted. It can be proved that the frictional resistance of the disk clutch can be assumed to have a moment arm which is equal to the mean radius of the disks, $\frac{R+r}{2}$. In other words, the frictional resistance may be assumed to be concentrated at a point which is at a distance, $\frac{R+r}{2}$, from the center of rotation. Since the moment of the frictional resistance must equal the torque transmitted,

$$P_t \times \frac{R+r}{2} = T \tag{137}$$

or dividing by the radius, $\frac{R+r}{2}$,

$$P_t = \frac{T}{\frac{R+r}{2}} \tag{138}$$

Since the frictional resistance is always equal to the total normal pressure multiplied by the coefficient of friction,

$$P_t = \mu P_n \tag{139}$$

It is evident from Fig. 83 that the contact surface lies in a plane that is perpendicular to the axis of the shafting. Therefore the total normal pressure at the contact surface is equal to the total axial pressure or,

$$P_n = P_a \tag{140}$$

The total normal pressure is of course equal to the area times the unit normal pressure, p_n. The area of the contact surface is equal to $\pi R^2 - \pi r^2$ or $\pi(R^2 - r^2)$. Hence

$$P_n = p_n \times \pi(R^2 - r^2) \tag{141}$$

From formulas (139) and (140), it follows that

$$P_t = \mu P_a \tag{142}$$

Multiplying both members of the preceding formula by $\frac{R+r}{2}$,

$$P_t \times \frac{R+r}{2} = \mu P_a \frac{R+r}{2}$$

Therefore from formula (137),

$$T = \mu P_a \frac{R+r}{2} \tag{143}$$

and since $P_a = P_n$, (formula 140)

$$T = \mu P_n \frac{R+r}{2} \tag{144}$$

The total axial pressure that is required to transmit a torque, T, can be found by solving formula (143) for P_a. This is accomplished by dividing both members of the formula by $\mu \times \frac{R+r}{2}$ which gives:

$$P_a = \frac{2T}{\mu(R+r)} \tag{145}$$

Example. A simple disk clutch as shown in Fig. 83 has an outside diameter of 12 inches and an inside diameter of 8 inches. The material of the contact surfaces and the conditions of operation permit the use of a coefficient of friction and an allowable normal pressure of 0.22 and 20 lb. per sq. in. respectively. Required: the axial pressure and the horsepower that can be transmitted at 1200 r.p.m.

Solution. Here $R=6$ in., $r=4$ in., and $p_n=20$ lb. per sq. in. Evaluating in formula (141),

$$P_n = p_n \times \pi(R^2 - r^2)$$
$$= 20 \times \pi(6^2 - 4^2) = 1257 \text{ lb.}$$

From formula (140)

$$P_a = P_n = 1257 \text{ lb.} \quad \textit{Ans.}$$

With $\mu = 0.22$ and the other values as given above, we have from formula (143)

$$T = \mu P_a \frac{R+r}{2}$$
$$= 0.22 \times 1257 \times \frac{6+4}{2} = 1383 \text{ in.-lb.}$$

Evaluating in formula (33), with $N=1200$ r.p.m.

$$H = \frac{T 2\pi N}{12 \times 33{,}000}$$
$$= \frac{1383 \times 2 \times 3.1416 \times 1200}{12 \times 33{,}000}$$
$$= 26.3 \text{ hp.} \quad \textit{Ans.}$$

Multiple-Disk Clutch. This type of friction clutch is of the same principle as the simple disk clutch but it employs several pairs of flat contact surfaces instead of just one pair. This is shown in Fig. 84, which presents a photograph and a sectional drawing of a

multiple-disk clutch as manufactured by the Dodge Manufacturing Corporation of Mishawaka, Ind. In this clutch, the disks, B, are held to the cylinder, A, by splines which permit the disks to have an axial motion relative to the cylinder but compel the disks to rotate with the cylinder. In a similar manner, the disks, D, are fastened to the casting, C, by feather keys so that they may be moved axially but must rotate with C. The disk, D, is connected to the toggle, E, and is moved axially thereby through the connection, F, of the toggle to the shift lever, G. In the sketch as shown the clutch is in engagement, the disks having been moved until their surfaces are in contact. It should be observed that the three disks, C and D, attached to one shaft and the two disks, B, attached to the other shaft furnish the clutch with four contact surfaces. The normal pressure set up by the toggle mechanism over the contact surfaces of alternate disks provides a frictional resistance which thus secures all parts of the clutch together and causes them to rotate as a unit. The wooden blocks of disks, B, can be replaced by parts using asbestos should service conditions warrant. The threaded sleeve, H, permits adjustment to take care of wear at the contact surfaces.

While the analysis of the multiple-disk clutch is similar to that of the simple disk, it must be recognized that when an axial pressure is introduced into the former, it will set up a normal pressure equal in magnitude to itself at every pair of contact surfaces. Therefore if we let m represent the number of pairs of contact surfaces, P_n the total normal pressure between one pair of contact surfaces, and P_m the total normal pressure of the n pairs of contact surfaces, we have

$$P_n = P_a, \text{ as in the simple disk clutch}$$

and

$$P_m = m \times P_n = m \times P_a \tag{146}$$

Since the area of one pair of contact surfaces is equal to $\pi(R^2 - r^2)$, the area of m pair will equal $m \times \pi(R^2 - r^2)$. Therefore if p_n is the unit normal pressure in lb. per sq. in., the total normal pressure of the m pairs of contact surfaces will be given by the formula

$$P_m = m \times p_n \times \pi(R^2 - r^2) = mp_n\pi(R^2 - r^2) \tag{147}$$

The frictional resistance then becomes $\mu \times P_m$, and since the moment arm of the latter is still $\frac{R+r}{2}$, the moment of the frictional resistance is equal to $\mu \times P_m \times \frac{R+r}{2}$. The latter as in the case of any friction

clutch must equal the torque transmitted, so that

$$T=\mu P_m\frac{R+r}{2} \tag{148}$$

Substituting for P_m in the above, its equal $m\times P_a$ from formula (**146**), we have

$$T=\mu m P_a\frac{R+r}{2} \tag{149}$$

If the above formula is compared with formula (**143**), it is seen that a multiple-disk clutch will transmit m times as much torque as a simple disk clutch which uses the same axial pressure and has the same inner and outer radii of contact surfaces.

Solving formula (**149**) for P_a by dividing both members of it by $\mu m\frac{R+r}{2}$, we obtain

$$P_a=\frac{2T}{\mu m(R+r)} \tag{150}$$

Comparing the above formula with formula (**145**), it will be seen that the axial pressure and hence the normal pressure per pair of contact surfaces of a multiple-disk clutch is $\frac{1}{m}$ of that of a similar simple disk clutch that transmits the same torque. Hence, in general, by increasing the number of pairs of contact surfaces, we lower the unit pressure required to transmit a given torque, and thus lengthen the life of the clutch plates. However a multiple-disk clutch with a large number of disks presents an undesirable characteristic in having a heavy rotating mass.

Example. A multiple-disk clutch employs four pairs of contact surfaces, whose outer diameter is 12 inches and whose inner diameter is 8 inches. If the coefficient of friction is assumed as 0.22. (*a*) find the axial pressure and the horsepower transmitted if the unit normal pressure is 20 lb. per sq. in. and the r.p.m. is 600; (*b*) find the axial pressure and the unit normal pressure in lb. per sq. in. if this clutch transmits 26.3 hp. at 1200 r.p.m.

Solution. (*a*) We shall first find the total normal pressure, P_m, of the m pairs of contact surfaces.

Here $m=4$, $p_n=20$ lb. per sq. in., $R=6$ in., and $r=4$ in. Substituting these values in formula (**147**),

$$\begin{aligned}P_m&=mp_n\pi(R^2-r^2)\\&=4\times20\times\pi(6^2-4^2)\\&=5027 \text{ lb.}\end{aligned}$$

To find P_a by applying formula (**146**)

$$P_m = m \times P_a$$

$$P_a = \frac{P_m}{m}$$

Evaluating $$P_a = \frac{5027}{4} = 1257 \text{ lb.} \quad \textit{Ans.}$$

Substituting our known values in formula (**148**), or in formula (**149**)

$$T = \mu P_m \frac{R+r}{2}$$

$$T = 0.22 \times 5027 \times \frac{6+4}{2} = 5530 \text{ in.-lb.}$$

Applying formula (**33**) with $N = 600$,

$$H = \frac{T 2\pi N}{12 \times 33{,}000}$$

$$= \frac{5530 \times 2 \times \pi \times 600}{12 \times 33{,}000} = 52.6 \text{ hp.} \quad \textit{Ans.}$$

(*b*) Here $N = 1200$ r.p.m., and $H = 26.3$ hp. Using formula (**34**)

$$T = \frac{12 \times 33{,}000 H}{2\pi N}$$

$$= \frac{12 \times 33{,}000 \times 26.3}{2\pi \times 1200} = 1383 \text{ in.-lb.}$$

With $T = 1383$ in.-lb., $\mu = 0.22$, $m = 4$, $R = 6$ in., and $r = 4$ in., substitute in formula (**150**) for P_a. Thus

$$P_a = \frac{2T}{\mu m (R+r)}$$

$$= \frac{2 \times 1383}{0.22 \times 4 \times (6+4)} = 314.2 \text{ lb.} \quad \textit{Ans.}$$

From formula (**146**), the total normal pressure

$$P_m = m \times P_a$$

$$= 4 \times 314.2 = 1257 \text{ lb.}$$

Applying formula (**147**) and solving for p_n,

$$P_m = m p_n \pi (R^2 - r^2)$$

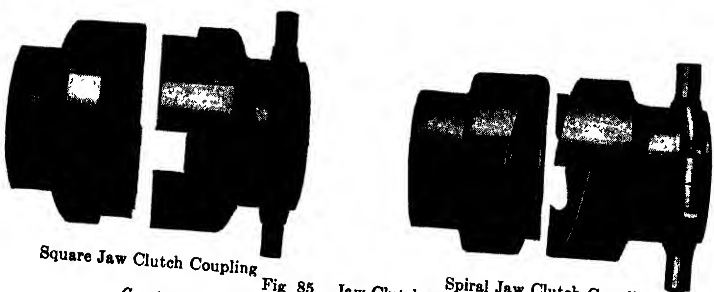
$$1257 = 4 \times p_n \times \pi \times (6^2 - 4^2)$$

$$p_n = \frac{1257}{4 \times \pi \times 20} = 5 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

(**Note.** Part (*b*) of the preceding example was inserted so that a comparison might be made with the example of the simple disk

clutch. It will be noted that under similar conditions of design and operation the unit normal pressure in this example of the multiple-disk clutch is only one-fourth as large as it was in the case of the simple disk clutch.)

Jaw Clutch. A jaw clutch is a device that through the direct contact of interlocking lugs or jaws permits one shaft to drive another. It is then a positive clutch rather than a friction clutch. As shown in Fig. 85, it consists of two halves, one of which is permanently fastened to the shaft by a sunk key while the other is fastened to its shaft by a feather key. Thus the latter can be moved along the shaft until its entering jaws slide into the mating pockets of the former.



Square Jaw Clutch Coupling Spiral Jaw Clutch Coupling

Fig 85 Jaw Clutches

Courtesy of Dodge Manufacturing Corp., Mishawaka, Indiana

The square jaw type is used where engagement and disengagement in motion and under load is not necessary. It is evident that this type will transmit power in either direction of rotation.

The spiral jaw type is made with either left hand or right hand spiral jaws since power can be transmitted by them in one direction only. This construction permits shifting more readily and is occasionally used where the clutch must be engaged and disengaged while in motion. Its use is limited to low speeds and moderate loads.

Jaw clutches are frequently applied to sprocket wheels, gears, and pulleys. In such a case, the non-sliding part is built integrally with the hub.

The design of this type of clutch must provide a sufficient area in shear at the place where the jaws meet the sleeve so that the moment of the safe shearing resistance will equal the torque transmitted. Likewise the area in crushing between the contact or driving surfaces must be so designed that the moment of the safe crushing

resistance will be equal to the torque transmitted. The moment arm in each case can be taken as the mean radius of the sleeve.

PROBLEMS

1. What is the difference between a shaft coupling and a clutch?
2. What kind of a coupling is used to connect shafts whose axes intersect?
3. What are the advantages of a flexible coupling?
4. Classify clutches.
5. A sleeve coupling is used to connect two 3-inch shafts whose safe shearing stress is 10,000 lb. per sq. in. (*a*) What torque is transmitted by the shafting? (*b*) To what torsional moment is the coupling subjected? (*c*) Find the outside diameter and the length of the coupling on the basis of the assumptions as given in the text material of this chapter. (*d*) What will be the induced stress, S_s, in the sleeve if the diameter of part (*c*) is used? (*e*) If a cast iron sleeve with an ultimate shearing stress of 20,000 lb. per sq. in. is used, what is the numerical value of the factor of safety? *Ans.* (*a*) 53,000 in.-lb. approximately. (*b*) 53,000 in.-lb. (*c*) $6\frac{1}{2}$ in.; $10\frac{1}{2}$ in. (*d*) 1021 lb. per sq. in. (*e*) 19.5.
6. A flange coupling is employed to connect two 3-inch shafts. The allowable shearing stress of the shafts is 10,000 lb. per sq. in. while that of the bolts is 6000 lb. per sq. in. Find the diameter of bolts if the number to be used is four. *Ans.* $\frac{7}{8}$ in.
7. The flange coupling of the preceding problem has a thickness of flange of $\frac{1}{2}$ inch. If the weaker material in crushing at the bearing surface has an ultimate compressive stress of 60,000 lb. per sq. in., find (*a*) the induced unit compressive stress, (*b*) the factor of safety in compression. *Ans.* (*a*) 6750 lb. per sq. in., (*b*) 8.9
8. A cast-iron cone clutch is used to connect two shafts which transmit 35 horsepower at 150 r.p.m. The diameter of the clutch is 24 inches, the angle of the clutch is 10 degrees. The allowable unit normal pressure and the coefficient of friction are to be assumed as 50 lb. per sq. in. and 0.12 respectively. It is required to find (*a*) the torque, T, to be transmitted in inch-pounds, (*b*) the frictional resistance, P_t, in pounds, (*c*) the total normal pressure, P_n, in pounds, (*d*) the area of the contact surfaces, A, in square inches, (*e*) the width of the clutch, W, in inches, (*f*) the axial pressure, P_a, in pounds. *Ans.* (*a*) 14,700 in.-lb. (*b*) 1225 lb. (*c*) 10,200 lb. (*d*) 204 sq. in. (*e*) 2.7 in., say $2\frac{3}{4}$ in. (*f*) 1775 lb.
9. A cone clutch employs a cone which is lined with a high grade of friction lining material whose coefficient of friction and unit normal pressure are taken under the working conditions as 0.25 and 10 lb. per sq. in. respectively. The clutch is used to connect two 1-inch shafts whose safe shearing stress is 10,000 lb. per sq. in. The angle of the clutch is 12 degrees and the width of the clutch is $2\frac{1}{2}$ inches. (*a*) What torque may be transmitted by the clutch? (**Note.** Use formula (**31**) here). (*b*) What horsepower is transmitted at 1000 revolutions per minute? (**Note.** Use formula (**33**) here.) (*c*) Find the radius of the clutch. (*d*) Find the axial pressure required. *Ans.* (*a*) 1960 in.-lb. (*b*) 31.1 (*c*) 7.05 in., say $7\frac{1}{8}$ in. (*d*) 229 lb.
10. A cone clutch for industrial use has a width of face of 2 inches and a mean diameter of 16 inches. The cup of the clutch is cast iron and the cone is faced with a fibrous material. These surfaces permit under the operating con-

ditions the assumption of a coefficient of friction of 0.25 and a normal pressure of 35 lb. per sq. in. The angle of the clutch, α, is equal to 10°. (*a*) Find the area of the contact surfaces. (*b*) Find the total normal pressure. (*c*) Find the frictional resistance or tangential force, P_t, by using formula (133). (*d*) What torque can the clutch safely transmit in in.-lb.? Use formula (29). (*e*) What horsepower may be transmitted at 350 r.p.m.? Use formula (33). (*f*) Find the axial pressure by using formula (134). *Ans.* (*a*) 100.5 sq. in. (*b*) 3517.5 lb. (*c*) 879.4 lb. (*d*) 7035 in.-lb. (*e*) 39+ hp. (*f*) 611 lb.

11. A simple disk clutch as illustrated in Fig. 83 has an outside diameter of 10 inches and an inside diameter of 7 inches. The material of the contact surfaces and the conditions of operation permit of a coefficient of friction and an allowable normal pressure of 0.20 and 25 lb. per sq. in. respectively. It is required to obtain the axial pressure and the horsepower that can be transmitted if the clutch operates at 1000 r.p.m. *Ans.* 1000 lb.; 13.5 hp.

12. What horsepower may the clutch of the preceding problem transmit if it operates at 2000 r.p.m.? *Ans.* 27 hp.

13. A multiple-disk clutch employs six pairs of contact surfaces whose outer diameter is 7 inches and whose inner diameter is 4 inches. With $\mu = 0.20$ $p_n = 25$ lb. per sq. in. and $N = 500$ r.p.m., it is required to find: (*a*) the total normal pressure, P_m, in pounds, (*b*) the axial pressure, P_a, in pounds, (*c*) the torque in in.-lb. (*d*) the horsepower that may be safely transmitted. *Ans.* (*a*) 3886 lb. (*b*) 648− lb. (*c*) 2137 in.-lb. (*d*) 17− hp.

14. If a simple disk clutch is used in the place of the multiple-disk clutch in the preceding problem, find the unit normal pressure, the dimensions of the clutch plates being the same. *Ans.* 150 lb. per sq. in.

CHAPTER VII

WRAPPING CONNECTORS AND THEIR PULLEYS, SHEAVES, OR SPROCKETS

Introduction. The transmission of power from one shaft to another is often accomplished by means of a continuous flexible connector or endless band such as a belt, rope, or chain. The connector runs over or is wrapped partially around various types of wheels which are keyed to the shafts. The surfaces of these wheels are so formed that each meets the requirements of the particular type of wrapping connector with which it is to be used. Thus a flat band or belt requires or dictates the cylindrical or nearly cylindrical surface of its so-called pulley. A V-belt necessitates, in most cases, the use of a properly designed grooved rim on its pulley or sheave, as does also a rope connector. A chain requires the use of a sprocket wheel which has teeth that are not unlike those of a gear.

Any wrapping connector, with the exception of the chain, depends upon a frictional resistance between itself and its pulley or sheave for the transmission of its power. Since the teeth of a sprocket project through the links of a chain, the latter becomes a positive drive and thus transmits its power and motion independent of the use of any frictional resistance. While transmitting power all wrapping connectors are subjected to a tensile stress. Each secures, theoretically, a constant angular velocity ratio for the pair of shafts it connects. Although they may be used to connect non-parallel shafts, their use with parallel shaft installations is more common.

Flat Belt. A flat belt is one with a relatively thin rectangular cross section, whose longer dimension, the width of the belt, is placed parallel to the axes of the pulleys over which it runs. The belt can be so placed upon the pulleys that either direction of rotation of the driven shaft is possible. When the open belt of Fig. 86 is used, the driven pulley rotates in the same direction as the driver. On the other hand, if the crossed belt installation of Fig. 87 is used, the driven pulley rotates in the opposite direction to that of the driver. The

open belt installation should be used whenever it is possible for there is some destructive action present at the crossing of the belt in the other installation. In both, the theoretical angular velocity or r.p.m. ratio of driver to driven or follower is inversely as the diameters or radii of the pulleys. This is proved in the text on Mechanism and is given by the following formula,

$$\frac{N_a}{N_b}=\frac{D_b}{D_a}=\frac{R_b}{R_a} \qquad \textbf{(151)}$$

in which: N_a = the r.p.m. of pulley, A.
N_b = the r.p.m. of pulley, B.
D_a = the diameter of pulley, A, in inches.
D_b = the diameter of pulley, B, in inches.
R_a = the radius of pulley, A, in inches.
R_b = the radius of pulley, B, in inches.

The actual angular velocity ratio will vary slightly from the theoretical due to the slippage and so-called creeping of the belt on the surface of the pulleys.

Formula (**151**) can be applied to the other types of wrapping connectors if the pitch diameters (or pitch radii) of their sheaves or sprockets are used therein.

Although flat belts may be used to connect other than parallel shafts, the parallel shaft installation is to be preferred and the others should be avoided whenever it is possible to do so.

Two intersecting shafts may be connected by a belt drive if two pulleys known as guide pulleys are used in addition to the two main pulleys of the shafts to be connected. These so-called guide pulleys must be carefully located for the belt must approach each in a direction perpendicular to its axis and the belt must leave the guide pulley in a direction perpendicular to the axis of the pulley to which it is being directed or guided. This is because the Law of Belting, which must be followed in every case, states that in a belt drive the side of the belt that approaches the pulley must be perpendicular to the axis of rotation of that pulley.

A belt drive may also be used to connect two shafts which are at right angles to each other. This is known as the quarter-twist belt drive. The pulleys must be so located that the law of belting will not be violated. The belt can run in but one direction, that is the

TABLE XIX—Thicknesses of Leather Belting

Name	Thickness in Inches
Medium single	5/32 to 3/16
Heavy single	3/16 to 7/32
Light double	15/64 to 17/64
Medium double	9/32 to 5/16
Heavy double	21/64 to 23/64

motion of the belt cannot be reversed, unless a guide pulley be used to direct the belt properly.

The most widely used material for flat belts is leather. Other materials used include cotton, rubber, and steel. Belts are classified according to the number of plies which are cemented or stitched together to give the belt its required thickness. Thus a single-ply leather belt is made of a single thickness of hide, while a double-ply belt is made of two thicknesses of hide that are cemented to each other to give a larger thickness to the belt. In order to obtain an endless belt of required length, the ends of the pieces of material to be used may be cemented or laced to each other, or the ends may be joined by some form of metallic fastener. The joint formed by cementing is as strong as the belt itself; a wire-laced joint is 80 to 90 per cent as strong as the belt; rawhide lacing gives 50 per cent while metal fastenings are even less efficient.

The standard widths of leather belting as approved by the American Leather Belting Association vary as follows:

by $\frac{1}{8}$ inch between widths of $\frac{1}{2}$ inch and 1 inch,
by $\frac{1}{4}$ inch between widths of 1 inch and 4 inches,
by $\frac{1}{2}$ inch between widths of 4 inches and 7 inches,
by 1 inch between widths of 7 inches and 12 inches.

Larger sizes are made to order.

The standard thicknesses recommended by the same association are given in Table XIX.

Example. A 15 hp. motor, whose speed is 1200 r.p.m. is belt-connected to the drive shaft of a machine. The pulley, A, on the armature shaft of the motor is 12 inches in diameter while that on the driven shaft, B, is 30 inches in diameter. Find (*a*) The speed of the belt in feet per minute. (*b*) The speed of the driven pulley in r.p.m. (*c*) The torque in the armature shaft in in.-lb. (*d*) The tangential force at the rim of the armature shaft pulley. (*e*) The

torque in the drive shaft in in.-lb. (*f*) The tangential force at the rim of the drive shaft pulley.

Solution. (*a*) In a belt drive, the driving pulley is assumed to give its rim velocity to the belt, which imparts this velocity to the rim of the driven pulley. Therefore the speed of the belt is equal to the velocity of the rim of either pulley. From Mechanism, the linear velocity, V_L, of any point on a rotating body is given by the formula

$$V_L = 2\pi RN = \pi DN$$

Applying this formula to the rim of the driving pulley, A, in which case $D_a = \frac{12}{12} = 1$ ft. and $N_a = 1200$ r.p.m., we have

$$V_L = \pi \times 1 \times 1200 = 3770 - \text{f.p.m.} \quad \textit{Ans.}$$

(*b*) Here, we have $N_a = 1200$ r.p.m., $D_a = 12$ in. and $D_b = 30$ in. Evaluating in formula (151)

$$\frac{N_a}{N_b} = \frac{D_b}{D_a} \text{ or } \frac{N_b}{N_a} = \frac{D_a}{D_b}$$

$$\frac{N_b}{1200} = \frac{12}{30}$$

$$N_b = 1200 \times \frac{12}{30} = 480 \text{ r.p.m.} \quad \textit{Ans.}$$

(*c*) Here, $H = 15$ hp., and $N_a = 1200$ r.p.m.

Applying formula (34)

$$T_a = \frac{12 \times 33{,}000 H}{2\pi N_a}$$

Evaluating, $$T_a = \frac{12 \times 33{,}000 \times 15}{2 \times \pi \times 1200} = 788 \text{ in.-lb.} \quad \textit{Ans.}$$

(*d*) Applying formula (40) with $R_a = \frac{12}{2} = 6$ in.

$$P_{ta} = \frac{T_a}{R_a}$$

$$= \frac{788}{6} = 131.3 \text{ lb.} \quad \textit{Ans.}$$

(*e*) Applying formula (34) with $H = 15$ hp. and $N_b = 480$ r.p.m.

$$T_b = \frac{12 \times 33{,}000 H}{2\pi N_b}$$

$$T_b = \frac{12 \times 33{,}000 \times 15}{2 \times \pi \times 480} = 1970 \text{ in.-lb.} \quad \textit{Ans.}$$

To check by applying formula (43), which states that the torques in the connected shafts are inversely proportional to the numbers of revolutions per minute.

$$\frac{T_b}{T_a}=\frac{N_a}{N_b}$$

$$\frac{T_b}{788}=\frac{1200}{480}$$

$$T_b=788\times\frac{1200}{480}=1970 \text{ in.-lb. which}$$

checks the above.

(**Note.** Since $\frac{N_a}{N_b}$ is the speed reduction factor, it will again be noted from the above that the speed reduction factor is equal to the torque multiplication factor, or that as the speed is reduced the torque is increased.)

(*f*) Applying formula (41), $P_{tb}=\frac{T_b}{R_b}$ in which $T_b=1970$ in.-lb. and $R_b=\frac{30}{2}=15$ in., we have

$$P_{tb}=\frac{1970}{15}=131.3 \text{ lb.} \quad \textit{Ans.}$$

(**Note.** From the above it is seen that the tangential force at the rim of the driver is equal to the tangential force at the rim of the driven. This is true for all wrapping connector mechanisms and this tangential force is known as the Effective Belt Pull.)

The Radian, a Unit of Angular Measure. In belt formulas a unit of angular measure called the radian is used to a great extent. It is a much larger unit of angular measure than a degree, so when an angle is measured in radians, it will be represented by a much smaller number than when measured in degrees. When an angle of one radian is placed so that its vertex is at the center of a circle (an angle so placed is called a central angle), its sides will naturally become radial lines of the circle. These radial sides will cut off or intersect a certain arc on the circumference of the circle. This arc, which is intersected by the radial sides of a central angle of one radian, will be exactly equal to the length of the radius of the circle, no matter what size of circle is used. Hence, there can be as many angles of one radian, or as many radians, at the center of a circle as there are arcs of a length equal to the radius in the circumference of

the circle. In other words if the circumference is divided by R, the radius of the circle, the result will be the number of arcs of this length in the circumference, and hence the number of radians in the whole central angle of 360 degrees at the center of the circle. Let us perform this division and obtain this relationship between radian and degree so that the measurement of an angle in one unit can be changed into the other unit. Thus, since the circumference of a circle is equal to $2\pi R$, dividing by R, we have $\frac{2\pi R}{R}=2\pi$, the number of radians in 360 degrees so that

$$2\pi \text{ radians}=360 \text{ degrees} \tag{a}$$

Dividing Step (a) by 2π

$$1 \text{ radian}=\frac{360}{2\pi} \text{ degrees} \tag{b}$$

Dividing Step (a) by 360

$$1 \text{ degree}=\frac{2\pi}{360} \text{ radians} \tag{c}$$

From Step (b) it is evident that the radians of an angle can be changed to degrees by multiplying by 360 over 2π. From Step (*c*) it is evident that the measurement of an angle given in degrees can be changed into radians by multiplying by 2π over 360.

Example. It is required to express in radian measure a central angle of 60 degrees.

Solution. From step (c) as previously given

$$1 \text{ degree}=\frac{2\pi}{360} \text{ radians.}$$

Therefore

$$\begin{aligned} 60 \text{ degrees} &= 60\times 1 \text{ degree} \\ &= 60\times\frac{2\pi}{360} \text{ radians} \\ &= 1.0472 \text{ radians.} \qquad \textit{Ans.} \end{aligned}$$

Example. How many degrees are there in a central angle of 2 radians?

Solution. From step (b) as previously given

$$1 \text{ radian}=\frac{360}{2\pi} \text{ degrees.}$$

Therefore

$$2 \text{ radians} = 2 \times 1 \text{ radian}$$

$$= 2 \times \frac{360}{2\pi} \text{ degrees}$$

$$= 114.6^\circ \quad \textit{Ans.}$$

Example. Let it be assumed that the angle of contact, α, between the belt and the smaller pulley of Fig. 86 is equal to 150° 27′. It is required to express α in the radian as a unit.

Solution. Since 1 degree = 60 minutes (1° = 60′)

$$\alpha = 150^\circ\ 27' = 150\tfrac{27}{60}^\circ = 150.45^\circ$$

$$= 150.45 \times \frac{2\pi}{360} \text{ radians}$$

$$= 2.626 \text{ radians} \quad \textit{Ans.}$$

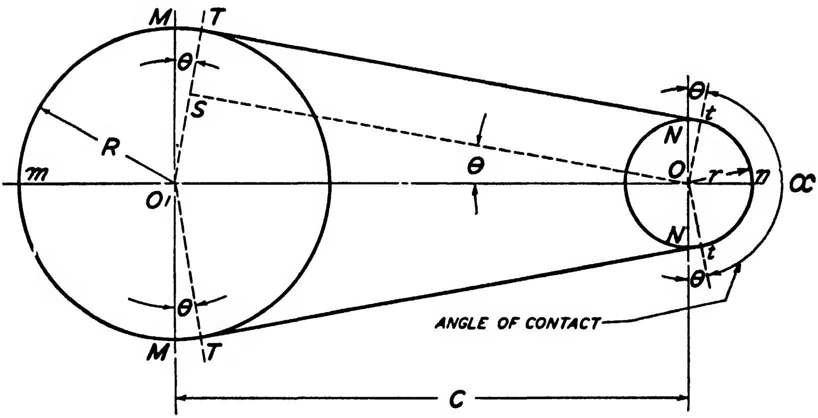


Fig. 86. Open Belt

Length of Open Belt. Fig. 86 illustrates an open belt stretched over two pulleys which are keyed to parallel shafts. For deriving the formula for the length of such a belt, the latter is shown in a taut position between the pulleys on both the upper and lower sides. Such is the condition of the belt when no motion is being transmitted. It will be noted that the belt is tangent to the pulley rims at points, t and T, and hence is perpendicular to the radii Ot and O_1T, drawn to these points of tangency. Let the radius of the smaller pulley be denoted by r, and the radius of the larger pulley by R. When a line

is drawn from O, the center of the smaller pulley parallel to tT, a rectangle, $OSTt$, is formed in which $OS=tT$. Also a right triangle, OSO_1, is formed, in which the hypotenuse, OO_1, is of a length equal to the distance between the centers of the pulleys, C, and the acute angle O_1OS, called θ (theta), is equal to the central angle, NOt, of the smaller pulley, and to the central angle, MO_1T, of the larger pulley. For two angles are equal when the sides of one are either parallel or perpendicular to the sides of the other.

Now in the triangle, OSO_1,

$$O_1S=O_1T-ST$$

But $\quad O_1T=R$ and $ST=Ot=r$

Therefore $\quad O_1S=R-r$

Also in the triangle OSO_1,

$$\sin\theta=\frac{O_1S}{OO_1}$$

Since $\quad O_1S=R-r$ and $OO_1=C$

$$\sin\theta=\frac{R-r}{C}$$

or

$$\theta=\sin^{-1}\frac{R-r}{C}$$

(The above step is read as follows: θ is the angle whose sine is $\frac{R-r}{C}$)

An inspection of the figure indicates that the length, L, of the open belt is given by the statement,

$$L=2tT+2(\text{arc } MT)+\text{arc } MmM+\text{arc } NnN-2(\text{arc } Nt)$$

In this statement,

$$tT=OS=\sqrt{OO_1^2-O_1S^2}=\sqrt{C^2-(R-r)^2}$$

arc $\quad MT=R\theta=R\times\sin^{-1}\frac{R-r}{C}$,

in which θ, or $\sin^{-1}\frac{R-r}{C}$, is in radians

arc $MmM=\pi R$, the half circumference of the larger pulley

arc $\quad NnN=\pi r$, the half circumference of the smaller pulley

and arc $Nt=r\theta=r\times\sin^{-1}\frac{R-r}{C}$,

in which θ, or $\sin^{-1}\frac{R-r}{C}$, is in radians.

Substituting these values in the above statement of L, we have

$$L=2\sqrt{C^2-(R-r)^2}+2R\sin^{-1}\frac{R-r}{C}+\pi R+\pi r-2r\sin^{-1}\frac{R-r}{C}$$

$$L=2\sqrt{C^2-(R-r)^2}+\pi R+\pi r+2R\sin^{-1}\frac{R-r}{C}-2r\sin^{-1}\frac{R-r}{C}$$

$$L=2\sqrt{C^2-(R-r)^2}+\pi(R+r)+2(R-r)\sin^{-1}\frac{R-r}{C} \qquad (152)$$

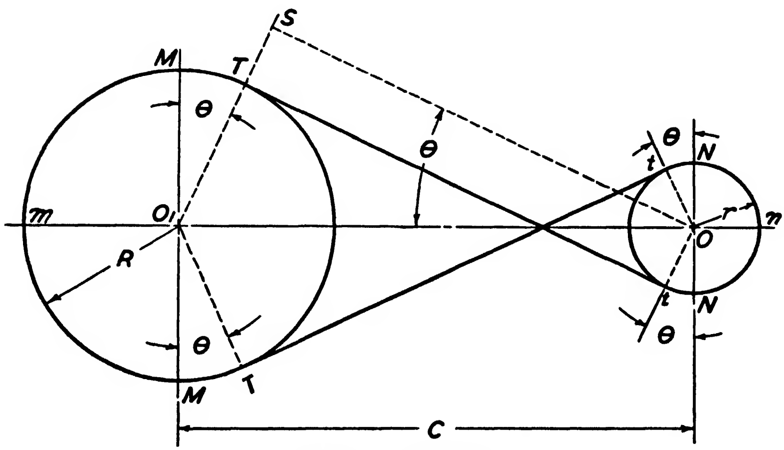


Fig. 87. Crossed Belt

Length of Crossed Belt. Fig. 87 shows a crossed belt stretched between two pulleys. In this case the notation used is identical to that used in the open belt of Fig. 86. As in the preceding case, if a line, OS, is drawn from the center, O, of the smaller pulley to the radius O_1T of the larger pulley, a rectangle, $OSTt$, is formed in which $OS=Tt$. Likewise, a right triangle, OSO_1, is formed in which angle O_1OS or

$$\theta=\angle NOt=\angle MO_1T$$

and hypotenuse, OO_1, $=C$, the distance between centers of the pulleys or their shafts. In this right triangle however,

$$O_1S=O_1T+TS=R+r,\text{ since } TS=Ot=r$$

and $$\sin\theta=\frac{O_1S}{OO_1}=\frac{R+r}{C}$$

or $$\theta=\sin^{-1}\frac{R+r}{C}$$

An inspection of Fig. 87 indicates that the length, L, of the crossed belt is given by the statement,

$$L = 2tT + 2(\text{arc } MT) + \text{arc } MmM + \text{arc } NnN + 2(\text{arc } Nt)$$

In this statement

$$tT = OS = \sqrt{OO_1^2 - O_1S^2} = \sqrt{C^2 - (R+r)^2}$$

$$\text{arc } MT = R\theta = R \sin^{-1} \frac{R+r}{C},$$

in which θ or $\sin^{-1} \frac{R+r}{C}$ is in radians

arc $MmM = \pi R$, the half circumference of the larger pulley
arc $NnN = \pi r$, the half circumference of the smaller pulley

$$\text{arc } NT = r\theta = r \sin^{-1} \frac{R+r}{C},$$

in which θ or $\sin^{-1} \frac{R+r}{C}$ is in radians

Substituting these values in the statement for L,

$$L = 2\sqrt{C^2 - (R+r)^2} + 2R \sin^{-1} \frac{R+r}{C} + \pi R + \pi r + 2r \sin^{-1} \frac{R+r}{C}$$

$$L = 2\sqrt{C^2 - (R+r)^2} + \pi(R+r) + 2(R+r) \sin^{-1} \frac{R+r}{C} \tag{153}$$

Formulas (152) and (153) show that the length of either an open belt or a crossed belt can be obtained if only the radii of the pulleys and the distance between their centers are known. In the application of either formula, the same unit of length must of course be used for each radius as well as for the distance between centers. This unit then becomes the unit in which the length of the belt will be expressed.

Example. It is required to find the length of an open belt in a power transmission installation in which the pulleys are 24 inches and 40 inches in diameter and the distance between their centers is 6 feet.

Solution. Here $R = \frac{40}{2} = 20$ in., $r = \frac{24}{2} = 12$ in., and $C = 72$ in.

Applying formula (152)

$$L = 2\sqrt{C^2 - (R-r)^2} + \pi(R+r) + 2(R-r)\sin^{-1} \frac{R-r}{C}$$

and evaluating,

$$L=2\sqrt{72^2-(20-12)^2}+\pi(20+12)+2(20-12)\sin^{-1}\frac{20-12}{72}$$

$$L=2\sqrt{5120}+\pi\times32+16\times\sin^{-1}\frac{1}{9}$$

Now $\sin^{-1}\frac{1}{9}$ (the angle whose sine $=\frac{1}{9}$) $=\sin^{-1} 0.1111=6°23'$. But this angle must always be in radians when used in either belt length formula, hence,

$$6°23'=6\frac{23}{60}\times\frac{2\pi}{360} \text{ radians}=0.1114 \text{ radians.}$$

Substituting this value of $\sin^{-1}\frac{1}{9}$, in the above,

$$L=2\sqrt{5120}+\pi\times32+16\times0.1114$$

$$L=143.10+100.53+1.78$$

$$L=245.41 \text{ in.}$$

$$=\frac{245.41}{12}=20.45 \text{ ft.} \quad \textit{Ans.}$$

Angle, or Arc of Contact. An inspection of Fig. 86 will show that the angle of contact α, between an open belt and the smaller pulley is less than the angle of contact between the belt and the larger pulley. Since the capacity of a belt depends on the extent of the angle of contact, the design of an open belt must be based on the former or the minimum angle (or arc) of contact. From Fig. 86, it is evident that the latter is given by the formula,

$$\alpha=180-2\theta, \text{ in degrees}$$

$$=180-2\sin^{-1}\frac{R-r}{C}, \text{ in degrees} \qquad \textbf{(154)}$$

From Fig. 87, it is evident that the angle of contact is the same for both pulleys when a crossed belt is used. Here,

$$\alpha=180+2\theta, \text{ in degrees}$$

$$=180+2\sin^{-1}\frac{R+r}{C}, \text{ in degrees} \qquad \textbf{(155)}$$

Should the minimum angle of contact for an open belt drive be smaller than desired, an idler pulley as shown in Fig. 88 can be employed. The use of such a pulley increases the angle of contact and maintains the proper tension in the belt.

The angle of contact in an open belt drive that is horizontal, or

nearly so, can be increased by having the direction of motion of the driver such as will create the tight or driving side of the belt on the lower side. The so-called slack side will then be on top and the sag in the belt will tend to increase the angle of contact and hence the capacity of the belt. Evidently (study Figs. 89 and 90) if the driving side of the belt is above the pulleys, the sag of the slack side will be on the bottom and the angle of contact will be decreased.

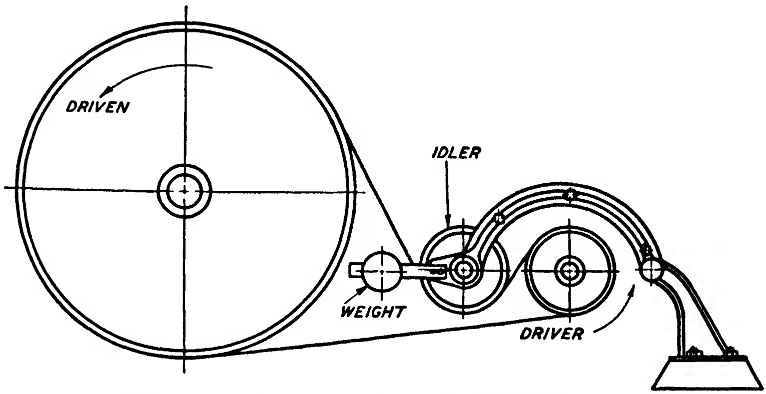


Fig. 88. Belt Tightener in Use with a Short-Center Belt Drive

Example. In the preceding example, we had the following given data: $R=20$ in., $r=12$ in., and $C=72$ in. It is now required to find the angle of contact α, between the belt and the smaller pulley in both degrees and radians.

Solution. Evaluating in formula (**154**),

$$\alpha = 180 - 2 \sin^{-1} \frac{R-r}{C}$$

$$= 180 - 2 \sin^{-1} \frac{20-12}{72}$$

$$= 180 - 2 \times 6°23'$$

$$= 167°14' \quad \textit{Ans.}$$

Since $360° = 2\pi$ radians,

$$1° = \frac{1}{360} \text{ of } 2\pi = \frac{2\pi}{360} \text{ radians}$$

Therefore: $167\frac{14°}{60} = 167\frac{14}{60} \times \frac{2\pi}{360}$ radians

$$= \frac{10{,}034}{60} \times \frac{2\pi}{360} = 2.92 \text{ radians} \quad \textit{Ans.}$$

Design of Flat Belt. When a belt is stretched over a pair of pulleys which are at rest, it is subjected to an initial tensile load or force throughout its entire length. This initial tension is therefore present in the belt on each side of the pulleys. This causes the belt to take a position relative to the pulleys as shown in Fig. 89. This initial tension in the belt also subjects the pulleys to a normal force

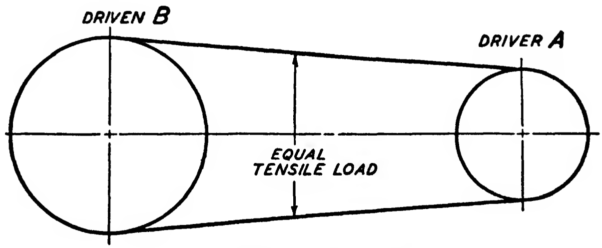


Fig 89. Belt at Rest

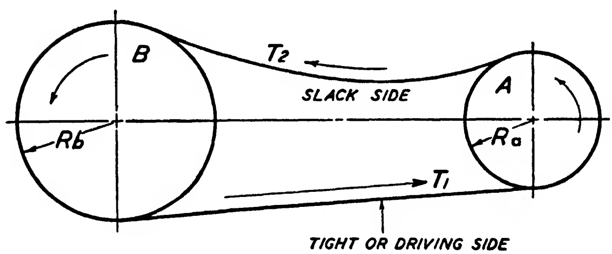


Fig 90. Belt in Motion

which sets up a frictional force or resistance between each pulley rim and the belt. Now as the driver, A, starts to rotate, this frictional force acts as a tangential force, P_t, at the rim of each pulley and the initial tension in the belt must be of such a magnitude that the frictional force will be sufficient to cause the belt to cling to the rims of the pulleys. This will permit the driver to increase the initial tension of the lower or tight side and decrease the initial tension of the upper side which will cause the latter to become somewhat slack, Fig. 90. This change in tension will proceed until the tension in the tight side becomes enough greater than that in the slack side to cause the driven pulley, B, to rotate. This will come to pass when the difference in

TABLE XX—Values of $e^{\mu\alpha}$ or $\frac{T_1}{T_2}$

Values of μ	Angle of Contact, α					
	72°	108°	144°	180°	216°	252°
0.25	1 37	1.60	1 87	2 19	2 57	3.00
0 30	1 46	1.76	2 13	2.57	3.10	3 74
0 35	1 55	1 93	2 41	3 00	3 74	4.66
0 40	1 65	2 13	2 73	3 51	4 52	5 81
0 45	1.76	2.34	3.10	4 11	5 45	7 24
0 50	1 87	2 57	3 51	4.81	6.59	9.00

the tensions becomes equal to that tangential force or effective belt pull which is required to overcome the resistance to motion of the driven pulley, or

$$T_1 - T_2 = P_t \tag{156}$$

where T_1 = tension in the tight side, in pounds.

T_2 = tension in the slack side, in pounds.

P_t = the effective belt pull, in pounds.

The effective belt pull as previously shown is the tangential force that occurs at the rim of each pulley. This is shown by formulas (39), (40), and (41) to be equal to the torque in either pulley shaft divided by the radius of the pulley on that shaft. P_t can also be obtained from formula (35), in the application of which, V_L is the speed of the belt in feet per minute.

Another relationship between the tensions, T_1 and T_2, is that given by the formula,

$$\frac{T_1}{T_2} = e^{\mu\alpha} \tag{157}$$

in which $e = 2.718$.

μ = the coefficient of friction between the belt and the pulley rim.

α = the angle of contact between the belt and the smaller pulley, in radians.

For our convenience, the numerical values of $\frac{T_1}{T_2}$ or $e^{\mu\alpha}$ are given in Table XX. To use the table, find the angle of contact α, in degrees by using formula (154) or (155). Locate in the table the angle that approximates α most closely. Under this angle, select the number to represent $e^{\mu\alpha}$ that lies opposite the coefficient of friction of the first column. For example suppose that α is found to be 150° and the

coefficient of friction to be used is 0.35. Since the nearest angle in the table to 150 degrees is 144 degrees, we have for the value of $e^{\mu\alpha}$, the number, 2.41, which lies opposite 0.35.

For angles and coefficients of friction not given exactly in the tables, a solution of formula (**157**) will of course yield a more accurate value of $e^{\mu\alpha}$.

It is evident that if a belt is subjected alternately to two tensions (or tensile loads), T_1 and T_2, it must be designed to resist safely the larger which is the load, T_1. Since the latter is a tensile load, formula (**5**) is applicable in this case so that,

$$T_1 = AS_t \tag{158}$$

Since $A = b \times t$

where b = the breadth of the cross-section of the belt.

t = the thickness of the cross-section of the belt.

We obtain by substituting this value of A in formula (**158**)

$$T_1 = btS_t \tag{159}$$

The selection of the value to be used for the working stress, S_t, should take into account not only the material of the belt but also the type of fastening employed and the wear of the belt. The lower the induced stress in a belt, the less the wear, and therefore the longer will be its period of service. With due consideration given to the latter, a working stress of about 300 pounds per square inch is advisable for oak-tanned leather belts with cemented joints or joints which are machine-laced with wire. With other types of joints, this value of S_t should be reduced by multiplying it by the ratio of the strength of the joint to the strength of the belt itself. Such a ratio is the efficiency of the joint. Since the average ultimate tensile strength of oak-tanned leather is about 3600 pounds per square inch, the above value of S_t indicates that a factor of safety of about 12 should be used in general for determining the working stress for other belts.

With every rotating body there is involved a force which acts outward or away from the center of rotation. This is called a centrifugal force. Since a belt surrounds a pulley, it is subjected to such a centrifugal force. The effect of this force is to create an added tensile load upon the belt and hence to cause the belt to need a larger cross-sectional area than is necessary for the major tensile load, T_1. It

can be proved that the tensile stress set up in a belt by centrifugal force alone is given by the following formula,

$$C_t = 0.013 V_s^2 \tag{160}$$

in which

C_t = tensile stress created by centrifugal force, in lb. per sq. in.

V_s = speed, or velocity of the belt, in feet per second.

In a belt drive, the linear velocities of the rims of the pulleys are equal to each other and are also equal to the speed of the belt. When the latter is less than 3000 feet per minute or 50 feet per second, the effect of centrifugal force upon the belt may be neglected. Hence formula (**159**) should be used. However if the speed of the belt is more than 50 feet per second, formula (**161**), which follows directly, should be used in the design of the belt. This formula is a modification of the belt design formula (**159**) in that $(S_t - C_t)$ replaces S_t. Thus

$$T_1 = bt(S_t - C_t) \tag{161}$$

in which the reduction in the stress, S_t, by an amount C_t, makes necessary a larger area than would result if formula (**159**) were used. The additional area created by formula (**161**) is that necessary to take care of the added tensile load on the belt created by centrifugal force.

Example. A belt runs over a 30-inch pulley which rotates at 150 r.p.m. (*a*) What is the speed of the belt in feet per minute and in feet per second? (*b*) Should the effect of centrifugal force be considered in this case in the design of the belt? (*c*) If 20 horsepower are transmitted, find the effective belt pull, the rim velocity, and the r.p.m. of the driven pulley.

Solution. (*a*) Here $D = 30$ in. $= 2.5$ ft. and $N = 150$ r.p.m. Since the speed of the belt is equal to the linear velocity of the rim of the pulley,

$$V_L = \pi DN$$
$$= \pi \times 2.5 \times 150 = 1177.5 \text{ f.p.m.} \quad \textit{Ans.}$$

or

$$V_s = \frac{1177.5}{60} = 19.6 \text{ f.p.s.} \quad \textit{Ans.}$$

(*b*) No, because the centrifugal effect at the above belt speed is negligible. *Ans.*

(*c*) *First method.* Evaluating in formula (**35**)

$$H=\frac{P_tV_L}{33{,}000}$$

with $H=20$ hp., and $V_L=1177.5$ f.p.m.

$$20=P_t\times\frac{1177.5}{33{,}000}$$

$$P_t=20\times\frac{33{,}000}{1177.5}=560.5 \text{ lb.} \quad \textit{Ans.}$$

Second method. To find the torque in in.-lb. in the shaft of the 30-inch pulley, by using formula (**34**).

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 20}{2\times\pi\times 150}=8400 \text{ in.-lb.}$$

applying formula (**39**)

$$P_t=\frac{T}{R}$$

$$=\frac{8400}{15}=560 \text{ lb., to check above.} \quad \textit{Ans.}$$

Example. The angle of contact between a belt and its smaller pulley is 144 degrees and the coefficient of friction between belt and pulley is 0.30. It is required to find the value of $\frac{T_1}{T_2}$ by using formula (**157**). Check the result with that given in Table XX.

Solution. The value of the angle of contact must first be changed from degrees to radians. Since 2π radians $=360$ degrees, this can be done by multiplying the value of the angle in degrees by $\frac{2\pi}{360}$, thus

$$\alpha=144 \text{ degrees}$$

$$=144\times\frac{2\pi}{360}=2.513 \text{ radians.}$$

With this value of α and with $\mu=0.30$,

$$\frac{T_1}{T_2}=e^{\mu\alpha}$$

$$=2.718^{0.30\times 2.513}=2.718^{0.754}$$

The above value is computed by logarithms as follows

$$\text{Log } 2.718=0.4342$$

$$\text{Log } 2.718^{0.754}=0.754\times\log 2.718$$

$$=0.754\times0.4342=0.3274$$

Therefore

$$2.718^{0.754}=\log^{-1} 0.3274=2.13-$$

or, $\frac{T_1}{T_2}=2.13$, which checks the value of Table XX. *Ans.*

Example. An 80-inch flywheel on a compressor is connected by a flat belt to a 20-inch pulley on the shaft of a 50-horsepower motor. The distance from center of flywheel to center of motor pulley is 8 feet. The motor operates at 800 r.p.m. If the coefficient of friction between belt and pulleys is assumed as 0.30 and the safe tensile stress of the belt is taken as 300 lb. per sq. in. find the width of the $\frac{9}{32}$-inch medium double belt to be used in this case.

Solution. Step 1. To obtain the speed of the belt. The speed of the belt is equal to the linear velocity of the rim of either pulley. Considering the velocity of the rim of the smaller pulley,

$$V=\pi DN$$

in which $D=20$ in. $=\frac{20}{12}$ ft., and $N=800$ r.p.m.

Evaluating, $V=\pi\times\frac{20}{12}\times800=4188.8$ f.p.m.

Step 2. To obtain the stress, C_t, due to the centrifugal effect. Since the speed of the belt is more than 3000 f.p.m. the centrifugal effect must be considered.

$$V_s=\frac{4188.8}{60}=69.8 \text{ f.p.s.}$$

Substituting this value in formula (**160**)

$$C_t=0.013V_s^2$$
$$=0.013\times69.8^2=63.3 \text{ lb. per sq. in.}$$

Step 3. To obtain the effective belt pull. First find the torque in the motor shaft by substituting the values of $H=50$ hp. and $N=800$ r.p.m. in formula (**34**).

$$T=\frac{12\times33{,}000\times H}{2\pi N}$$

$$=\frac{12\times33{,}000\times50}{2\times\pi\times800}=3940 \text{ in.-lb. nearly.}$$

Applying formula (**39**), in which $r=\frac{20}{2}=10$ in.

P_t, the effective belt pull,

$$=\frac{T}{r}$$

$$=\frac{3940}{10}=394 \text{ lb.}$$

Step 4. To obtain the angle of contact between the belt and the smaller pulley. Here $R=40$ in., $r=10$ in., and $C=8$ ft. $=96$ in. Applying formula (**154**)

$$\alpha=180-2\sin^{-1}\frac{R-r}{C}$$

$$=180-2\sin^{-1}\frac{40-10}{96}$$

$$=180-2\sin^{-1}0.3125$$
$$=180-2\times18°12.5'$$
$$=143°35'$$

Step 5. To obtain the value of $e^{\mu\alpha}$ from Table XX.
From the table, for $\mu=0.30$ and $\alpha=144°$ nearly,

$$e^{\mu\alpha}=2.13$$

Step 6. To obtain T_1, the tension in the tight side of the belt. Substituting the value of P_t, the effective belt pull, in formula (**156**),

$$T_1-T_2=P_t$$
$$T_1-T_2=394$$

From formula (**157**)

$$\frac{T_1}{T_2}=e^{\mu\alpha}=2.13$$

or

$$T_2=\frac{T_1}{2.13}$$

Substituting this value of T_2 in the preceding statement of formula (**156**)

$$T_1-\frac{T_1}{2.13}=394$$

$$2.13\,T_1-T_1=2.13\times394$$
$$1.13\,T_1=839.2$$

$$T_1=\frac{839.2}{1.13}=743- \text{ lb.}$$

Step 7. To find the width of the belt, b. Here $T_1=743$ lb., $t=\frac{9}{32}$ in., $S_t=300$ lb. per sq. in. and $C_t=63.3$ lb. per sq. in.

Applying formula (**161**),

$$T_1 = bt(S_t - C_t)$$

and evaluating $743 = b \times \frac{9}{32}(300 - 63.3)$

Multiplying both members of the above equation by 32,

$$32 \times 743 = b \times 9(300 - 63.3)$$

$$23{,}776 = 2700b - 569.7b$$

$$23{,}776 = 2130.3b$$

$$b = \frac{23{,}776}{2130.3} = 11.2- \text{ in., say } 12 \text{ in.} \quad \textit{Ans.}$$

(**Note.** It will be noted that a 12-inch width had to be selected as the widths from 7 inches to 12 inches vary by 1 inch, and of course, the next larger standard size was selected.)

Adjustment of Belt Tension. New belts when placed in service are subject to considerable stretching, the extent of which is less noticeable after the belt has been in service for some time. The stretching of a belt increases its length and hence decreases its tension. In order to maintain the tension in a belt, it is therefore necessary either to shorten the belt, increase the distance between centers of the pulleys, or employ the use of a third pulley as a tension pulley or belt tightener. The shortening of a belt can be accomplished from time to time in the case of belts with laced or hinged joints by opening up the joint and cutting a small piece from the belt. This will return the belt to its initial length. The distance between centers of the pulleys can be altered by some provision in the installation such as an adjusting screw which permits one of the pulleys to be relocated with respect to the other. The idler pulley of Fig. 88 automatically takes up any slack of the belt that may be due to stretching. The belt tighteners of Figs. 91 and 92 employ geared mechanical adjustment for their idler or tension pulleys. The rack and pinion type of belt tightener of Fig. 91 has cast-iron guides and iron work ready to attach to wooden supports as shown. The belt tightener of Fig. 92 is provided with a frame which is designed to permit the tightener to be bolted to the floor.

Pulleys. The design or selection of the proper pulleys to be used in a belt drive demands careful consideration. The driving and driven pulleys must have diameters which will give the desired r.p.m. ratio, for as we have seen, the r.p.m. ratio is the inverse ratio of the

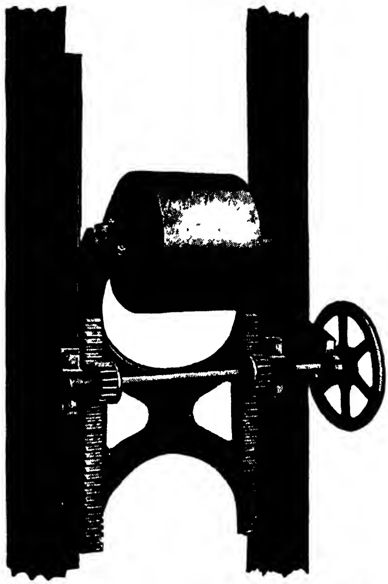


Fig 91. Rack and Pinion Type Belt Tightener
Courtesy of Link-Belt Company, Chicago, Ill.

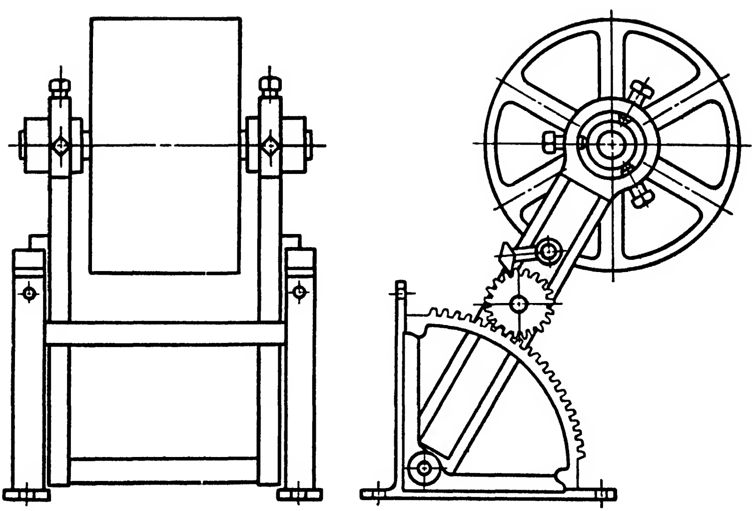


Fig 92. Floor Belt Tightener

diameters, (or radii). Also the sizes of the pulleys are factors in the determination of the belt speed; hence, diameters must be used that will create a belt speed which is desirable. Furthermore, the smaller pulley must be large enough to cause no damage to the belt as the latter is bent around it. For a thick belt when bent around a relatively small pulley is subjected to a destructive action. Pulleys, when installed, must be in perfect alignment, so that the belt will remain thereon.

The materials of which pulleys are made are as follows: (*a*) Cast Iron; (*b*) Steel; (*c*) Wood; (*d*) Paper.

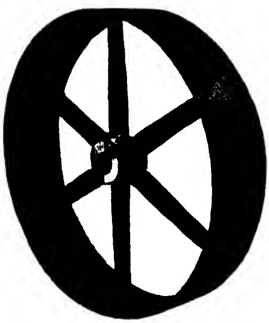


Fig 93 Solid Cast-Iron Pulley

Fig 94 Split Cast-Iron Pulley

Courtesy of Link-Belt Company, Chicago, Ill

Cast Iron Pulleys. The most common material used in the manufacture of pulleys is cast iron. Since this material is comparatively weak in tension, too high a rim speed must be avoided. For with such a rim speed, a large centrifugal force is set up and under the tension thus created the pulley is liable to burst. The maximum rim or belt speed should be considered as about 5000 feet per minute, and above 3500 feet per minute, careful balancing of the pulley is required.

Varying thicknesses in any casting are responsible for an unequal cooling which tends to set up within the casting an internal stress of unknown magnitude. This must be carefully watched in the casting of a pulley. It can be reduced or minimized by permitting the casting to cool slowly in the foundry sand, or by proper annealing after its removal from the sand.

Cast-iron pulleys are classified by their manufacturers as light-weight when constructed for use with thin belts and hence built to

service light loads. They are classified as heavy duty when they are to be used with heavy belts and submitted to heavy service. They are made solid as in Fig. 93 or split as in Fig. 94. Solid pulleys are lower in first cost, but this advantage may be more than offset by the saving in installation and maintenance expense afforded by the use of split pulleys, since the latter are more easily mounted on their shafts. The hub of either a solid or a split pulley is provided with a key-way which should be located opposite an arm.

Cast-iron pulleys are made in a great many sizes, from small diameters to very large and from narrow rims to wide ones. Small pulleys generally are made with a web connection between the rim and the hub. As the pulleys increase in size this web is replaced by arms, the number of which varies from four to eight depending on the diameter of the pulley. A double set of arms is provided when the rim width of a pulley is more than 20 inches. For the exact sizes of pulleys, the student should refer to the catalog of any manufacturer of power-transmission machinery. In fact, one can obtain a great deal of valuable information relative to all machine elements from the various catalogs of manufacturers of machine elements.

Proportions of Cast-Iron Pulleys. A belt has a tendency to climb to the highest point on a pulley. Therefore in order to keep the belt on the pulley, such a high point is purposely created at the center of the rim by increasing its thickness, as shown in Fig. 95. A pulley with such a rim is said to have a Crown. The taper which provides the height of the crown, C, should not exceed $\frac{1}{8}$ inch per foot of face, F. Instead of coming to a point as in the figure, some pulley rims are rounded to provide the same amount of crowning, C. Instead of being crowned, some pulleys are flanged at the edges to hold the belt on the pulley; but flanged pulley rims chafe and wear the edges of the belt.

The inside of the rim of a cast-iron pulley, as well as the outside of the hub should have a taper of $\frac{1}{2}$ inch per foot to permit easy withdrawal from the foundry mould. A theoretical calculation for the thickness of the rim may give a thickness that could not be cast in the foundry and the section in that case will have to be increased. A section as light as can be cast will usually be found abundantly strong for the forces it has to resist. A fair assumption of the rim thickness, t, is $\frac{3}{16}$ inch for lightweight pulleys of a diameter of 10

inches and $\frac{3}{8}$ inch for heavy-duty pulleys of the same diameter. The thickness in each of these cases may be increased $\frac{1}{16}$ inch for each 10-inch increase in diameter. The face of the pulley is given by formulas proposed by C. G. Barth as

$$F = 1\tfrac{3}{32}b + \tfrac{3}{16} \qquad (162)$$

to

$$F = 1\tfrac{3}{16}b + \tfrac{3}{8} \qquad (163)$$

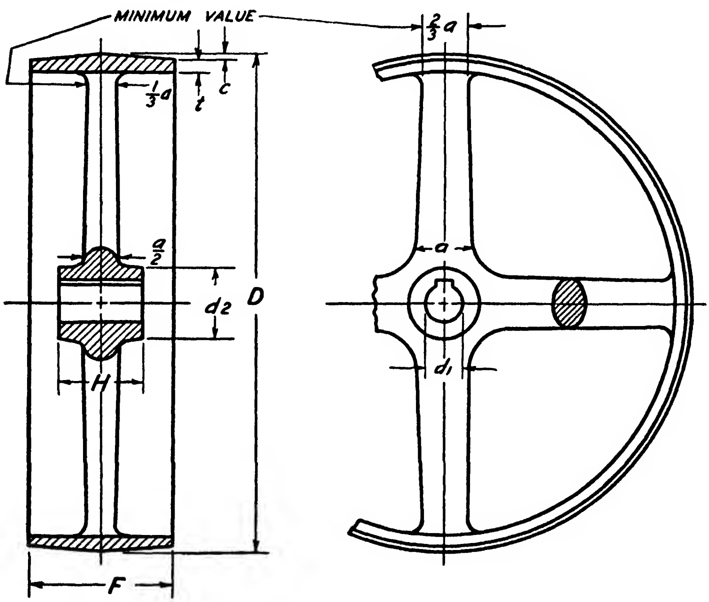


Fig. 95. Cast-Iron Pulley

where F = width of rim or face of pulley in inches
b = width of belt in inches.

The diameter and length of the hub of a pulley is proportioned to pulley diameter, face, bore, and conditions of service. The Dodge Manufacturing Corporation of Mishawaka, Indiana, in their catalog, D-30-Jr, state that for iron pulleys "the standard length of hub is $\frac{1}{20}$ of pulley diameter in inches plus $\frac{1}{2}$ of face width for single arm pulleys, or plus $\frac{3}{4}$ of face width for pulleys with two sets of arms. This length may vary $\frac{1}{4}$ inch to $\frac{3}{4}$ inch either way from standard length." A formula that gives good average values in most cases is as follows:

$$H=\frac{\pi}{2}d_1 \tag{164}$$

in which H = length of hub in inches
d_1 = diameter of shaft in inches

The diameter, d_2, of the hub may be obtained from the formula:

$$d_2=1.5d_1+1 \text{ in.} \tag{165}$$

If the above formula produces a diameter of hub that is more than twice the diameter of the shaft, as it will for relatively small shafts, the following formula should be used:

$$d_2=2d_1 \tag{166}$$

The arms of a cast-iron pulley are usually elliptical in section, with the major axis of the ellipse equal to twice the minor axis. Since the pulley is subjected to a tangential force or effective belt pull, P_t, at its rim, the arms may be assumed to take this tangential force as an end load. Hence they act as cantilever beams supported at the center of the pulley. Due to the fact that the belt wraps itself around the pulley rim through approximately 180 degrees, one-half of the arms are assumed to carry the load, so that the

$$\text{end load per arm}=\frac{P_t}{\frac{n}{2}}=\frac{2P_t}{n}$$

where n = the number of arms.

Since in a cantilever beam, the maximum bending moment, M, is at the support, we have from formula (23)

$$M=\frac{2P_t}{n}\times R$$

in which the length of the beam is taken as the radius, R, of the pulley, and P of the formula is replaced by $\frac{2P_t}{n}$.

Applying the beam design formula (27),

$$M=SZ$$

and substituting the value of M as given above,

$$\frac{2P_t}{n}\times R=SZ$$

The section modulus, Z, for an ellipse whose major axis is a and minor axis is $\frac{a}{2}$, is found in Table III to be $\frac{\pi a^3}{64}$. Substituting this value in the above equation

$$\frac{2P_t}{n}\times R=S\times\frac{\pi a^3}{64}$$

Multiplying both members of this equation by $64\times n$,

$$2\times P_t\times R\times 64=S\times\pi a^3\times n$$

Dividing by $S\times\pi\times n$,

$$a^3=\frac{2\times P_t\times R\times 64}{S\times\pi\times n}=\frac{P_tR}{Sn}\times\frac{128}{\pi}$$

$$a=\sqrt[3]{\frac{P_tR}{Sn}\times\frac{128}{\pi}}=\sqrt[3]{\frac{P_tR}{Sn}}\times\sqrt[3]{\frac{128}{\pi}}$$

$$a=3.44\sqrt[3]{\frac{P_tR}{Sn}}=3.44\sqrt[3]{\frac{T}{Sn}} \qquad \textbf{(167)}$$

The unit working stress, S, for cast-iron arms can be assumed as 1800 to 3000 lb. per sq. in., depending upon the grade of cast iron used.

The dimensions of the arm as found from formula (**167**) are those which the arm would have if it extended to the center of the pulley. It is on the side of safety to give the arm these dimensions at the hub. The dimensions of the smaller section at the rim can then be obtained by allowing a taper of $\frac{3}{8}$ inch per foot but such dimensions should not be less than $\frac{2}{3}$ of those at the hub.

Example. It is required to obtain the dimension at the hub of the standard elliptical arms of a cast-iron pulley, which has a diameter of 30 inches, carries 6 arms, and transmits 20 horsepower at 150 r.p.m. Assume the unit working stress of cast iron as 2000 lb. per sq. in.

Solution. In solving such an example as this, it is much better to use the analysis which produced formula (**167**) than to evaluate directly in the latter formula. We shall here solve by analysis and then check the solution by evaluating in the formula.

First method. Step 1. Find the torque in the pulley shaft by using formula (**34**), in which $H=20$ hp. and $N=150$ *r.p.m.*

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 20}{2\times\pi\times 150}=8408 \text{ in.-lb.}$$

Step 2. Obtain the effective belt pull by using formula (39) with $R=\frac{30}{2}=15$ in.

$$P_t=\frac{T}{R}$$

$$P_t=\frac{8408}{15}=560.5 \text{ lb.}$$

Step 3. Obtain the end load per arm, assuming that one-half the arms, or 3 arms, carry the total load, P_t.

$$\text{Load per arm}=\frac{P_t}{\frac{n}{2}}$$

$$=\frac{560.5}{3}=187- \text{ lb.}$$

Step 4. Obtain the maximum bending moment by applying formula (23), in which $L=R=15$ in.

$$M=PL$$

$$=187\times15=2805 \text{ in.-lb.}$$

Step 5. Here $M=2805$ in.-lb., $S=2000$ lb. per sq. in. and Z (from Table III) $=\frac{\pi a^3}{64}$. Applying formula (27)

$$M=SZ$$

and evaluating, $2805=2000\times\frac{\pi a^3}{64}$

$$2805\times64=2000\times\pi a^3$$

$$a^3=\frac{2805\times64}{2000\times\pi}=28.6$$

$$a=\sqrt[3]{28.6}=3.06 \text{ in., say } 3\tfrac{1}{8} \text{ in.} \quad \textit{Ans.}$$

Therefore $$\frac{a}{2}=\frac{3\frac{1}{8}}{2}=1\tfrac{9}{16} \text{ in.} \quad \textit{Ans.}$$

(**Note.** $3\frac{1}{8}$ in. was chosen as the value of a so that $\frac{a}{2}$ would not be expressed in any fraction with a unit less than the sixteenth. Always keep the material and method of construction in mind when increasing the theoretical answer to its corresponding practicable value.)

Second method. Step 1. Find the torque, T, as in the first method

$$T = \frac{12 \times 33{,}000 H}{2\pi N}$$

$$= \frac{12 \times 33{,}000 \times 20}{2 \times \pi \times 150} = 8408 \text{ in.-lb.}$$

Step 2. Find the width of the arms by evaluating in formula (**167**)

$$a = 3.44 \sqrt[3]{\frac{T}{Sn}}$$

$$= 3.44 \sqrt[3]{\frac{8408}{2000 \times 6}}$$

$$= 3.06 \text{ in., say } 3\tfrac{1}{8} \text{ in.} \quad \textit{Ans.}$$

Example. In the case of the pulley of the preceding example, find (*a*) the diameter, d_1, of the shaft for which the pulley hub is to be bored, assuming $S_s = 7000$ lb. per sq. in., (*b*) the diameter of the hub, (*c*) the length of the hub, (*d*) the face of the pulley, assuming that the width of belt is eight inches.

Solution. (*a*) Here $S_s = 7000$ lb. per sq. in. and T (from the preceding problem) $= 8408$ in.-lb., Z_p, from Table IV, $= \frac{\pi d_1^3}{16}$, Using formula (**31**)

$$T = S_s Z_p$$

$$8408 = 7000 \times \frac{\pi d_1^3}{16}$$

$$d_1 = \sqrt[3]{\frac{8408 \times 16}{7000 \times \pi}} = \sqrt[3]{6.1} = 1.82 \text{ in., say } 1\tfrac{7}{8} \text{ in.}$$

or $1\frac{15}{16}$ in., the latter is the more standard. *Ans.*

(*b*) From formula (**166**) using $d_1 = 1\frac{15}{16}$ in.

$$d_2 = 2d_1$$

$$= 2 \times 1\tfrac{15}{16} = 3\tfrac{7}{8} \text{ in.} \quad \textit{Ans.}$$

(*c*) From formula (**164**)

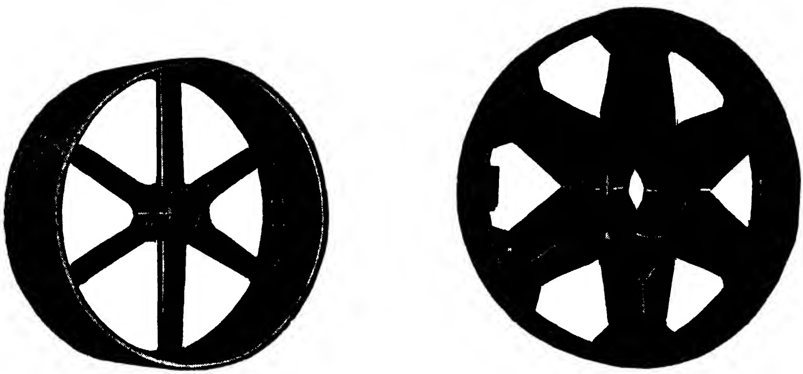
$$H = \frac{\pi}{2} d_1$$

$$= \frac{\pi}{2} \times 1\tfrac{15}{16} = 3.04 \text{ in., say } 3\tfrac{1}{16} \text{ in. or } 3\tfrac{1}{8} \text{ in.} \quad \textit{Ans.}$$

(*d*) From formula (163)

$$F = 1\tfrac{3}{16}b + \tfrac{3}{8}$$
$$= 1\tfrac{3}{16} \times 8 + \tfrac{3}{8} = 9\tfrac{7}{8} \text{ in.} \quad \textit{Ans.}$$

Steel Pulleys. Steel pulleys as shown in Fig. 96 are fabricated from pressed-steel sheets and for that reason have great strength and durability. They also possess the important characteristic of lightness, being only 40 per cent to 60 per cent as heavy as cast-iron pulleys of the same capacity. They present a coefficient of friction with leather belting which is at least equal to that obtained by cast iron pulleys.



Standard Type — Heavy Duty Type

Fig 96 Steel Split Pulleys

Courtesy of The American Pulley Co., Philadelphia, Pa

The rim of a steel pulley is generally formed in two halves, each of which may be made up of one or more sections. These are formed in dies under heavy hydraulic pressure. The arms are pressed separately and are so narrow that they cut the air rather than displace it. This reduces the losses due to air fanning to a minimum, which is a considerable factor when a pulley is run at high speeds. The rim sections and arms together with the separately stamped hub are riveted and bolted together thus forming a split pulley. The clamping action of the hub holds the pulley to its shaft so that it is unnecessary to key the pulley upon its shaft except for the most severe service. Steel pulleys are generally equipped with interchangeable bushings to permit their use with shafts of different sizes.

Wood Pulleys. Wood is another material which is used in the construction of transmission pulleys. It provides a lighter pulley

and one which offers a higher coefficient of friction than those made of cast iron or steel.

Wood pulleys are made both solid and split. A solid wood motor pulley is shown in Fig. 97. They are generally constructed of selected maple which is laid up in segments and glued together under heavy pressure. They are kept from absorbing moisture by protective coatings of shellac or varnish, so that warping will not occur. Wood pulleys are made with cast-iron hubs with keyways, or have adjustable bushings which prevent relative rotation between them and the shaft by the frictional resistance set up.

Tight and Loose Pulleys. A combination of pulleys (Fig. 98) consisting of one tight pulley and one loose pulley is often employed on the shaft of a machine which uses power intermittently. The tight pulley is keyed to the shaft of the machine while the loose pulley rotates freely thereon. Hence the belt runs over the tight pulley if power is to be served to the machine and when the latter is to be thrown out of service the belt is shifted by means of a belt shifter to the loose pulley. In this manner the stopping of one machine does not interfere with other machines which are driven by the same line shaft.

The loose pulley is generally somewhat smaller in diameter than the tight pulley. This lessens the tension in the belt while it is running idle. A collar is provided to hold the loose pulley in position. The bearing in its hub must be carefully designed and provision made for its proper lubrication.

Stepped Cone Pulleys. A machine can be driven at several different speeds when stepped cone pulleys like unto that illustrated in Fig. 99 are used. The pulleys are arranged on the driving and driven shafts so that the smallest step of one pulley is opposite the largest step of the other. As the belt is shifted, it engages pairs of steps of different diameters and in this manner may secure as many different speeds for the machine as there are pairs of steps. The diameters of the pairs of steps which work together must be so selected that they will give the desired r.p.m. ratios and permit the use of the same belt (length).

Rope Drives. Although rope drives have been superseded of late to a great extent by other methods of transmitting power, certain conditions arise which still make a rope drive feasible and economical.

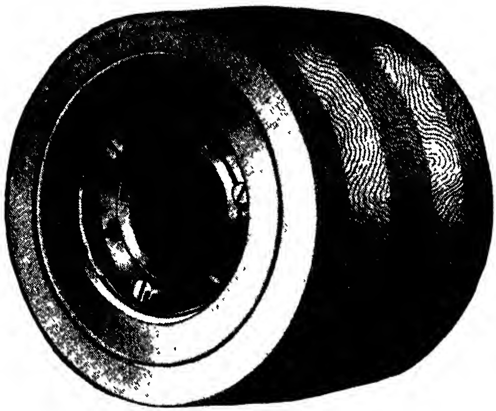


Fig 97. Solid Wood Motor Pulley
Courtesy of Dodge Manufacturing Corp , Mishawaka, Indiana

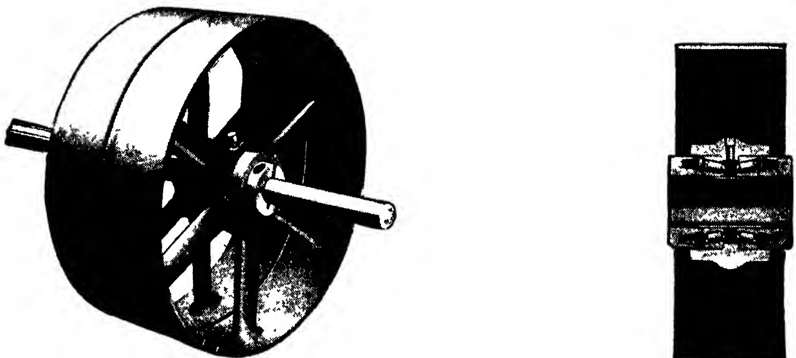


Fig 98 Left, Tight and Loose Pulleys Right, Loose Pulley Is Mounted on Timken Roller Bearings
Courtesy of Dodge Manufacturing Corp , Mishawaka, Indiana

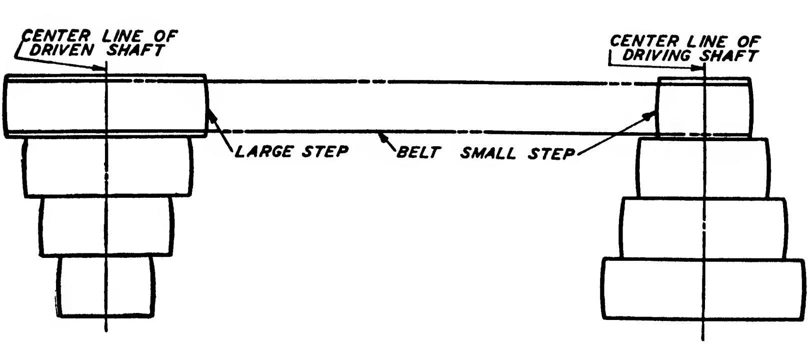


Fig 99. Stepped Cone Pulley

This is particularly true when a considerable horse power is to be transmitted between a pair of shafts where distance between centers is rather large, say from 50 to 100 feet or more.

Two kinds of rope are used in power transmission, namely manila and cotton rope. Manila rope is stronger and more durable than cotton rope, but the rough hemp fiber of which it is composed causes the rope to wear and chafe internally when it is bent around its sheave, (grooved pulley). To overcome this, the rope fibers are lubricated with some suitable lubricant such as tallow or graphite, which also aids the rope in withstanding exposure to the weather. The softness and smoothness of the cotton fiber prevents any internal damage or deterioration in the cotton rope, hence lubrication with the

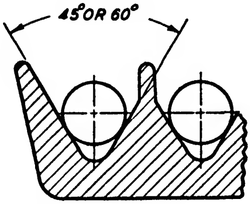


Fig. 100 Textile Rope Laboring Sheave

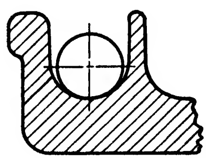


Fig. 101 Textile Rope Idler Sheave

latter is unnecessary except to permit less external wear between the rope and the grooves of its sheave.

As with any wrapping connector, the bending of a rope about its sheave sets up a bending stress which decreases as the diameter of the sheave increases. For this reason, it is advisable to use a sheave whose diameter is about 40 times the diameter of the rope it carries. If this is adhered to, an allowable working stress of $200d^2$ pounds per rope can be used in design for either a manila or cotton rope. (d is the nominal diameter of the rope.) This gives a factor of safety of 35 or more for manila ropes and a trifle less for cotton ropes. The latter can accept this somewhat lower factor of safety due to the inherent smoothness of fiber to which reference has already been made.

The sheaves with which ropes are used are in reality pulleys with grooved rims. The grooves are finished smooth so as to minimize the external wear that would result from contact with a rough surface. A section of such a rim is shown in Fig. 100. The grooves not only hold the rope on the pulley but also cause a wedge-like action to

take place thus increasing the power that can be transmitted by providing a greater frictional resistance between the rope and the sheave. Since this wedge-like action is to some extent destructive, a sheave when used only as an idler or tension sheave is grooved as in Fig. 101.

Systems of Rope Driving. There are two systems of rope driving. One is known as the Multiple, or English, and the other as the Continuous, or American, system.

The Multiple System consists of one or more independent ropes running side by side in the grooves of the sheaves. Each rope is supposed to transmit the same amount of power. This requires that each rope be under the same initial tension, a condition that is dif-

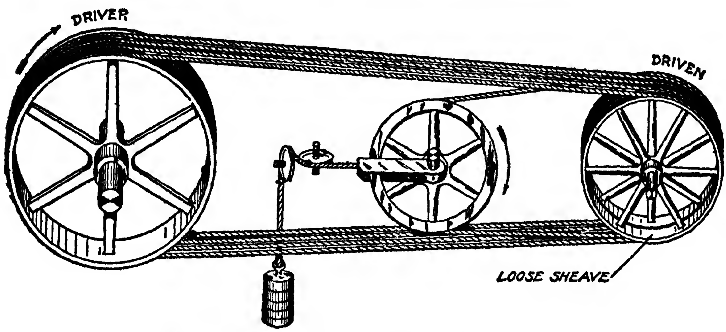


Fig 102 Continuous Rope Drive, American System

ficult to obtain, and hence a disadvantage of the system. A distinct advantage however is that if one rope breaks, the drive can still continue to function. With this system, it is possible to distribute power to different machines from the same driving sheave.

The Continuous System makes use of only one rope which is wound around the driving and driven pulley several times. In this system, Fig. 102, the rope is conducted from the outside groove of one sheave wheel to the inside groove of the other by means of the use of a traveling tension carriage or jockey. This carriage also serves to maintain a uniform tension throughout the rope. It is arranged to travel back and forth, automatically regulating the slack. In this manner the stretch in the rope and the inequalities in the load are taken care of, a distinct advantage of this system.

Design of Rope Drives. The design of a rope drive is very similar to the design of a belt drive. The power transmitted sets up

tensions, (or tensile loads) in the rope whose relationships are expressed by the same formulas as for belts.

These formulas are:

$$T_1 - T_2 = P_t \qquad \text{(Formula (156))}$$

and

$$\frac{T_1}{T_2} = e^{\mu\alpha} \qquad \text{(Formula (157))}$$

In these formulas, T_1 and T_2 are the total tensions which are set up in the n ropes of the drive, whether these n ropes are individual ropes of the English system, or the n times that a continuous rope of the American system is wound around the pulley. In the case of either cotton or manila ropes, the above ratio of the tensions can be taken as

$$\frac{T_1}{T_2} = 3 \qquad (168)$$

This value of the ratio takes into consideration the added frictional resistance obtained through the wedge-like action of the rope with the groove of its sheave. It further assumes that the angle of contact, α, is approximately 180 degrees as is generally the case.

From formula (168), $T_2 = \frac{T_1}{3}$

Substituting this value of T_2 in formula (156),

$$T_1 - \frac{T_1}{3} = P_t$$

from which

$$3T_1 - T_1 = 3P_t$$

$$2T_1 = 3P_t$$

$$T_1 = \frac{3P_t}{2} \text{ lb.} \qquad (169)$$

The effect of centrifugal force is to subject the rope to a tensile load which is in addition to the maximum tension T_1, which it must carry. This centrifugal effect is given by the formula

$$C_1 = 0.0105 d^2 V_s^2 \text{ pounds} \qquad (170)$$

in which

C_1 = centrifugal effect in pounds for each one of the n ropes used
d = nominal diameter of the rope
V_s = speed of the rope in ft. per sec.

If C_1 is the centrifugal effect for one rope, then the centrifugal

effect, C_n, of the n ropes of the drive will be given by the formula

$$C_n = n \times C_1 \text{ pounds} \tag{171}$$

The total tensile load that the rope drive, consisting of n ropes, must carry is the sum of the tension, T_1, due to the power transmitted and the tension, C_n set up by centrifugal force. This is $T_1 + C_n$ or $T_1 + nC_1$.

If we let t represent the number of pounds of tensile load that one rope can safely carry, the number of ropes, n, that the drive must contain can be found by dividing the total tensile load, $T_1 + nC_1$, by t.

Thus $$n = \frac{T_1 + nC_1}{t}$$

Multiplying by t $$nt = T_1 + nC_1$$

from which $$nt - nC_1 = T_1$$

$$n(t - C_1) = T_1$$

Dividing by $(t - C_1)$ $$n = \frac{T_1}{t - C_1} \tag{172}$$

The allowable or safe tensile load that may be assumed for either a cotton or a manila rope is given in terms of its diameter.

$$t = 200d^2, \text{ pounds per rope} \tag{173}$$

By substituting in formula (172), the values of t and C_1, as given in formulas (173) and (170) respectively, we have

$$n = \frac{T_1}{200d^2 - 0.0105d^2V_s^2} \tag{174}$$

Example. How many $1\frac{1}{2}$-inch manila ropes are necessary for a rope drive that is to transmit 260 horsepower at a belt speed of 3500 feet per minute?

Solution. Step 1. To obtain the total effective belt pull, P_t, of this rope drive. Here $H = 260$ hp. and $V_L = 3500$ f.p.m.
Evaluating in formula (35)

$$H = \frac{P_t V_L}{33{,}000}$$

$$260 = \frac{P_t \times 3500}{33{,}000}$$

$$P_t = \frac{33{,}000 \times 260}{3500} = 2451.4 \text{ lb.}$$

Step 2. Find the tension, T_1, in the tight side of the drive.

From formula (**169**)

$$T_1 = \frac{3P_t}{2}$$

$$= \frac{3 \times 2451.4}{2} = 3677.1 \text{ lb.}$$

Step 3. Find the allowable tensile load, t, for a $1\frac{1}{2}$-inch manila rope.

From formula (**173**) $t = 200d^2$

$$t = 200 \times (1\tfrac{1}{2})^2 = 450 \text{ lb.}$$

Step 4. Find the centrifugal effect, C_1, in pounds per rope.

Here $d = 1\frac{1}{2}$ in. and $V_s = \frac{3500}{60} = 58.3$ f.p.s.

Substituting these values in formula (**170**),

$$C_1 = 0.0105d^2V_s^2$$

$$= 0.0105 \times (1\tfrac{1}{2})^2 \times 58.3^2$$

$$= 80.3 \text{ lb.}$$

Step 5. Find the number of ropes, n. Here $T_1 = 3677.1$ lb., $t = 450$ lb., and $C_1 = 80.3$ lb.

Evaluating in formula (**172**),

$$n = \frac{T_1}{t - C_1}$$

$$= \frac{3677.1}{450 - 80.3} = 9.9 \text{ ropes, say 10 ropes.} \quad \textit{Ans.}$$

Wire Rope. Wire rope is made by twisting small steel wires into strands and then in turn twisting several of these strands about a hemp core or center. The number of wires used in a strand and the number of strands that are woven into the rope are given in the name of the rope. Thus a 6-7 wire rope is one that has 7 wires in each of its 6 strands while a 6-19 rope has 19 wires in each of its 6 strands. The former evidently contains 42 individual wires while the latter contains 114 wires. While a great many different types of ropes are made, and can be found in the catalogs of wire rope manufacturers, the most common types are probably the 6-7, 6-19, and 6-37. For a given diameter of rope, the larger the number of wires used therein, the greater will be its flexibility and the smaller may be the diameter of the sheave or drum with which it is to be used.

The bending stress set up in a wire rope is a considerable factor.

Manufacturers, in listing their ropes, give the ultimate stress per rope and the minimum diameter of sheave with which each rope should be used. Smaller diameters than those listed would use up, in bending, too much of the available strength of the rope and thus leave too small an amount for the useful task that is to be performed. It will be found that these minimum diameters suggest the following relationships between themselves and the diameters of their ropes.

$D=85d_r$ for a 6-7 rope.

$D=48d_r$ for a 6-19 rope.

$D=30d_r$ for a 6-37 rope.

in which D=diameter of sheave or drum in inches.

d_r=diameter of rope in inches.

The bending stress, S_b, of a wire rope may be obtained from the following formula,

$$S_b=12{,}000{,}000\ \frac{d_w}{D},\ \text{lb. per sq. in.} \tag{175}$$

in which D is as given above and d_w is the diameter of each of the individual wires of which the rope is composed.

$d_w=\frac{1}{9}d_r$, for a 6-7 rope.

$d_w=\frac{1}{15}d_r$, for a 6-19 rope.

$d_w=\frac{1}{21}d_r$, for a 6-37 rope.

The sheave of a wire rope is provided with a wide groove so that the rope may rest upon the bottom of the groove. As a rule the bottom of the groove is provided with an insert of rubber, leather, or wood. This avoids excessive wear of the rope and increases the frictional resistance between rope and sheave. A rope drum employed in a hoist may be either spirally grooved or plain.

Example. A 6-19 wire rope with a nominal diameter of $\frac{3}{4}$-inch is listed by its manufacturers as having an ultimate strength of 46,000 pounds. If this rope is used with a drum whose diameter is 48 times the diameter of the rope, what load may be hoisted safely?

Solution. In the case of wire ropes which are used for hoisting, a factor of safety of at least 6 should be employed, so that in this case the total available or safe strength of the rope $=\frac{46{,}000}{6}=7667$ lb. (not lb. per sq. in.)

In a 6-19 rope there are

$$6\times 19=114 \text{ wires.}$$

The diameter of each wire in a 6-19 rope, as previously stated, is

$$d_w=\tfrac{1}{15}d_r=\tfrac{1}{15}\times\tfrac{3}{4}=0.05 \text{ in.}$$

It follows that the area of each wire will be

$$a=\frac{\pi d_w^2}{4}=\frac{\pi\times 0.05^2}{4}=0.00196 \text{ sq. in.}$$

Hence the total area of the rope is given by

$$A=114a=114\times 0.00196=0.224 \text{ sq. in.}$$

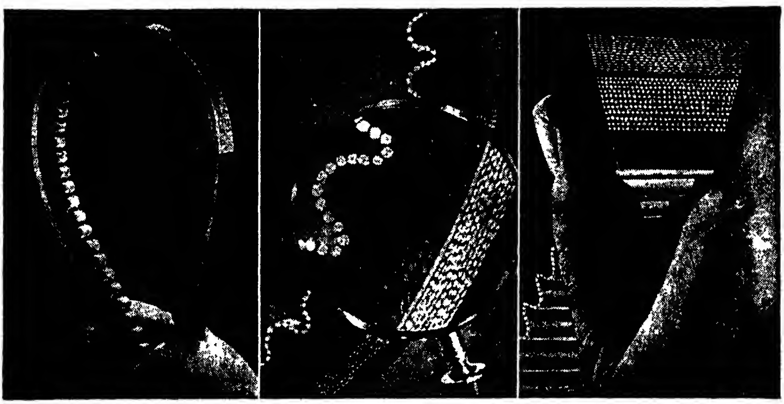


Fig 103 Dayton Cog-Belt, a V-Belt
Courtesy of The Dayton Rubber Mfg Co, Dayton, Ohio

Applying formula (175), in which
D, the drum diameter $=48d_r=48\times\tfrac{3}{4}=36$ in.,
we obtain for the bending stress in the rope

$$S_b=12{,}000{,}000\ \frac{d_w}{D}$$

$$=12{,}000{,}000\times\frac{0.05}{36}=16{,}667 \text{ lb. per sq. in.}$$

The available strength of the rope that is used up in bending will equal the area, A, of the rope times S_b, and

$$A\times S_b=0.224\times 16{,}667=3733 \text{ lb. (not lb. per sq. in.)}$$

That part of the total available strength of the rope which is not used up in bending becomes available for raising the load, W. From the first step of this solution, the total available strength of the rope

is equal to 7667 lb. Therefore

$$W = 7667 - 3733 = 3934 \text{ lb.} \quad \textit{Ans.}$$

V-Belt Drive. This mode of power transmission derives its name from the trapezoidal cross section of the belt which it employs. Such a cross section is shown at the right in Fig. 103. This figure portrays the Dayton Cog-Belt, manufactured by the Dayton Rubber Mfg. Company of Dayton, Ohio. Fig. 104 pictures this same belt cut so as to show its three prime sections. The outer section of the belt is called the Tension Section, by its manufacturer, due to the fact that it is under tension when flexed over the pulley. It consists of bias-cut fabric which is said to provide just the right amount of "give." The middle section is called the Neutral Section. It is the so-called "strength" section and consists of pre-stretched cord cable fabric. It is applied under tremendous tension and is mainly responsible

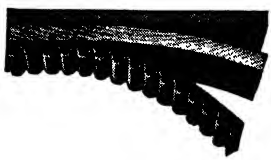


Fig. 104. V-Belt, Cut to Show Three Prime Sections
Courtesy of The Dayton Rubber Mfg Co, Dayton, Ohio

according to its manufacturer for the non-stretching feature of the belt. The inner section is called the Compression Section due to the action therein when the belt is flexed over the pulley. It is the Cogged surface of this section that gives the belt its name. It provides the flexibility which together with the tough fiber and rubber composition of this section insures a cross-sectional rigidity and prevents distortion so that the sides of the belt will have their areas in full contact with the sides of the pulley grooves. The wedge-like action between belt and groove secured by such contact sets up a relatively high frictional resistance. This characteristic of the V-Belt drive is the main reason for its many advantages for transmitting power under certain conditions of installation.

Other manufacturers of V-Belts make similar claims to the above characteristics for their products, and obtain them through a variety of internal constructions. In fact most manufacturers have several different types of construction, each of which is designed to meet certain specific operating conditions.

V-belts, as indicated by their cross sections, operate with grooved sheaves of the types shown in Figs. 105 and 106. They may operate individually as in the fan-belt drive of an automobile where a small amount of power is to be transmitted, or collectively as a multiple-drive, by which as much as 500 horsepower can be transmitted. As a multiple-drive, their installation is similar to that of a multiple-rope drive. The V-belt drive can be used where small center distances are necessarily employed even though the velocity ratio between driver and driven is relatively high. For, due to the

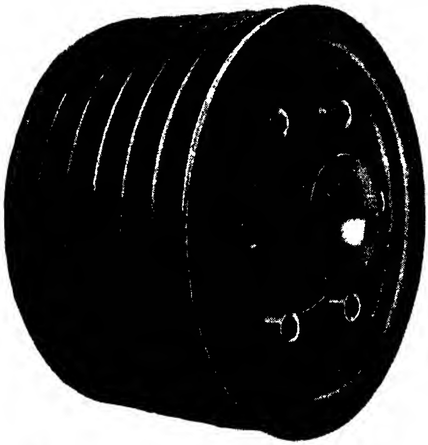


Fig 105. V-Belt Sheave with Demountable Rim
Courtesy of Dodge Manufacturing Corp., Mishawaka, Indiana

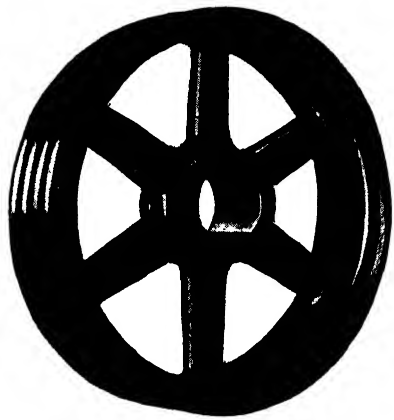


Fig. 106. Cast-Iron Sheave for V-Belt
Courtesy of The American Pulley Co., Philadelphia, Pa

wedge-like action of the belt in the grooves of its sheave, the arc of contact with the smaller pulley need not be as large as when a flat belt is employed. V-belts operate at high or low speeds transmitting full power without much slippage. They insure smooth easy starting, readily absorb shock and vibration, and are silent and clean. Neither lubrication nor belt dressing need be employed. With a V-Belt installation the belt tensions are less than those in a similar flat belt installation due to the greater frictional resistance created by the belt riding in the grooves of the sheaves. Hence the pressures on the bearings which support the shafts of the pulleys are materially reduced. This tends to longer bearing life and hence reduced maintenance expense. Typical V-belt drives are illustrated in Figs. 107 and 108. In Fig. 107 both large and small pulleys are grooved, but

in Fig. 108, the larger pulley is plain and uncrowned. The latter is called a V-Flat Drive and is possible where a short center distance and a fairly high velocity ratio present a large arc of contact with the larger pulley.

Fig. 107. Typical Installation of V-Belt Drive
Courtesy of The American Pulley Co , Philadelphia, Pa.

Chain-Drive. In all of the previous drives mentioned in this chapter, power is transmitted from one shaft to another through a frictional resistance which is set up between the wrapping connector and its pulley. In such cases, slipping occurs to at least some small extent. This interferes with the angular velocity ratio between the driver and its follower. When it is imperative that the angular velocity ratio be constant, some form of positive drive must be

Fig. 108. V-Flat Drive on Air Compressor—an Application of the "D-V" Drive
Courtesy of Dodge Manufacturing Corp., Mishawaka, Indiana

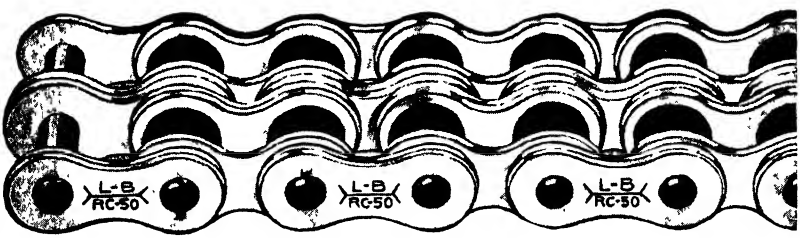


Fig 109 Double Roller Chain
Courtesy of Link-Belt Company, Chicago, Ill.

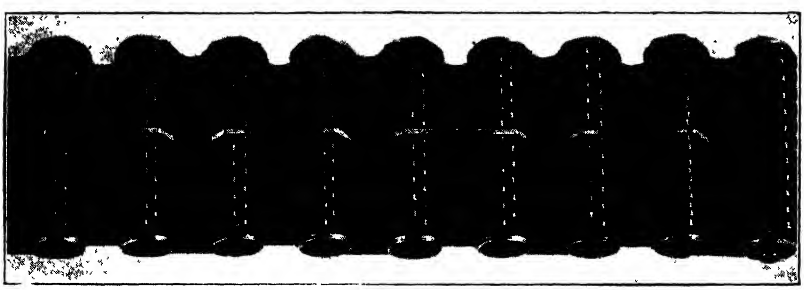


Fig. 110 Silent Chain
Courtesy of Link-Belt Company, Chicago, Ill

employed. The chain is that type of a wrapping connector which secures positive driving through its use with toothed wheels called sprockets. A chain-drive is used in preference to gears when the distance between centers of the shafts to be connected makes a gear-drive unfeasible. With a chain-drive, the mean angular velocity ratio between driver and follower is constant and is inversely as the number of teeth on the sprockets. But there is a very slight variation in the angular velocity ratio over an angular displacement or movement of the sprocket equal to the central angle included between the center lines of a pair of adjacent teeth. This variation becomes even smaller as the distance between teeth (the pitch of the sprocket) is reduced, so that with a fair number of teeth on a sprocket, the variation in speed is of no practical importance.

Chains may be classified as follows: (*a*) hoisting and hauling chains, (*b*) elevator and conveyor chains, (*c*) power-transmission chains.

The two main types of chains that are used for power transmission are the Roller Chain and the Silent Chain.

A double roller chain is shown in Fig. 109. Where the power requirement is less, a single roller chain can be used. On the other hand, if the power requirement is too great for a double roller, a triple or quadruple chain may be used. Such a chain operates well at speeds as high as 1000 feet per minute. Since a chain is composed of a great number of small parts, close attention must be paid to its lubrication and cleaning. Where it is exposed to considerable dirt and dust, a casing should be used. Such a casing for the chain affords protection to the employees as well. Periodically the chain should be removed from the sprockets and cleaned in a bath of gasoline or kerosene. After this it should be well drained and its lubricant restored by its being placed in a bath of hot grease or oil until the lubricant has had plenty of time in which to penetrate to all bearing surfaces.

The so-called Silent Chain of Fig. 110 is an improvement upon the previous type. It gets its name from the fact that it is much less noisy in operation than other types of chains. It can be used at higher speeds and can transmit a greater horsepower. The Link-Belt Silverstreak Silent Chain is made in various sizes to transmit power up to 2000 horsepower and over. Silent chains permit speed reduc-

tions of 15 to 1 and short distances between centers. They usually operate in a casing which contains the oil for automatic lubrication of the drive. A silent chain drive is illustrated in Fig. 111.

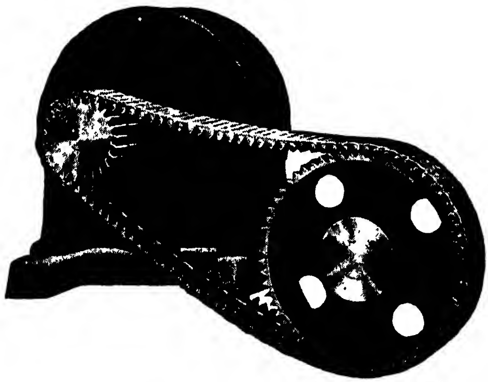


Fig. 111. Silent Chain Drive
Courtesy of Link-Belt Company, Chicago, Ill

PROBLEMS

1. State the relationship between the angular velocities and diameters of the driver and driven pulleys of a belt drive.

2. What materials are used in the construction of belts?

3. It is required to find the length of an open belt which is used to connect a 20-inch pulley to a 36-inch pulley, the distance between centers being 5 feet. *Ans.* 209+ in.

4. In the preceding problem, the 20-inch pulley makes 720 revolutions per minute. (*a*) What is the speed of belt in feet per minute? (*b*) What is the r.p.m. of the 36-inch pulley? *Ans.* (*a*) 3770 − f.p.m. (*b*) 400 r.p.m.

5. Find the arc of contact between the belt and the smaller pulley in Problem 3. Obtain the answer in both degrees and radians. *Ans.* 164° 41′; 2.874 radians.

6. A pulley, 24 inches in diameter, transmits 50 horsepower at 600 r.p.m. The arc of contact between the belt and pulley is 144 degrees, the coefficient of friction between belt and pulley is 0.35, and the safe working stress of the belt is 300 lb. per sq. in. It is required to find (*a*) the speed of the belt in feet per minute, (*b*) the tangential force at the rim of the pulley, (*c*) the effective belt pull, (*d*) the value of $e^{\mu\alpha}$, (*e*) the value of T_1, the tension in the tight side, (*f*) the centrifugal effect, C_1, in lb. per sq. in., (*g*) the width of belt to be used if its thickness is $\frac{1}{4}$ inch. *Ans.* (*a*) 3770 f.p.m. (*b*) 437.6 lb. (*c*) 437.6 lb. (*d*) 2.41. (*e*) 748 lb. (*f*) 51.3 lbs. per sq. in. (*g*) 12 in.

7. A cast-iron pulley is 22 inches in diameter and carries four standard elliptical arms. The pulley transmits 25 horsepower at 300 r.p.m. It is required to find (*a*) the diameter of the shaft (or bore) if S_s is taken as 7000 lb. per sq. in., (*b*) the dimensions of the arms at the hub, assuming the allowable working stress in the arms to be 2000 lb. per sq. in., (*c*) the face of the pulley, assuming that it

carries a 7-inch belt (Use formula (163)), (*d*) the length of the hub, (*e*) the diameter of the hub. *Ans.* (*a*) $1\frac{5}{8}$ in. (*b*) $1\frac{1}{2}$ in. by 3 in. (*c*) $8\frac{13}{16}$ in. (*d*) $2\frac{5}{8}$ in. (*e*) $3\frac{1}{4}$ in.

8. What is a V-flat drive?

9. When should a multiple V-belt drive be used in place of a flat belt drive?

10. When should a flat belt drive be used instead of a multiple V-belt drive?

11. When should a chain drive be selected instead of a V-belt drive?

12. Does a wire rope come into contact with the bottom of the groove or with the sides of the groove of its sheave?

13. Find, (*a*) the number of wires, (*b*) the diameter of each wire, and (*c*) the area of a $\frac{1}{2}$-inch 6-37 wire rope. *Ans.* (*a*) 222 wires (*b*) 0.0238 in. (*c*) 0.099 sq. in.

14. How large a drum or sheave should be used with a $\frac{5}{8}$-inch 6-19 wire rope? *Ans.* 30 in.

15. A 6-19 wire rope with a nominal diameter of $\frac{1}{2}$ inch is listed by its manufacturer as having an ultimate strength of 25,000 pounds. If this rope is used with a drum whose diameter is 48 times the diameter of the rope, what load may be safely hoisted? Let the factor of safety $F=6$. *Ans.* 2560 lb.

16. How many 1-inch manila ropes are necessary for a rope drive that is to transmit 150 horsepower at a belt speed of 4000 f.p.m? *Ans.* 12.1 ropes, say 12 or 13 ropes.

17. The diameter of a sheave is taken as 40 times the diameter of the rope to be used therewith. If the speed of the rope is 4000 feet per minute and the diameter of the rope is 1 inch, find the r.p.m. of the sheave. *Ans.* 382 r.p.m.

18. How do the English and American systems of rope drive differ from each other?

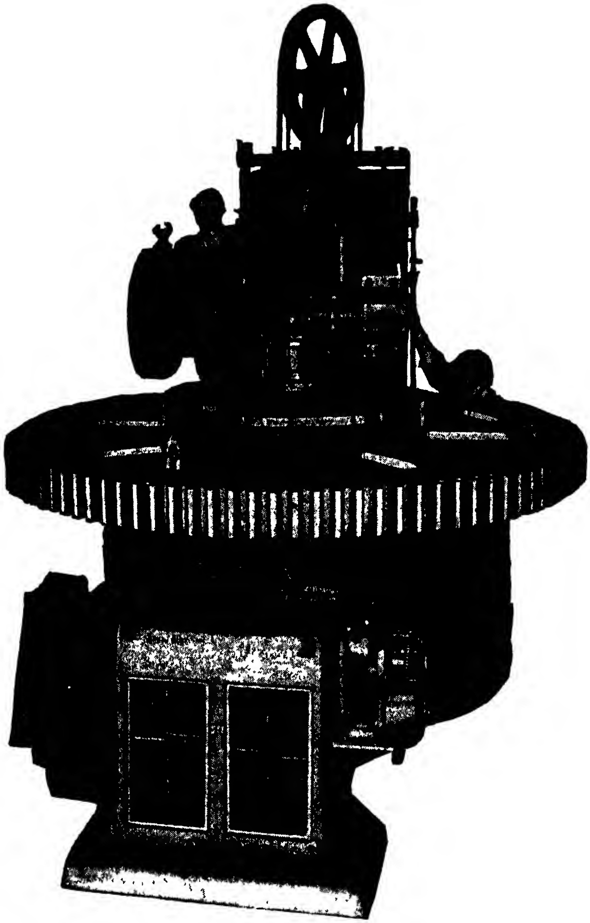
19. Is the groove of an idler pulley of a textile rope drive designed to secure a wedge-like action between itself and the rope?

20. Give a classification of chains.

21. Name two kinds of chains that are used for transmitting power.

22. What kind of a wrapping connector mechanism may be used when positive driving must be secured?

23. Two shafts, *A* and *B*, are connected by a chain drive. Shaft *A* carries a 15-tooth sprocket and rotates at 1000 r.p.m. Shaft *B* carries a 150-tooth sprocket. It is required to find the rotative speed of shaft *B*. *Ans.* 100 r.p.m.



144-INCH SPUR GEAR CUTTER

Tnis machine is operating on a large spur gear 142 inches in diameter, 4 C P. This is a very difficult cutting operation as the gear is eccentric in 8 points and has a wobble of ¾ inch for equalizing purposes.

Courtesy of Foote Bros. Gear and Machine Corp , Chicago, Illinois

CHAPTER VIII

FRICTION DRIVES AND SPUR AND BEVEL GEARS

Cylindrical Friction Wheels. Two parallel shafts, when not too far apart, may be connected by a pair of right circular cylinders as shown in Fig. 112. From a theoretical standpoint, these cylinders will have pure rolling and will establish between themselves or their shafts a constant angular velocity ratio. Actually, however, since this is a friction drive, there will be some slipping between the contact surfaces of the cylinders, which will cause the angular velocity ratio to deviate a trifle from its theoretical value. When such a deviation is not permissible, a positive drive rather than a friction drive should be used.

Since cylindrical frictions theoretically satisfy the conditions of pure rolling (see text on Mechanism), the linear velocities of their points (or lines) of contact are equal. In Fig. 112, the point of contact is called P. (Note that it is the projection of the line of contact.) If this point is considered as a point of cylinder A, whose r.p.m. is N_a, its linear velocity is equal to $2\pi R_a N_a$. On the other hand, if P is taken as a point of B, its linear velocity is equal to $2\pi R_b N_b$. Since these linear velocities must be equal,

$$2\pi R_a N_a = 2\pi R_b N_b$$

$$R_a N_a = R_b N_b$$

Dividing both members of the equation by $R_a \times N_b$,

$$\frac{N_a}{N_b} = \frac{R_b}{R_a} \quad \text{(Compare with formula [36])} \qquad \textbf{(176)}$$

The above formula states that when two shafts are connected by a pair of rolling cylinders, their r.p.m.'s or angular velocities vary inversely as the radii of the cylinders.

It is evident from Fig. 112, that the sum of the radii of the cylinders is equal to the distance between centers; or

$$R_a + R_b = C \qquad \textbf{(177)}$$

Formulas (**176**) and (**177**) are a pair of simultaneous equations, by means of which the diameters or radii of the cylinders may be found

if the rotative speeds of the shafts and the distance between centers are known. This is demonstrated in the following example:

Example. It is required to connect two parallel shafts, A and B,

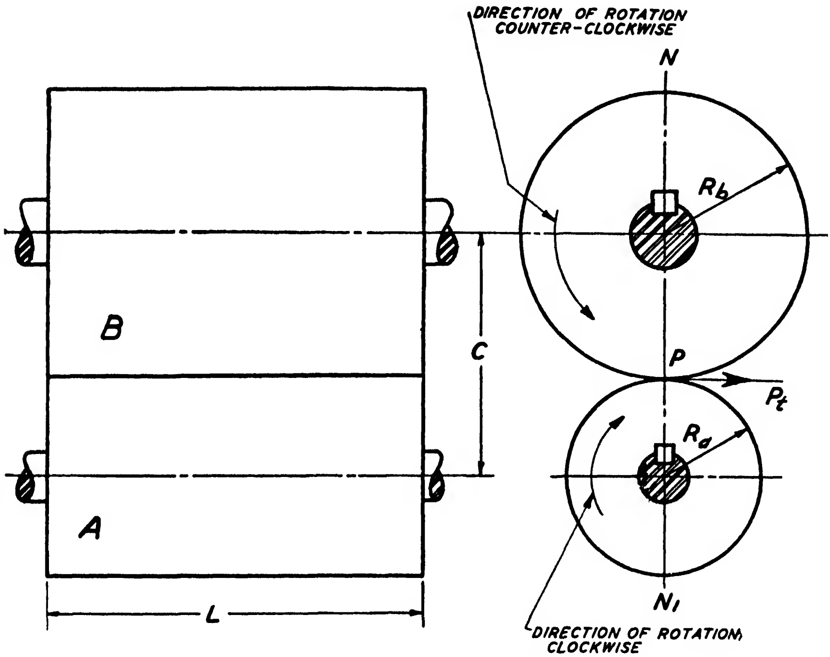


Fig 112. Cylindrical (or Spur) Friction Wheels

by a pair of rolling cylinders. What diameters must these cylinders have if it is given that

$$N_a = 300 \text{ r.p.m.}$$
$$N_b = 180 \text{ r.p.m.}$$

and
$$C = 18 \text{ in.}$$

Solution. Step 1. Evaluating in formula (**176**)

$$\frac{R_b}{R_a} = \frac{N_a}{N_b} = \frac{300}{180} = \frac{5}{3}$$

$$3R_b = 5R_a$$

or
$$R_b = \frac{5R_a}{3}$$

Step 2. Evaluating in formula (**177**)

$$R_a + R_b = 18$$

Step 3. Substituting in the latter equation, the value of R_b as obtained in Step 1.

$$R_a+\frac{5R_a}{3}=18$$

$$3R_a+5R_a=3\times18$$

$$8R_a=54$$

$$R_a=\frac{54}{8}=6.75 \text{ in.}$$

Therefore, $D_a=2R_a=2\times6.75=13.5$ in. *Ans.*

Step 4. Substituting the value of R_a in the result of Step 1,

$$R_b=\frac{5}{3}\times6.75=\frac{33.75}{3}=11.25 \text{ in.}$$

Therefore $D_b=2\times R_b=2\times11.25=22.5$ in. *Ans.*

Transmission of Power by Cylindrical Frictions. In Fig. 112, let us assume that A is the driver and B is the driven member of the mechanism. Now if A is to drive B, there must be set up along the common contact element a frictional force, P_t, which is shown by this figure to act tangentially with respect to the cylinders. From the fundamental torque formula (39), we have the following: $P_t=\frac{T_a}{R_a}$, in which T_a is the torque in shaft A. Since the magnitude of the turning moment of this frictional force with respect to the axis of the driven member must be sufficient to overcome the resistance to motion of the latter, it is evident from formula (29) that $T_b=P_tR_b$ in which T_b is the torque that is necessarily set up in shaft B if power is to be transmitted.

But P_t is here a frictional force and hence is dependent upon the materials of which the surfaces of contact are composed, and the total normal pressure between these surfaces; or

$$P_t=\mu P_n$$

from which $$P_n=\frac{P_t}{\mu} \quad \text{(See formula [133])}$$

The length, L, of the face of the contact surfaces will depend upon the amount of normal pressure that the material can safely withstand per inch of length. If P_1 represents this allowable pressure in pounds per inch of face, then

$$P_n=LP_1 \tag{178}$$

TABLE XXI

Material	μ	P_1 Allowable Pressure
Leather on Cast Iron	0 15 to 0 25	200 lb. per inch
Wood on Cast Iron	0.15 to 0 30	150 lb. per inch
Cork on Cast Iron	0 20	50 lb. per inch
Sulphite Fiber on Cast Iron	0.25	150 lb. per inch
Tarred Fiber on Cast Iron	0 25	250 lb. per inch
Cast Iron on Cast Iron	0 1 to 0 15	400 lb. per inch

The material of both contact surfaces of a friction drive may be cast iron but more often, in order to obtain a higher coefficient of friction, the surface of the driver is of such material as leather, wood, cork, sulphite fiber, etc., which is used in conjunction with a cast-iron surface for the driven member. The surface of the driven member should always be of the harder material so that little wear will occur during slippage. The values of the coefficient of friction, μ, and the allowable normal pressure, P_1, in lb. per in. of length of face, are given in Table XXI.

Example. Referring to the preceding example, let it be assumed that 10 horsepower are transmitted and that the materials used are wood on cast iron with $\mu = 0.20$ and $P_1 = 150$ lbs. per in. It is required to find the length, L, of the cylinder.

Solution. Step 1. To find the torque in shaft A. Here $H = 10$ hp, $N_a = 300$ r.p.m.

Applying formula (34),

$$T_a = \frac{12 \times 33{,}000H}{2\pi N_a}$$

and evaluating, $T_a = \dfrac{12 \times 33{,}000 \times 10}{2 \times \pi \times 300} = 2101$ in.-lb.

Step 2. To find the frictional resistance or tangential force, P_t. Here T_a is as found in Step 1 and R_a (from preceding example) = 6.75 in.

Applying formula (39) and evaluating therein,

$$P_t = \frac{T}{R}$$

$$= \frac{2101}{6.75} = 311.3 \text{ lb.}$$

Step 3. To find the total normal pressure, P_n. Here $\mu = 0.20$

Applying formula (133) $P_n = \frac{P_t}{\mu}$

$$= \frac{311.3}{0.20} = 1556 \text{ lb.}$$

Step 4. To find the length of face. Here $P_1 = 150$ lb. per in. and $P_n = 1556$ lb.

Applying formula (178),

$$P_n = LP_1$$

$$1556 = L \times 150$$

$$L = \frac{1556}{150} = 10.4 \text{ in., say } 10\tfrac{1}{2} \text{ in.} \quad \textit{Ans.}$$

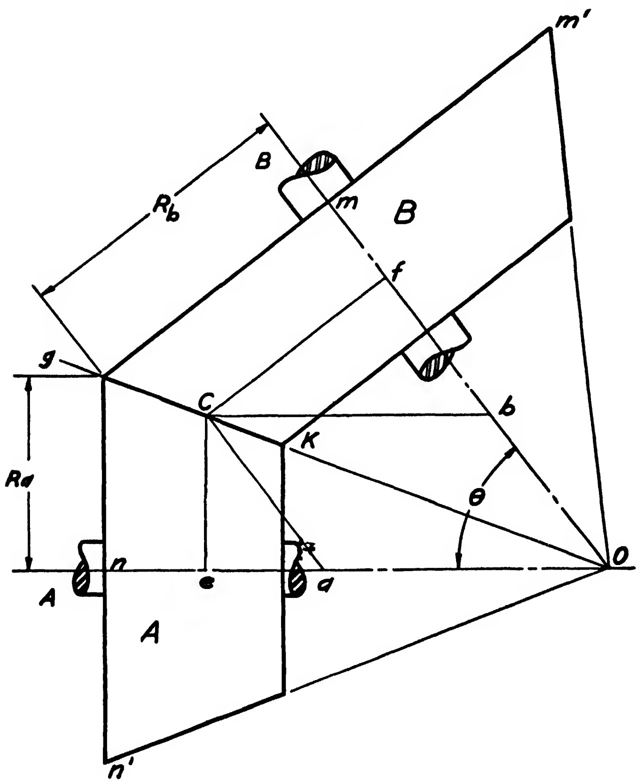


Fig. 113. Bevel Friction Wheels

Bevel Friction Wheels. Two shafts whose axes intersect at some angle, θ, as in Fig. 113, can be connected by a pair of right circular

cones or their frusta. Such rolling conical frusta which are always used in practice instead of the entire cones, are known as Bevel Friction Wheels. They can be designed for any angular velocity ratio and any angle, θ, between the shafts. While from a theoretical standpoint they maintain a constant angular velocity ratio together with pure rolling, their driving action is not positive, but depends wholly on the frictional resistance which is set up along their common contact element, kg. Hence, while transmitting power, some slippage is bound to occur which, as in rolling cylinders, will alter to some extent their angular velocity ratio.

Graphical Construction of Bevel Frictions. Let it be required to construct two rolling cones that may be used to connect a pair of intersecting shafts having given the angle of intersection, θ, and the r.p.m.'s of the shafts. Referring to Fig. 113, the axes of the shafts are laid out as shown intersecting at point, O. From the latter point, distances Oa and Ob are taken on the axes of A and B respectively, so that $\frac{Oa}{Ob}=\frac{N_a}{N_b}$. Lines ac and bc are next drawn parallel to the axes of the shafts so that a parallelogram, $Oacb$, is formed. The diagonal, OC, of this parallelogram is now drawn. This diagonal is the common contact element of the pair of rolling cones that are to be used under the conditions as given. To locate a pair of frusta of these cones, take any point, g, on the diagonal. Lay off any distance, gk, to represent the common contact element of the frusta. From both g and k drop perpendiculars to the axes A and B, extending these perpendiculars beyond the axes (as in the case of gm to m' so that $mm'=gm$) to form the completed frusta, A of shaft A and B of shaft B.

That these cones will give the desired r.p.m. ratio can be proved in the following manner. In the similar right triangles, ace, and bcf,

$\frac{ce}{cf}=\frac{ac}{bc}$, (corresponding sides of similar triangles are proportional)

But $\frac{ac}{bc}=\frac{Ob}{Oa}$, (opposite sides of a parallelogram are equal)

Since $\frac{Ob}{Oa}=\frac{N_b}{N_a}$, (by construction)

Therefore $\frac{ce}{cf}=\frac{N_b}{N_a}$, (things equal to the same thing are equal to each other)

The distances ce and cf are in reality corresponding radii of the

cones A and B, respectively, for they are drawn to the same point on the common contact element. Therefore it is proved that the radii of the cones are inversely proportional to the r.p.m.'s and hence will give the desired angular velocity ratio. The cones are given however by their outer or base radii. It may be noted therefore that since triangle Ogm is similar to triangle Ocf and triangle Ogn is similar to triangle Oce, we have $\frac{gn}{gm}=\frac{ce}{cf}=\frac{N_b}{N_a}$. But gn and gm are the radii of the bases of A and B respectively.

Therefore
$$\frac{R_a}{R_b}=\frac{N_b}{N_a} \tag{179}$$

Friction Drives Versus Positive Drives. Both of the preceding direct contact mechanisms, cylindrical (or spur) and bevel friction wheels, produce theoretically pure rolling and a constant angular velocity ratio between driver and follower. But the action between driver and follower in both mechanisms is dependent on friction, that is, neither mechanism affords positive driving between driver and follower and hence, as has been stated, some slippage is possible. This slippage would temporarily destroy the pure rolling and cause a deviation in the velocity ratio. To have positive driving, the common normal drawn through the point of contact, as line NN_1, Fig. 112, must not pass through the permanent center of driver or follower, as it is seen to do in this figure. (See also line NN_1, Fig. 117.) Hence the outlines of the contact surfaces of a positive drive direct contact mechanism, must be of such a shape that will keep the common normal from passing through these centers. Furthermore, if the contact surfaces are designed so as to cause the common normal through their point of contact to cut the line of centers at the same fixed point at every phase or instant, a constant angular velocity ratio will be afforded by the mechanism. This velocity ratio between driver and follower will be inversely as the segments into which the line of centers is cut by the common normal. Therefore a direct contact mechanism can be designed which will give positive driving accompanied by a constant velocity ratio from which no deviation is possible, since the action between driver and follower is no longer dependent on friction.

Positive drive direct contact mechanisms which will give a con-

stant velocity ratio are in reality obtained through the alteration of the surfaces of friction drives, so that interlocking members, called teeth, are provided, and the resulting links are called Toothed Wheels or Gears. The former surfaces of the friction members, although completely destroyed, are retained in a sense as the basis of the design and measurement of the resulting gears, and are known as the Pitch Surfaces of the gears. The transverse sections of these surfaces are called the Pitch Circles of the gears and in the design of the

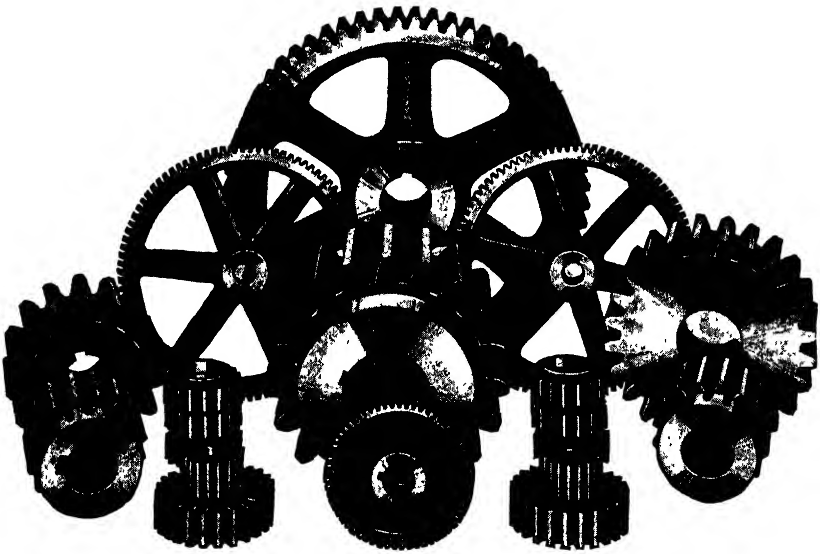


Fig. 114. Spur Gears
Courtesy of Foote Bros. Gear and Machine Corp., Chicago, Ill.

latter, these pitch circles are tangent to each other as they formerly were in friction drives. Toothed wheels, resulting from the alteration of cylindrical frictions (right circular) are known as Spur Gears, (See Fig. 114) while those resulting from the alteration of bevel frictions are known as Bevel Gears, (See Fig. 115) and the alteration is so effected that the gears retain the constant angular velocity ratio of their corresponding friction members. Their pitch surfaces will theoretically retain their pure rolling action, but the actual contact members which are now the interlocking teeth will have an action with each other which is a mixture of rolling and sliding.

Law of Gearing. It is evident from the preceding article that since the engaging teeth are to secure a constant velocity ratio for the gears, the outlines or profiles of these teeth must be so formed that the common normal to the profiles at the point of contact always intersects the line of centers at the same point. It is also evident that since this velocity ratio is to be the inverse ratio of the radii of the pitch circles of the gears, the segments of the line of centers that are cut by the common normal must be these pitch circle radii, or pitch radii as they are called. Therefore, the profiles of engaging

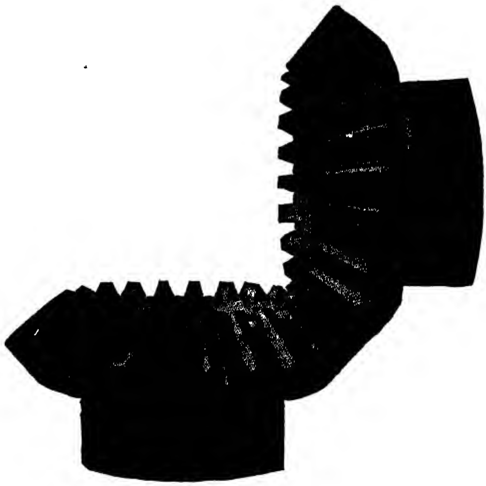


Fig 115. Bevel Gears
Courtesy of Foote Bros Gear and Machine Corp , Chicago, Ill.

gear teeth must be so formed that the common normal to them at their point of contact must always cut the line of centers of the gears at the point of tangency of their pitch circles, the so-called Pitch Point. This statement is known as the Law of Gearing.

Gear Tooth Profiles. There are a great number of curves which can be used for the outlines or profiles of gear teeth in that they will obey the law of gearing. However from a commercial standpoint, only two are employed. These are the Cycloidal and the Involute curves which give rise to the so-called Cycloidal and Involute Systems. Of these two systems, the involute has so many advantages that it is used almost exclusively in modern gear practice. Fig. 116 is an illustration of the teeth of an involute spur gear. Information relative

to the layout of these teeth is to be found in the text on Mechanism.

Definitions of Terms used with Gears. (See Fig. 116) The Pitch Circle is the most important circle of a gear. The size of a gear is dictated by the diameter of the pitch circle. Thus a 12-inch gear is one which has a pitch circle whose diameter is 12 inches. This diameter is referred to as the pitch diameter. The pitch circle is the

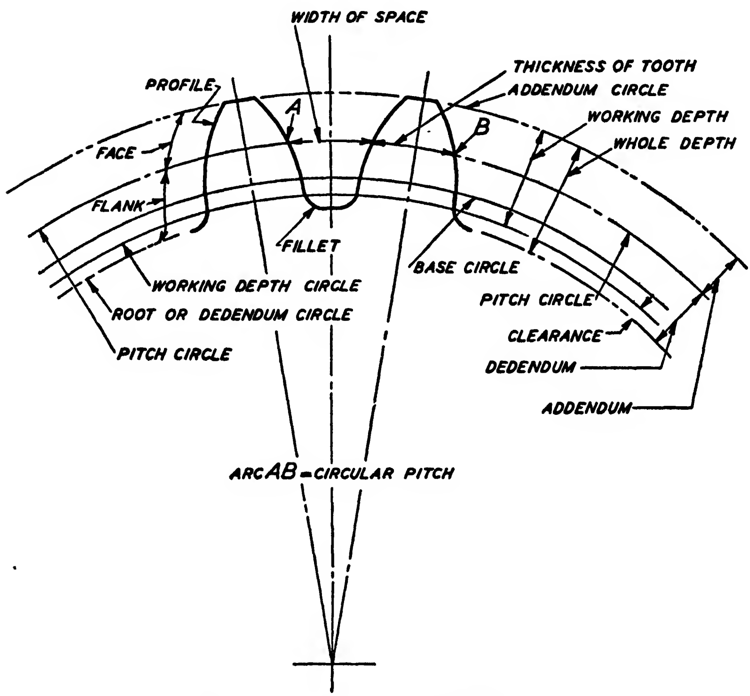


Fig. 116. Teeth of an Involute Spur Gear

circle on which and from which practically all the dimensions of the gear teeth are taken.

The Addendum Circle is the circle that bounds or limits the height of the teeth. It is the outermost circle of the gear and its diameter is often called the outside diameter.

The Root or Dedendum Circle is the circle from which the teeth protrude or extend. It bounds the bottom of the teeth and hence the spaces between the teeth.

The Base Circle is one that occurs only in the involute system of

gears. It is the circle from which the involute curve of the tooth profile is generated.

The Circular or Circumferential Pitch is the distance measured in inches from a point on one tooth to the corresponding point on the next or adjacent tooth, the distance being measured along and hence being an arc of the pitch circle. It is evident that any two gears that will mesh or work together must have the same circular pitch.

The Diametral (di-am′-e-tral) Pitch is the ratio of the number of teeth to the number of inches in the pitch diameter. In other words it is the number of teeth per inch of diameter of the gear. Thus a 2-pitch gear is one whose diametral pitch is 2, and therefore the gear has 2 teeth for every inch of its pitch diameter. Gears which work together must have the same diametral pitch.

The Thickness of Tooth is its thickness measured along the pitch circle.

The Width of Space is its width measured along the pitch circle. It is evident that the thickness of tooth plus the width of space is equal to the circular pitch.

Backlash is the difference between the width of space and the thickness of tooth. Theoretically, for well formed or cut teeth, the width of space is equal to the thickness of tooth and hence the backlash is equal to zero. But from a practical standpoint, the width of space is made slightly greater than the thickness of tooth to provide for irregularities in the form or spacing of the teeth, so that there will be no possibility of binding as the tooth of one gear enters the space of the other.

The Addendum is the radial distance from the pitch circle to the addendum circle.

The Dedendum or *Root* distance is the radial distance from the pitch circle to the dedendum or root circle.

The Whole Depth of a tooth is the distance from the addendum circle to the dedendum circle. It is evidently equal to the sum of the addendum and dedendum distances.

The Clearance is a radial distance equal to the difference obtained by subtracting the addendum from the dedendum. It is the distance between the top of the mating tooth and the bottom of the space of the other gear into which the tooth projects.

The Working Depth is equal to two times the addendum.

The Face of the Tooth is that part of the profile between the pitch and addendum circles.

The Face of the Gear is its width measured along a tooth of a straight-tooth gear.

The Flank of the Tooth is that part of the profile between the pitch and dedendum circles.

Relation between the Diametral and Circular Pitches.

Let P_d = diametral pitch,

P_c = circular pitch,

D_p = the pitch diameter,

and t = the number of teeth.

From the definition of diametral pitch, we obtain,

$$P_d=\frac{D_p}{t} \tag{180}$$

from which

$$t=P_dD_p \tag{181}$$

From the definition of circular pitch, we obtain

$$tP_c=\pi D_p \tag{182}$$

or

$$t=\frac{\pi D_p}{P_c} \tag{183}$$

Since these two values of t as given in formulas (181) and (183) are equal to each other,

$$P_dD_p=\frac{\pi D_p}{P_c}$$

Dividing by D_p,

$$P_d=\frac{\pi}{P_c} \tag{184}$$

from which,

$$P_c=\frac{\pi}{P_d} \tag{185}$$

Formulas (184) and (185) state that π divided by one pitch yields as its quotient the other pitch.

From formula (182), we obtain by dividing both members of the equation by π,

$$D_p=\frac{tP_c}{\pi} \tag{186}$$

This above formula states that the diameter of a pitch circle of a gear is equal to the product of the number of teeth and the circular pitch divided by 3.1416.

Example. Find the circular pitch of a 3-pitch gear.

Solution. Here $P_d=3$

From formula (185), we have

$$P_c=\frac{\pi}{P_d}=\frac{\pi}{3}=1.0472 \text{ in. } \quad \textit{Ans.}$$

Example. Find the diametral pitch of a gear whose circular pitch is $1\frac{1}{2}$ inches.

Solution. Here $P_c=1\frac{1}{2}$ in.

From formula (184) we have,

$$P_d=\frac{\pi}{P_c}=\frac{3.1416}{1.5}=2.0944 \quad \textit{Ans.}$$

Example. A 36-tooth spur gear has a circular pitch of $\frac{3}{4}$ inch. Find the pitch diameter of the gear.

Solution. Here $P_c=\frac{3}{4}$ in. and $t=36$ teeth

Applying formula (186)

$$D_p=\frac{tP_c}{\pi}$$

$$=\frac{36\times\frac{3}{4}}{\pi}=8.594 \text{ in. } \quad \textit{Ans.}$$

Example. How many teeth has a 2-pitch spur gear whose pitch diameter is 14 inches?

Solution. Here $P_d=2$ and $D_p=14$ in.

From formula (181) $\quad t=P_dD_p$

$$=2\times14=28 \text{ teeth. } \quad \textit{Ans.}$$

Example. A 3-pitch spur gear has 60 teeth. Find the pitch diameter of the gear.

Solution. Here $P_d=3$, and $t=60$

Evaluating in formula (181),

$$t=P_dD_p$$

we have $\quad 60=3D_p$

$$D_p=\frac{60}{3}=20 \text{ in. } \quad \textit{Ans.}$$

Pressure Angle, or Angle of Obliquity. Fig. 117 is an illustration of two involute spur gears in mesh with each other. With the direction of rotation as given in the figure, one pair of teeth is just about to make contact at point *a*, another pair is in contact at the pitch point, *P*, and still another pair is just leaving contact with each other

at point, b. It will be observed that their pitch circles are tangent to each other at the pitch point, P. The line NN_1 is drawn through the pitch point making an angle of $14\frac{1}{2}$ degrees with the common tangent to the pitch circles. The base circle of each gear is concentric with its pitch circle and is drawn tangent to the line NN_1. Hence the

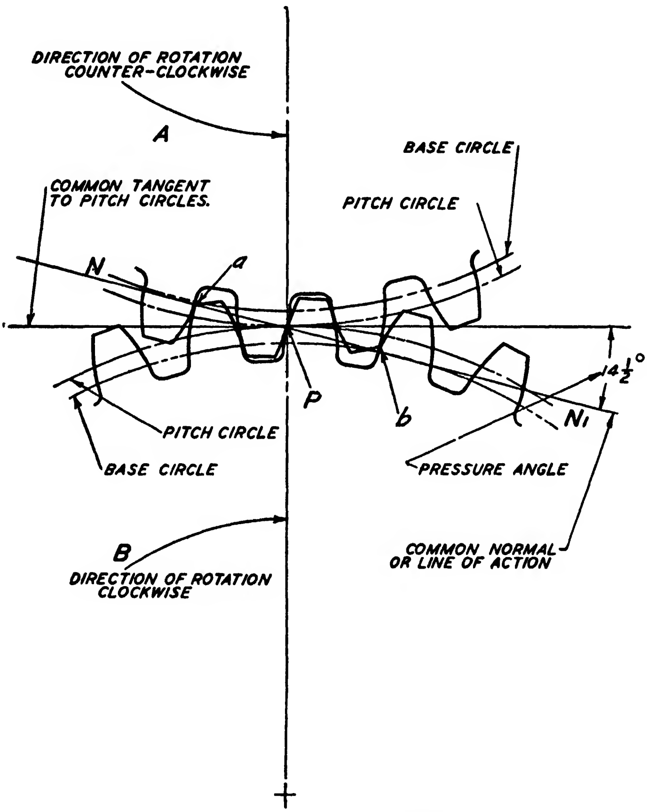


Fig. 117. Involute Spur Gears

base circles are determined by the line NN_1. Now when a line has pure rolling with a circle, any point of the line generates a curve which is called an involute of the circle and it is a characteristic of the involute curve, that its generating line is at all times normal to the involute. Therefore since NN_1 is in a position to roll with these base circles, any point of the line will generate simultaneously an involute of each circle as the circles rotating about their respective centers

roll with the line. The pair of involute curves thus generated are used for the profiles of the teeth of the meshing gears. Hence when the teeth are in contact, their point of contact is a position of the common generating point of their involute profiles. But the common generating point is a point of the line NN_1. Therefore as action between such a pair of teeth continues, their point of contact is always on the line NN_1. Since the latter is normal to the involute curves that are generated by it, the teeth have NN_1, as their common normal or line of action as long as they are in contact with each other. In this manner, gears with teeth so formed, called involute gears, obey the law of gearing; for NN_1, their line of action cuts the line of centers at the pitch point, P, at every phase. The angle which this line of action makes with the common tangent to the pitch circles is known as the Pressure Angle or the Angle of Obliquity. It is evident that in the involute system of gears, the pressure angle is constant. Standard involute gears generally use a pressure angle of $14\frac{1}{2}$ degrees, although an angle of 20 degrees is also used to quite an extent.

Cast Gear Teeth. Gear teeth may be either cast or cut. Cast tooth gears are made of cast iron, cast steel, cast steel alloys, or bronze. Manufacturers generally furnish gears of steel or steel alloys either with or without heat treatment. If one gear of a pair of engaging gears is made of bronze, sparking is prevented. Cast iron gears operating with cast steel pinions (small gears) tend to equalize resistance to wear. Cast teeth are not as reliable and free from noise as cut teeth and hence are used for low speeds. They are, as a rule, of the involute form, although occasionally gears with a low number of teeth are made cycloidal. Since the circular pitch is more convenient to the pattern maker, it is always used for cast gear teeth. It is generally made to vary by $\frac{1}{8}$-inch increments.

The tooth proportions or dimensions that are often followed for cast gear teeth are as follows:

Addendum $=0.32\ P_c$.
Dedendum $=0.39\ P_c$.
Working depth $=0.64\ P_c$.
Clearance $=0.07\ P_c$.
Whole depth $=0.71\ P_c$.
Thickness of tooth $=0.48\ P_c$.

Width of space $=0.52\ P_c$.

Backlash $=0.04\ P_c$.

Pressure Angle (for involute system) $=14\frac{1}{2}°$.

Radius of fillet $=\frac{1}{7}$ the distance between teeth measured on the addendum circle.

Example. Find the dimensions of the teeth of a cast spur gear whose circular pitch is $1\frac{1}{4}$ inches.

Solution.

Addendum	$=0.32\ P_c=0.32\times1.25=0.40$ in.
Dedendum	$=0.39\ P_c=0.39\times1.25\times0.4875$ in.
Working depth	$=0.64\ P_c=0.64\times1.25=0.80$ in.
Clearance	$=0.07\ P_c=0.07\times1.25=0.0875$ in.
Whole depth	$=0.71\ P_c=0.71\times1.25\times0.8875$ in.
Thickness of tooth	$=0.48\ P_c=0.48\times1.25=0.60$ in.
Width of space	$=0.52\ P_c=0.52\times1.25=0.65$ in.
Backlash	$=0.04\ P_c=0.04\times1.25=0.05$ in.

Example. The gear of the preceding example has 32 teeth. Find (*a*) the pitch diameter, (*b*) the addendum circle diameter, (*c*) the root circle diameter of the gear.

Solution. (*a*) Here $P_c=1\frac{1}{4}$ in. and $t=32$ teeth.

Applying formula (186)

$$D_p=\frac{tP_c}{\pi}$$

Evaluating in the above

$$D_p=\frac{32\times1\frac{1}{4}}{\pi}=\frac{40}{\pi}=12.732 \text{ in.} \quad \textit{Ans.}$$

(*b*) Since the diameter of the addendum circle is equal to the diameter of the pitch circle plus 2 times the addendum,

the addendum circle diameter $=12.732+2\times0.40=13.532$ in. *Ans.*

(*c*) Since the diameter of the root circle is equal to the pitch diameter minus 2 times the dedendum,

the root circle diameter $=12.732-2\times0.4875=11.757$ in. *Ans.*

Cut Gear Teeth. Cut gear teeth may be metallic or non-metallic. The metallic are made of cast iron, bronze, and carbon or alloy cast or forged steel with or without special heat treatment or case-hardening. The more care that is taken in the cutting of metallic gear teeth, the smoother and hence the less noisy they will be in operation. To

cut down the noise still further, so-called Silent Gears, are made from materials such as Micarta, Rawhide, etc. The base material of Micarta is a fabric woven to a definite weight and thickness. This fabric is carefully impregnated with a binder and is then subjected to heat and pressure. The result is a material which is claimed to produce silent, efficient, and economical gears. (See Fig. 118) Rawhide has long been used in manufacturing gears and pinions to work with

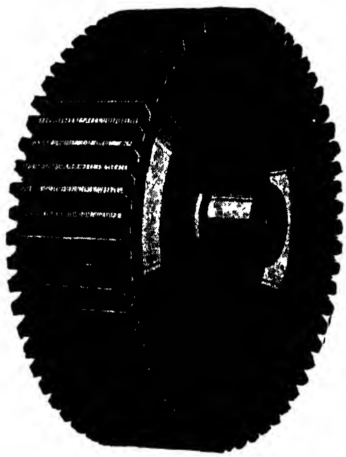


Fig. 118. Non-Metallic Cut Spur Gear; Silent Gear with Steel Center
Courtesy of Foote Bros. Gear and Machine Corp., Chicago, Ill

metallic gears where quiet operation is desired. Gears made of such a material as rawhide require metal flanges which reach to the top of the teeth and provide protection and support. (See Fig. 119)

There are several methods by which the cutting of gear teeth may be accomplished. Two of these methods are shown in Figs. 120 and 121. In Fig. 120, a spur gear is being cut by a form cutter while in Fig. 121, a spur gear is being cut or generated by a cutter-gear.

The diametral pitch is the accepted pitch for cut gears. It has been standardized to vary

by $\frac{1}{4}$ increments from 1-pitch to 4-pitch
by $\frac{1}{2}$ increments from 4-pitch to 6-pitch
by 1 increments from 6-pitch to 16-pitch
by 2 increments from 16-pitch to 32-pitch

Note that the larger the diametral pitch, the smaller the teeth.

Form cutters for each of the above pitches are obtainable for both the standard involute and the cycloidal systems of gears. To cut a complete set of involute gears of a given pitch requires only

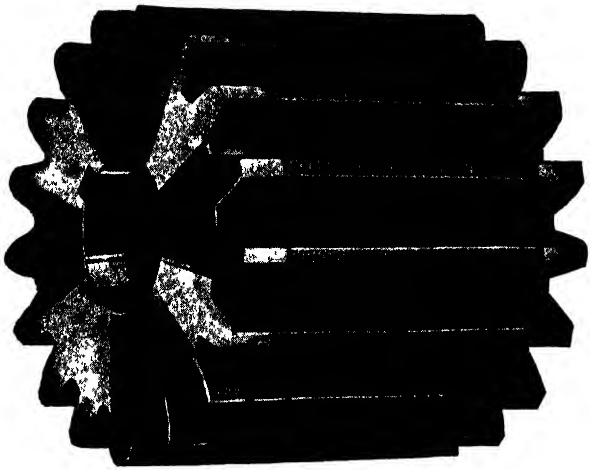


Fig. 119. Non-Metallic Cut Spur Gear. Note Brass Flanges and Laminations Used
Courtesy of Foote Bros. Gear and Machine Corp., Chicago, Ill

Fig. 120. Gear Cutting Machine Using Form Cutter
The use of a stocking cutter for roughing, combined with a finishing cutter for finishing, is clearly shown on this machine
Courtesy of Brown & Sharpe Mfg. Co., Providence, R. I.

eight cutters, while twenty-four cutters are required with the cycloidal system. This fact plus the other inherent advantages of the involute system makes the latter dominate the field.

Table XXII gives the standard Brown and Sharpe cutters.

The tooth proportions for $14\frac{1}{2}$-degree involute and cycloidal teeth are given as follows:

TABLE XXII

Cutter Number	Number of Teeth
1	135 teeth to a rack inclusive
2	55 teeth to 134 teeth inclusive
3	35 teeth to 54 teeth inclusive
4	26 teeth to 34 teeth inclusive
5	21 teeth to 25 teeth inclusive
6	17 teeth to 20 teeth inclusive
7	14 teeth to 16 teeth inclusive
8	12 teeth to 13 teeth inclusive

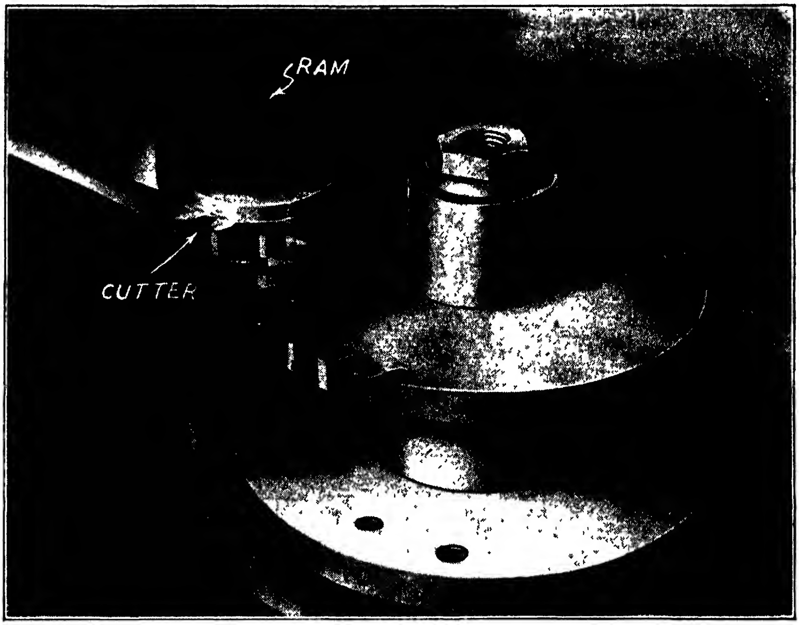


Fig. 121. Cutting or Generating the Teeth of a Spur Gear
Courtesy of The Fellows Gear Shaper Co., Springfield, Vermont

$$\text{Addendum} = \frac{1}{P_d}$$

$$\text{Dedendum} = \frac{1.157}{P_d}$$

$$\text{Working depth} = \frac{2}{P_d}$$

$$\text{Clearance} = \frac{0.157}{P_d}$$

$$\text{Whole depth} = \frac{2.157}{P_d}$$

$$\text{Thickness of tooth} = 0.5\ P_c$$

$$\text{Width of space} = 0.5\ P_c$$

Backlash = 0, theoretically,

(however, in practice a small amount of backlash equal to $\frac{0.04}{P_d}$ is actually used which of course changes slightly the values given above for the thickness of tooth and width of space).

Pressure angle (for involute system) = $14\frac{1}{2}$ degrees.

Radius of fillet = $\frac{1}{7}$ the distance between teeth measured on the addendum circle.

Example. The following data is given for a cut spur gear: diametral pitch = 2.5. number of teeth = 35.

It is required to find the pitch diameter, circular pitch, and the dimensions of the teeth of this gear.

Solution. Here $P_d = 2.5$ and $t = 35$ teeth.

Applying formula (181),

$$t = P_d D_p$$

$$35 = 2.5 \times D_p$$

$$D_p = \frac{35}{2.5} = 14 \text{ in.}$$

From formula (185)

$$P_c = \frac{\pi}{P_d}$$

$$= \frac{3.1416}{2.5} = 1.2566 \text{ in.}$$

$$\text{Addendum} = \frac{1}{2.5} = 0.4 \text{ in.}$$

$$\text{Dedendum} = \frac{1.157}{2.5} = 0.4628 \text{ in.}$$

$$\text{Working depth} = \frac{2}{2.5} = 0.80 \text{ in.}$$

$$\text{Clearance} = \frac{0.157}{2.5} = 0.0628 \text{ in.}$$

$$\text{Whole depth} = \frac{2.157}{2.5} = 0.8628 \text{ in.}$$

Thickness of tooth = $0.5P_c = 0.5 \times 1.2566 = 0.628$ in.
Width of space = $0.5P_c = 0.5 \times 1.2566 = 0.628$ in.
Backlash = 0, theoretically.

Interchangeable Sets of Spur Gears. An interchangeable set of spur gears is a group of gears of different numbers of teeth so designed that any two gears of the group will work properly together.

Since the angular velocity ratio between driver and follower with any type of gear is inversely as the number of teeth, a variety of velocity ratios is obtainable through the use of the various pairs of gears that may be selected from an interchangeable set of spur gears. All members of an interchangeable set of involute spur gears must have:

1. The same diametral pitch and hence the same circular pitch.
2. The same angle of obliquity.

All members of an interchangeable set of cycloidal spur gears must have:

1. The same diametral pitch and hence the same circular pitch.
2. The faces and flanks of their teeth generated by the same rolling or generating circle, the diameter of which is equal to the radius of the pitch circle of the smallest gear of the set.

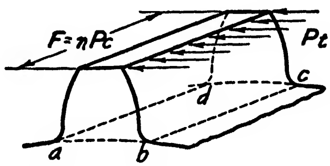


Fig 122 Action of Tooth as a Beam

Design of Spur Gear Teeth. In designing the teeth of a gear, it is assumed that a gear tooth acts as a cantilever beam which is subjected to an end load equal to the tooth pressure, P_t, as shown in Fig. 122. The dangerous section or supporting section of the tooth therefore becomes the root area, *abcd,* where the tooth joins the rim of the gear. This area must then be large enough to withstand safely the load, P_t. The root area, *abcd,* is a rectangle (in the case of a spur gear) whose thickness, *ab,* is the thickness of the tooth at the root circle and whose length, *bc,* is the face of the gear, F. The length F is taken as some number, n, times the circular pitch of the gear or

$$F=nP_c.$$

For cast spur gears at ordinary speeds,

$$n=2 \text{ to } 3.$$

For cut spur gears

$$n=3 \text{ to } 4.$$

Since the thickness, *ab,* depends upon the form of the tooth and upon the thickness of tooth on the pitch circle (and therefore on the

circular pitch, for the thickness of the tooth is given in terms of the circular pitch) and the length of tooth, bc, depends upon the circular pitch, it is evident that the creation of a safe root area depends upon the proper selection of a circular (or diametral) pitch that is large enough to stand up under the forces involved in the design.

An equation or formula which will provide such a pitch has been derived by Mr. Wilfred Lewis. It is universally accepted and is known as the Lewis Equation. It is given as follows:

$$P_t = SFP_c y \tag{187}$$

in which P_t = tooth pressure in pounds
S = stress in pounds per square inch
F = length of tooth in inches
P_c = circular pitch in inches
y = a constant which is based on the shape of the profile of the tooth and depends on the number of teeth of the gear

The value of the form factor, y, for either a standard $14\frac{1}{2}$-degree involute or cycloidal tooth profile may be obtained from Table XXIII. These values as given in the table have been computed from the formula:

$$y = \left(0.124 - \frac{0.684}{t}\right)$$

in which as before t = the number of teeth.

Since $$P_t = \frac{T}{R_p} \qquad \text{(See formula (39))} \qquad (a)$$

in which R_p = the radius of the pitch circle

and $$t\,P_c = \pi D_p = 2\pi R_p \qquad \text{(See formula (182))} \qquad (b)$$

from which by dividing both members of the equation by 2π,

$$R_p = \frac{t\,P_c}{2\pi} \qquad (c)$$

we have by substituting this value of R_p in the step (a)

$$P_t = \frac{T}{\frac{t\,P_c}{2\pi}} = \frac{2\pi T}{t\,P_c} \qquad (d)$$

But $$F = nP_c \qquad (e)$$

Substituting the values of P_t and F of steps (d) and (e) respectively in equation (187), we have

$$\frac{2\pi T}{tP_c} = S\,n\,P_c^2 y \tag{f}$$

Multiplying by tP_c,

$$2\pi T = S\,n\,P_c^3 t y \tag{g}$$

Dividing by $S\,n\,t\,y$

$$P_c^3 = \frac{2\pi T}{S\,n\,t\,y} \tag{h}$$

Therefore $$P_c = \sqrt[3]{\frac{2\pi T}{S\,n\,t\,y}} = \sqrt[3]{2\pi} \times \sqrt[3]{\frac{T}{S\,n\,t\,y}}$$

or $$P_c = 1.84\sqrt[3]{\frac{T}{S\,n\,t\,y}} \tag{188}$$

A formula for the diametral pitch, P_d, can be obtained from step (g) above by substituting therein for P_c, its equal, $\frac{\pi}{P_d}$

Thus we obtain $$2\pi T = Sn\left(\frac{\pi}{P_d}\right)^3 t y$$

or $$2\pi T = Sn\frac{\pi^3}{P_d^3}\, t y$$

Multiplying by P_d^3 and dividing by $2\pi T$

$$P_d^3 = \frac{S\,n\,\pi^2 t\,y}{2T} = \frac{\pi^2}{2} \times \frac{S\,n\,t\,y}{T}$$

Therefore $$P_d = \sqrt[3]{\frac{\pi^2}{2}} \times \sqrt[3]{\frac{S\,n\,t\,y}{T}}$$

$$P_d = 1.7\sqrt[3]{\frac{S\,n\,t\,y}{T}} \tag{189}$$

In both formulas (188) and (189), T is the torque in inch-pounds and the other notation is as previously defined. These formulas are convenient forms of the Lewis Equation to use in finding the required pitch when the torque, T, and number of teeth, t, are known. The value of the stress, S, to be used depends on the material and the linear velocity in feet per minute at the pitch circle. Values of S for the different materials and velocities are given in Table XXIV.

As previously stated, two gears which work with each other must have the same circular and therefore the same diametral pitches. If the gears are of the same material, the smaller gear will tend to

TABLE XXIII—Values of Form Factor, y

No. of Teeth	y	No. of Teeth	y	No. of Teeth	y	No. of Teeth	y
12	.067	22	.093	45	.1080	120	.1180
13	.071	23	.094	50	.1100	140	.1190
14	.075	24	.096	55	.1120	160	.1197
15	.078	26	.098	60	.1130	180	.1202
16	.081	28	.100	65	.1140	200	.1206
17	.084	30	.101	70	.1144	250	.1213
18	.086	33	.103	75	.1150	Rack	.1240
19	.088	36	.105	80	.1155		
20	.090	39	.107	90	.1164		
21	.092	40	.107	100	1170		

TABLE XXIV—Values of the Safe Stress, S, to be used in the Lewis Equation

Velocity in f.p.m.	Safe Stress, S, in Pounds per Square Inch				
	Cast Iron	Semi-Steel	Cast Steel	Machine Steel	Chrome Nickel
100	6900	12,800	17,100	21,400	85,700
200	6000	11,200	15,000	18,800	75,000
300	5300	10,000	13,400	16,700	67,000
400	4800	9,000	12,000	15,000	60,000
600	4000	7,500	10,000	12,500	50,000
900	3200	6,000	8,000	10,000	40,000
1200	2700	5,000	6,700	8,300	33,000
1500	2300	4,300	5,700	7,100	28,600
1800	2000	3,750	5,000	6,300	25,000
2100	1800	3,300	4,400	5,500	22,200

be weaker than the larger. Therefore in designing the gears, the pitch that the smaller gear requires should be determined and used for the pair. If the circular pitch is to be used, the theoretical result obtained by using formula (188) should be raised in value to the next $\frac{1}{8}$-in. increment as a standard value, thus increasing the size of the teeth and making them somewhat safer. If the diametral pitch is to be used, the standard that is just lower than the theoretical result of formula (189) should be adopted; for teeth increase in size as the diametral pitch is made smaller. In some designs, different materials are used for a pair of gears which are to work together. In such a case, should the material of the smaller gear have the greater strength, the pitch for each should be determined and that pitch adopted for the pair which will create the larger teeth.

Example. A cut semi-steel (high grade cast iron) spur gear has $14\frac{1}{2}$-degree involute teeth, a diametral pitch of 4, a pitch diameter of 8 inches, and a length of tooth of $2\frac{1}{2}$ inches. What horsepower can it transmit safely at 300 revolutions per minute?

Solution. Step 1. To find the circular pitch

Since $$P_d = 4$$

from formula (**185**), $$P_c = \frac{\pi}{P_d}$$

$$= \frac{\pi}{4} = 0.7854 \text{ in.}$$

Step 2. To find the velocity, V_L, at the pitch line in feet per minute, and the safe stress, S, in lb. per sq. in. Here $D = 8$ in. $= \frac{2}{3}$ ft. and $N = 300$ r.p.m.

Since $V_L = \pi DN$, we have upon evaluating,

$$V_L = \pi \times \tfrac{2}{3} \times 300 = 628.3 \text{ f.p.m.}$$

It will be noticed that this velocity lies between 600 f.p.m. and 900 f.p.m. in Table XXIV, and that for semi-steel there is a difference in stress of 1500 lb. per sq. in. for this difference in velocity of 300 f.p.m. Therefore for $(628.3 - 600)$ f.p.m., there will be a difference of stress, $\frac{28.3}{300} \times 1500 = 141.5$ lb. per sq. in. Since the stress decreases as the velocity increases, the interpolation has given

$$S = 7500 - 141.5 = 7358.5 \text{ lb. per sq. in.}$$

Step 3. To find the number of teeth, t, and the form factor, y. Here $P_d = 4$ and $D_p = 8$ in.

Applying formula (**181**)

$$t = P_d D_p$$

$$= 4 \times 8 = 32 \text{ teeth}$$

From Table XXIII, by interpolation,

$$y = 0.101 + \tfrac{2}{3}(0.103 - 0.101) = 0.1023$$

Step 4. To find the tooth pressure, P_t. Here $S = 7358.5$ lb. per sq. in., $F = 2.5$ in., $P_c = 0.7854$ in., $y = 0.1023$

Evaluating in formula (**187**),

$$P_t = SFP_c y$$

$$= 7358.5 \times 2.5 \times 0.7854 \times 0.1023$$

$$= 1478.1 \text{ lb.}$$

Step 5. To find the horsepower, H.

From formula (**35**),

$$H = \frac{P_t V_L}{33{,}000}$$

$$= \frac{1478.1 \times 628.3}{33{,}000} = 28.1 \quad \textit{Ans.}$$

Example. A pair of cast-iron involute spur gears is to be designed

to transmit 15 horsepower. The smaller gear (the pinion) is to have 24 teeth and a rotative speed of 200 r.p.m. It is required to obtain the circular pitch of this pair of gears assuming that the length of tooth is equal to three times the circular pitch.

Solution. The pitch to be used will be the pitch of the pinion. From the statement of the example, we have $H=15$ hp., $t=24$ teeth, $N=200$ r.p.m., $n=3$, and for a gear of 24 teeth, $y=0.096$ (from Table XXIII)

Since formula (188) applies in this case, it will be necessary to obtain the torque, T, in in.-lb., and the stress, S, in lb. per sq. in. The former can be obtained as follows by evaluating in formula (34):

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 15}{2\times\pi\times 200}=4729 \text{ in.-lb.}$$

The stress to be used in formula (188) cannot be determined at the beginning of the solution for it depends upon the pitch line velocity, which cannot be determined. Since the data for evaluation in the second member of formula (188) is known with the exception of the stress, a trial value of the latter will enable us to make a tentative solution of the formula, from which an approximate value of P_c will be obtained. This approximate value will yield in turn a fair idea of the size of the pinion we are to use, and hence allow us to obtain an approximate value of the speed at the pitch line of the pinion and its accompanying stress. Hence let us use as a trial value, $S=4000$ lb. per sq. in., which is an average value for cast iron as may be noted from Table XXIV.

Evaluating in formula (188).

$$P_c=1.84\sqrt[3]{\frac{T}{Snty}}$$

$$=1.84\sqrt[3]{\frac{4729}{4000\times 3\times 24\times 0.096}}$$

$$=1.84\sqrt[3]{\frac{4729}{27{,}648}}=1.84\times\sqrt[3]{0.171}$$

To solve the above by the aid of logarithms.

$$\log 0.171=9.2330-10$$

$$\log \sqrt[3]{0.171} = \tfrac{1}{3} \text{ of } \log 0.171$$
$$= \tfrac{1}{3} \text{ of } (9.2330-10)$$
$$= \tfrac{1}{3} \text{ of } (29.2330-30)$$
$$= 9.7443-10$$
$$\log 1.84 = 0.2648$$

$$\log (1.84 \times \sqrt[3]{0.171}) = \log 1.84 + \log \sqrt[3]{0.171}$$

For the logarithm of a product is equal to the sum of the logarithms of the individual factors of that product.

$$\text{Hence } \log = (1.84 \times \sqrt[3]{0.171})$$
$$= 0.2648 + (9.7443-10)$$
$$= 0.0091$$

Therefore $\qquad P_c = \log^{-1} 0.0091 = 1.02$ in.

We shall now obtain the stress that goes with the pitch line velocity of a pinion which has 24 teeth and the above pitch. Since the linear velocity of a point in rotation is equal to the circumference of the path of the point multiplied by its r.p.m. we have

$$V_L = tP_cN \text{ inches per minute}$$

for $\qquad t \times P_c =$ the circumference of the pitch circle

Therefore $\qquad V_L = \dfrac{tP_cN}{12}$ feet per minute $\qquad$ **(190)**

Evaluating in formula (190)

$$V_L = \frac{24 \times 1.02 \times 200}{12} = 408 \text{ f.p.m.}$$

From Table XXIV, the stress that corresponds to 408 f.p.m. is practically 4800 lb. per sq. in. Recalculating in the Lewis equation on the basis of this stress,

$$P_c = 1.84 \sqrt[3]{\frac{4729}{4800 \times 3 \times 24 \times 0.096}}$$
$$= 1.84 \sqrt[3]{0.143} = 1.84 \times 0.52$$
$$= 0.96 \text{ in., say 1 in.} \quad \textit{Ans.}$$

It will be seen that the pitch line velocity for a 24-tooth gear having a 1-inch pitch yields the same (or nearly the same) stress as has been used in the previous solution of the pitch. Therefore this is the final solution and the pitch of the pair of gears in this case is 1 inch.

If the stress obtained after the second solution had not been within about 100 pounds of the stress used therein, the new stress would have been used in another calculation and the resulting pitch checked as before.

Example. A cut involute semi-steel spur gear has 20 teeth whose face length is equal to 3.25 times the circular pitch. The gear transmits 30 horsepower at 720 revolutions per minute. It is required to obtain the diametral pitch of this gear.

Solution. In this example, we have given: $t=20$ teeth, $n=3.25$, $H=30$ hp., $N=720$ r.p.m. For a gear of 20 teeth, $y=0.090$ (from Table XXIII) and the torque can be obtained by using formula (34),

$$T=\frac{12\times 33{,}000H}{2\pi N}$$

$$=\frac{12\times 33{,}000\times 30}{2\times\pi\times 720}=2627 \text{ in.-lb.}$$

Since the velocity at the pitch line of this gear cannot as yet be obtained, a trial value of the safe stress, S, must be assumed. This assumption will permit us to obtain a tentative value of the diametral pitch, P_d, on the basis of which a more accurate value of S can be obtained. From Table XXIV we shall assume as an average value of S for semi-steel, 7500 lb. per sq. in.

Evaluating in formula (189),

$$P_d=1.7\sqrt[3]{\frac{Snty}{T}}$$

$$=1.7\sqrt[3]{\frac{7500\times 3.25\times 20\times 0.090}{2627}}$$

$$=1.7\sqrt[3]{16.7}=1.7\times 2.54=4.3, \text{ say } 4.$$

If $\qquad P_d=4,$

$$D_p=\frac{t}{P_d}=\frac{20}{4}=5 \text{ in.}$$

Therefore the linear velocity at the pitch line will be

$$V_L=\pi D_p N$$

$$=\pi\times\tfrac{5}{12}\times 720=942.5 \text{ f.p.m.}$$

For this linear velocity, Table XXIV yields a stress for semi-steel equal to approximately 5800 lb. per sq. in.

Recalculating for P_d on the basis of this new stress,

$$P_d = 1.7\sqrt[3]{\frac{5800\times3.25\times20\times0.090}{2627}}$$

$$=1.7\sqrt[3]{13}=1.7\times2.35=4-,\text{ say }4.$$

It is evident that this value of P_d is based on the stress which corresponds to its gear's pitch line velocity. Hence no more calculations for P_d need be made and therefore

$$P_d=4. \quad \textit{Ans.}$$

Example. Find the circular pitch and face of the gear of the preceding example.

Solution. Applying formula (**185**)

$$P_c=\frac{\pi}{P_d}=\frac{\pi}{4}=0.7854\text{ in.} \quad \textit{Ans.}$$

Since the face, F, is equal to n times the circular pitch

$$F=3.25\times0.7854=2.55\text{ in., say }2\tfrac{9}{16}\text{ in.} \quad \textit{Ans.}$$

Proportions for Spur Gear Parts. (See Fig. 123) The thickness of the rim of a spur gear may be taken in general equal to the thickness of the teeth on the root circle. A more definite statement which will produce about the same result is to make the rim thickness equal to 0.65 P_c. This will provide a rim which will stand up safely under the loads to which it is subjected and at the same time will be stiff enough to be practicable from the standpoint of casting and machining. The rim is often provided with a centrally located bead whose radial thickness is made equal to 0.5 P_c. It is evident that the length of the rim taken parallel to the axis of the gear is the same as the face of the gear tooth.

The rims of large gears are connected to their hubs by means of spokes or arms. The number of arms used depends upon the size (or pitch diameter) of the gear. Gears up to 20 inches in diameter carry four arms. As the diameter increases above 20 inches, the number of arms is made progressively larger until for diameters of 80 inches or more, ten arms are used. When gears are too small to carry arms, the gears are either made solid or a web connection is used. Round cored holes are often employed in the web construction. These lessen the gear weight and produce an appearance similar to the use of arms.

The arms of commercial gears are usually made elliptical in

section although for very large gears subjected to heavy loads, other sections may be used. The elliptical section creates a good appearance and is moulded easily. The standard elliptical section as in pulley arms has its minor axis equal to one-half its major axis. The design of gear arms is the same as that of pulley arms, (see chapter dealing with pulleys) the arms being considered as cantilever beams with a length equal to the radius of the gear. The end load on each

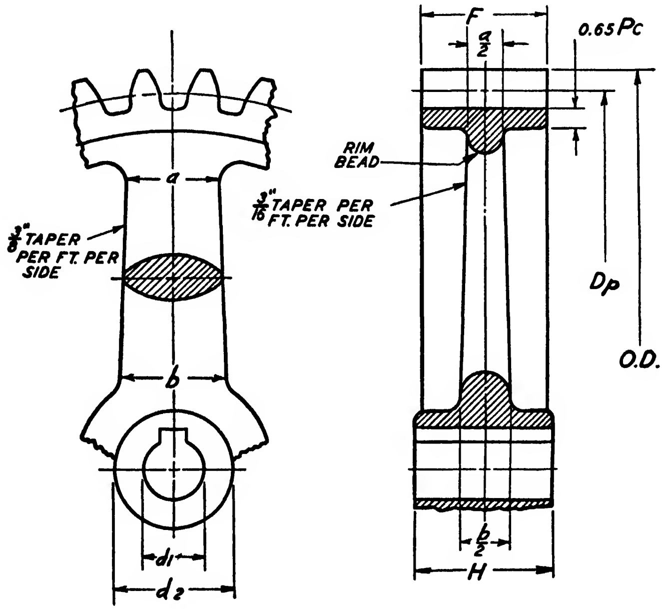


Fig. 123. Spur Gear

arm is generally taken as the tooth pressure divided by one-half the number of arms; some authorities however assume, due to the stiffness of the rim, that each arm carries its proportionate part of the tooth pressure in which case the end load per arm is equal to the tooth pressure divided by the number of arms. The arms are tapered as shown in Fig. 123.

The length of the hub, H, is taken equal to $1\frac{1}{4}$ times the diameter of bore, d_1, unless this gives a hub whose length is less than the face of the gear, F. In such a case the length of the hub should be made equal to the face. So it can be stated that the minimum length of the hub is equal to the face of the gear. Hub

lengths as here provided will be long enough to prevent the gear from rocking on the shaft. The diameter of the hub, d_2, is equal to $1\frac{1}{2}$ to 2 times the diameter of the bore, d_1.

Bevel Gears. Toothed wheels which are used to connect two shafts whose axes intersect are called Bevel Gears. (See Fig. 115) The latter provide positive driving and a constant angular velocity ratio between the shafts. The angular velocity ratio between driver

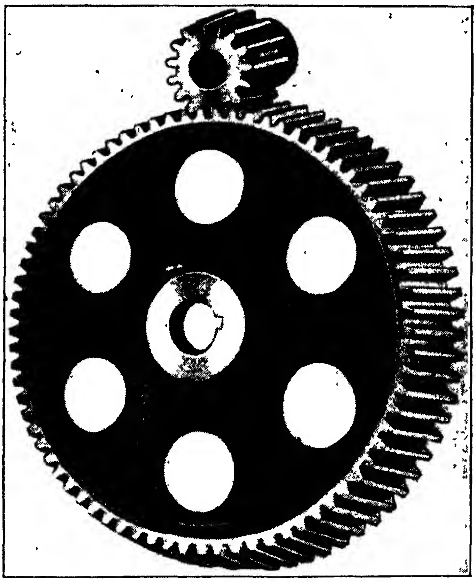


Fig. 124. Angular Gear and Pinion
Courtesy of Foote Bros. Gear and Machine Corp., Chicago, Ill.

and driven bevel gears is inversely as the numbers of teeth or inversely as the pitch circle diameters, as is the case with spur gears. As previously stated in this chapter, bevel gears result theoretically from the alteration of the surfaces of rolling cones. These conical surfaces then become the pitch surfaces of the bevel gears. Hence the teeth of bevel gears follow along a conical surface which causes the teeth to get progressively smaller from one end to the other. All elements of these teeth if extended would pass through the point of intersection of the axes of the gears, at which point the apex of each pitch surface is located. It is due to this fact that bevel gears are made in pairs only, and not in interchangeable sets. The size of a

bevel gear is given by the pitch diameter of its outer or larger pitch circle.

It has become customary to call a pair of bevel gears by the name of Miter gears when their shafts make an angle of 90 degrees with each other and have the same rotative speed. It is evident from this statement that a pair of miter gears must be of the same size and have the same number of teeth. Bevel gears whose axes intersect at any other angle than 90 degrees are known as Angular gears. (See Fig. 124.)

Bevel Gear Terms. Most terms used with bevel gears are the same as those used with spur gears and have the same interpretation in either case. The following definitions are given to clarify some terms previously used and to introduce others which are peculiar to bevel gears. Reference should be made to Fig. 125.

The addendum, pitch and root circles of a bevel gear are those located at the outer end of the bevel gear. Hence when the diameter of one of these circles is given, it is understood to be so located. Notice the addendum circle diameter, *O.D.*, and the pitch circle diameter, D_p, of Fig. 125.

The cone distance is the slant height of the pitch cone of a bevel gear.

The mounting distance is the distance from the center line of one gear to the end of the hub of the mating gear.

The backing, X, is the distance from the (outer) pitch circle of a gear to the end of its own hub.

The back cone of a bevel gear is a cone whose elements are perpendicular to the elements of the pitch cone at the outer end of the gear. Therefore both the back cone and pitch cone have the pitch circle of the gear for a base.

The face, F, of the gear is the length of its tooth measured parallel to the pitch element.

The pitch, either the diametral or circular, of a bevel gear always refers to that of the large end of the tooth or gear.

The apex, O, of the pitch cone is called the cone center. It is the intersection of the axes of the mating gears.

The angle between the addendum element of a tooth and the axis of the gear is called the face angle. The face angle of the pinion in Fig. 125 is the angle, *aoe*.

The angle between the pitch element of a tooth and the axis of the gear is called the pitch angle. The pitch angle of the pinion in Fig. 125 is the angle, *boe.*

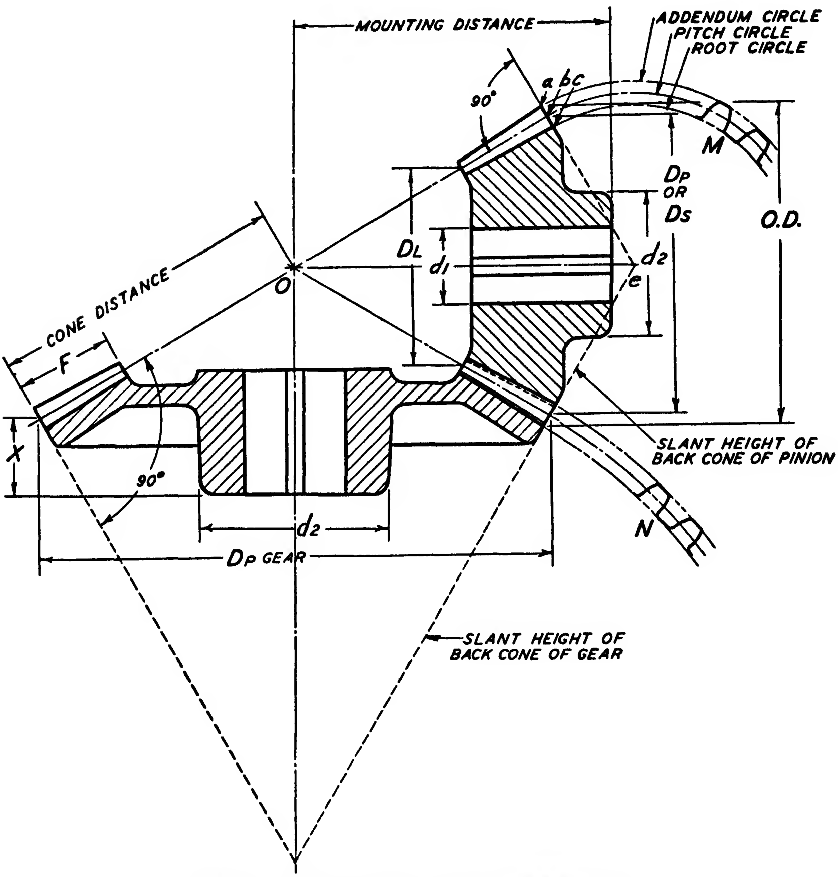


Fig. 125 Sectional View of Bevel Gear and Pinion

The angle between the root element of a tooth and the axis of the gear is called the root angle or cutting angle. The root angle of the pinion in Fig. 125 is the angle, *coe.*

Form of Bevel Gear Teeth. The true form of the large end of a bevel gear tooth is obtained by viewing it along the pitch element, *bo*, in which case it appears as given at *M* (or *N*) in Fig. 125. The outer end of a bevel gear tooth theoretically lies in a spherical surface

whose radius is equal to the slant height of its pitch cone, or the cone distance. Since a spherical surface is difficult to develop, the large end of the tooth is assumed to lie in a conical surface, called the back cone, and hence the large end has the same form as that of a spur gear tooth whose pitch radius is equal to the slant height of the back cone. It will be noticed then that the teeth at M in Fig. 125 are constructed in the same manner as spur gear teeth having the distance eb as a pitch radius, a pitch equal to the pitch of the bevel gear, and a number of teeth that a spur gear of such a pitch radius and pitch necessarily would have. This number of teeth on the imaginary or equivalent spur gear is evidently larger than the number of teeth on the corresponding bevel gear because the pitch radius of the spur is always greater than the pitch radius of the bevel. (The hypotenuse of a right triangle is longer than either leg.) The number of teeth on the imaginary spur gear is often referred to as the Formative Number of Teeth of the bevel gear.

Design of Bevel Gear Teeth. The strength of a bevel gear tooth is calculated in a manner similar to that of a spur gear tooth by using a modified form of the Lewis Equation, formula (187). This modification is made necessary by the fact that the supporting section (the section at the rim) of a bevel gear tooth is trapezoidal rather than rectangular (as in a spur gear) and that the pitch and pitch diameter of a bevel gear are taken at the large end. The modified form of the Lewis Equation is as follows:

$$P_t = SFP_c y \frac{D_L}{D_s} \tag{191}$$

in which

P_t = tangential force or tooth pressure in pounds at the pitch circle of the large end

S = stress in pounds per square inch

F = face of tooth in inches

P_c = circular pitch at large end in inches

y = form factor

D_s = pitch diameter at large end in inches

D_L = pitch diameter at small end in inches

The ratio, $\frac{D_L}{D_s}$, should not be less than $\frac{2}{3}$.

Formula (187) and those derived from it, formulas (188) and (189),

may be used in the case of bevel gears and will introduce little error if the pitch therein is taken as the mean pitch, that is, the pitch midway between the large and small ends. Having obtained this mean pitch, the pitch at the large end, which is the pitch of the gear, must then be calculated on the basis of the number of teeth and the diameter of the large end.

Proportion of Bevel Gear Parts. The length of tooth, or face, of a bevel gear should not be greater than $\frac{1}{3}$ the cone distance.

The arms of a bevel gear, transmitting as they do the load from the tooth to the shaft, may be designed in the same way as the arms of spur gears and pulleys. It is customary to assume that the load is carried by one-half of the number of arms. In addition to the section of the arm thus created in its action as a cantilever beam, it is well to attach a rib thereto to take care of the side thrust that is present in all bevel gears. Bevel gears are also made solid like unto the pinion of Fig. 125 or use a web connection between rim and hub as shown in the gear of the same figure. When a web is used, it should be backed up by ribs extending from the rim to the hub. The web may be lightened by the use of cored holes.

An important dimension of bevel gears is the backing, X. (See Fig. 125.) This dimension can be obtained from the following formulas, which have been taken from Catalog No. 204-13 of the Foote Bros. Gear and Machine Corporation of Chicago.

Backing for bevel gears and pinions:

$$\text{Backing of pinion in inches} = \frac{\frac{1}{4} \textit{ of pitch diameter of gear}}{(\textit{ratio of gear diam. to pinion diam.}) + 1}$$

$$\text{Backing of gear in inches} = \frac{\textit{pitch diam. of gear}}{4} - \text{backing of pinion.}$$

For miter gears:

$$\text{Backing of gear in inches} = \frac{\textit{pitch diameter}}{8}$$

The hubs of bevel gears have to be fully as long as those of spur gears, because there is a greater tendency to rock on the shaft due to the side thrust from the teeth. The hub of course must be long enough to permit of the backing dimension as obtained from the preceding formulas.

Any dimensions of parts not specified here can be taken the same as for spur gears.

Especial attention must be paid to the rigidity of the supporting shafts and bearings of bevel gears. Bearings should always be close to the hubs of the gears. Provision should be made so that the gears can be properly located and adjusted and the gear teeth properly lubricated.

PROBLEMS

1. Two parallel shafts are connected by a pair of cylindrical friction wheels, A and B. It is required to find the r.p.m. of B having given the following data: The rotative speed of A, N_a, $=450$ r.p.m. The diameter of A, D_a, $=8$ inches. The diameter of B, D_b, $=15$ inches. *Ans.* 240 r.p.m.

2. The materials of the surfaces of the cylinders used in the preceding problem are tarred fiber on cast iron, and hence from Table XXI, μ, the coefficient of friction is equal to 0.25 and P_1, the allowable pressure, is equal to 250 pounds per inch of length of the line of contact of the cylinders. If 10 horsepower is transmitted by these friction cylinders, find (*a*) the torque, T_a, in the shaft of cylinder A, (*b*) the torque, T_b, in the shaft of cylinder B, (*c*) the frictional resistance, P_t, (*d*) the length, L, of the cylinders. *Ans.* (*a*) 1400 in.-lb. (*b*) 2626 in.-lb. (*c*) 350 lb. (*d*) $5\frac{5}{8}$ in. nearly.

3. Two parallel shafts are at a distance of 12 inches between centers. The rotative speed of the driving shaft is 300 revolutions per minute while that of the driven shaft is 100 revolutions per minute. Find the diameters of the pair of rolling cylinders that will connect these shafts and secure the desired velocity ratio.

Ans. $\begin{cases} \text{Diameter of cylinder on driving shaft} = 6 \text{ in.} \\ \text{Diameter of cylinder on driven shaft} = 18 \text{ in.} \end{cases}$

4. Two cylinders, A and B, are keyed to shafts which are 14 inches between centers. Cylinder A has a rotative speed of 250 r.p.m., while cylinder B has a rotative speed of 100 r.p.m. Find the diameters of the cylinders. *Ans.* $D_a = 8$ in. $D_b = 20$ in.

5. Two bevel frictions, A and B, have rotative speeds of 180 and 300 r.p.m. respectively. If the outer diameter of A is 6 inches, find the outer diameter of B. *Ans.* 10 inches.

6. State the Law of Gearing.

7. What kinds of curves are used for the profiles of gear teeth?

8. The diametral pitch of a gear is 2. Find its circular pitch. *Ans.* 1.5708 in.

9. What is the diametral pitch of a gear whose circular pitch is $\frac{3}{4}$ in.? *Ans.* 4.1888.

10. A 2-pitch spur gear carries 30 teeth. Find the pitch diameter of the gear. *Ans.* 15 in.

11. A cast spur gear has 26 teeth and a circular pitch of $1\frac{1}{2}$ inches. The following dimensions of this gear are required: (*a*) addendum, (*b*) root or dedendum, (*c*) whole depth of tooth, (*d*) working depth of tooth, (*e*) clearance, (*f*) backlash, (*g*) thickness of tooth, (*h*) width of space, (*i*) pitch diameter, (*j*) outside or addendum circle diameter, (*k*) root circle diameter. *Ans.* (*a*) 0.48 in. (*b*) 0.585 in. (*c*) 1.065 in. (*d*) 0.96 in. (*e*) 0.105 in. (*f*) 0.06 in. (*g*) 0.72 in. (*h*) 0.78 in. (*i*) 12.414 in. (*j*) 13.374 in. (*k*) 11.244 in.

12. A cut spur gear has a pitch diameter of 10 inches and a diametral pitch of 4. The following information relative to this gear is required: (*a*) number of

teeth, (*b*) circular pitch, (*c*) addendum, (*d*) dedendum, (*e*) clearance, (*f*) whole depth, (*g*) working depth, (*h*) thickness of tooth, (*i*) width of space, (*j*) backlash, (*k*) outside diameter, (*l*) root diameter. *Ans.* (*a*) 40 teeth (*b*) 0.7854 in. (*c*) 0.25 in. (*d*) 0.289 in. (*e*) 0.039 in. (*f*) 0.539 in. (*g*) 0.50 in. (*h*) 0.3927 in. (*i*) 0.3927 in. (*j*) 0 (*k*) 10.50 in. (*l*) 9.422 in.

13. What is an interchangeable set of spur gears?

14. The face of a gear is $4\frac{1}{2}$ inches in length. If the circular pitch of the gear is $1\frac{1}{2}$ inches, what value of n was used in its design? *Ans.* 3.

15. It is required to obtain the circular pitch of a 36-tooth cast-iron spur gear which transmits 20 horsepower at 150 revolutions per minute. It is assumed that the face of the gear is $2\frac{3}{4}$ times the circular pitch. The teeth of the gear are of the involute system, the angle of obliquity being equal to $14\frac{1}{2}$-degrees. *Ans.* $1\frac{1}{8}$ in.

16. It is required to obtain the diametral pitch of a 16-tooth cast steel spur gear whose teeth are of the standard $14\frac{1}{2}$-degree involute form. The gear transmits 15 horsepower at 1000 revolutions per minute. It is to be assumed that $n = 3.5$. *Ans.* 6.

17. It is required to find the diametral pitch of a semi-steel spur gear which is to transmit 25 horsepower at 300 r.p.m. Assume 36 standard involute teeth on the gear and a value of n equal to 4. *Ans.* 4.5

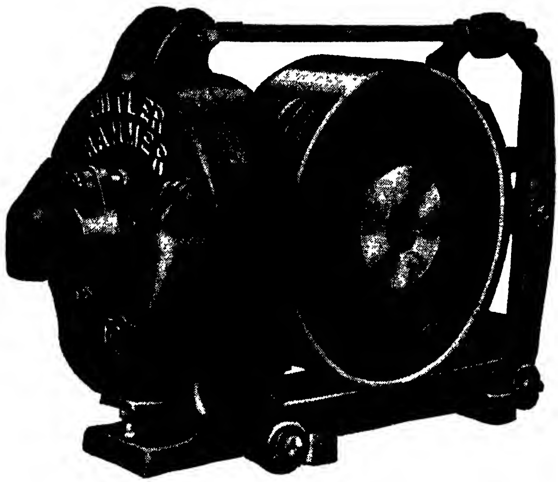
(Note. The theoretical answer for the diametral pitch in this problem is 4.7. This is not a standard pitch. Hence it will be noticed that in selecting the diametral pitch to be used, the next lower standard value, 4.5, is taken. In this manner, a somewhat larger and hence safer tooth is actually used and so the factor of safety is increased rather than decreased, which is always the custom in design. Why is the tooth of a 4.5 pitch gear larger than the tooth of a gear whose diametral pitch is 4.7?)

18. A cut semi-steel spur gear has $14\frac{1}{2}$-degree involute teeth, a diametral pitch of 3, a pitch diameter of 12 inches, and a length of tooth of $3\frac{1}{2}$ inches. It runs at 250 r.p.m. Required (*a*) the circular pitch of the gear, (*b*) the pitch line velocity in feet per minute, (*c*) the tooth stress in lb. per sq. in. based on the preceding velocity, (*d*) the number of teeth, (*e*) the form factor, y, for the above number of teeth, (*f*) the tooth pressure, P_t [Use formula (**187**)], (*g*) the torque in the gear's shaft (Use $T = P_t R$ where R is the pitch radius), (*h*) the horsepower that can be transmitted safely. [Find by formula (**35**) and then check by formula (**33**)]. *Ans.* (*a*) 1.0472 in. (*b*) 785.4 f.p.m. (*c*) 6575 lb. per sq. in. (*d*) 36 teeth (*e*) 0.105 (*f*) 2531 lb. (*g*) 15,180 in.-lb. approx. (*h*) 60.2 hp.

19. Determine the backing, X, for a miter gear whose pitch diameter is 10 inches. *Ans.* $1\frac{1}{4}$ in.

20. What is the so-called formative number of teeth of a bevel gear?

21. State the difference between an angular gear and a miter gear.



MAGNETIC BRAKE

Courtesy of Cutler-Hammer, Inc., Milwaukee, Wisconsin

CHAPTER IX

MISCELLANEOUS DETAILS OF DESIGN

Efficiency of Machines. In order that a machine may do its work or serve its purpose, energy must be supplied thereto. As stated heretofore, this energy is used by the machine in two ways;

1. in its own operation,
2. in doing the work or performing the task for which it was designed.

Efficiency in general may be defined as $\frac{output}{input}$, in which ratio both numerator and denominator must be in like units. Therefore the efficiency of a machine is the ratio of the work done by it to the total energy supplied to keep the machine operating at a constant rate. It follows then that for a machine to have a high efficiency, it must use in its own operation but a relatively small quantity of the energy that it receives, so that the major portion of the latter can be used in performing its useful or designated work. In other words, the higher the efficiency of a machine, the less the power required and hence the less the cost of operation will be.

If we let

e = the mechanical efficiency
E = the energy equivalent of the useful work done, in ft.-lb. per minute
E_a = the energy received by the machine in ft.-lb. per minute

then
$$e=\frac{E}{E_a} \tag{192}$$

or
$$E_a=\frac{E}{e} \tag{193}$$

Since power is defined as the rate at which work is done, formula (**192**) indicates that the mechanical efficiency of a machine is equal to the ratio of its theoretical horsepower to its actual

horsepower. In the preceding statement, it is evident that the theoretical horsepower is that power required by the machine for doing only the useful work it is to do, while the actual horsepower, as the name implies, is the power the machine actually receives and hence is larger than the theoretical horsepower by an amount equal to that power required by the machine merely for its own operation. It follows then that if

H = the theoretical horsepower of the machine

H_a = the actual horsepower of the machine

$$e=\frac{H}{H_a} \tag{194}$$

or

$$H_a=\frac{H}{e} \tag{195}$$

In the greater part of our study of Machine Design, the theoretical horsepower is used. It is therefore the theoretical horsepower, H, that is obtained by using such formulas as formulas (32), (33), and (35). If the efficiency, e, of a machine is known or assumed, the actual horsepower can be obtained from any one of formulas (32), (33), and (35) by including in the denominator of the second or right-hand member of the formula, the factor, e. This can be demonstrated as follows:

from formula (35) $$H=\frac{P_tV_L}{33{,}000}$$

Substituting this value of H in formula (195)

$$H_a=\frac{\frac{P_t\,V_L}{33{,}000}}{e}$$

$$=\frac{P_tV_L}{33{,}000}\times\frac{1}{e}$$

$$=\frac{P_tV_L}{33{,}000\,e} \tag{196}$$

Example. A load of 4000 pounds is lifted by a hoist which has a drum whose diameter is 30 inches. If the drum makes 20 r.p.m., find (*a*) the theoretical horsepower, H, of the hoist, (*b*) the actual horsepower, H_a, assuming the efficiency of the hoist to be 0.75. (This efficiency expressed in percentage would be given as 75 per cent.)

Solution. (*a*) It will be necessary to obtain the speed at which the load is lifted. This can be accomplished by assuming that the speed of lift is equal to the linear velocity of the rim of the drum. Hence, from the subject of Mechanism

$$V_L = \pi DN$$

in which D, the diameter of the drum = 2.5 ft. and N, the r.p.m. of the drum, = 20.

Evaluating in the preceding formula,

$$V_L = \pi \times 2.5 \times 20 = 157.08 \text{ f.p.m.}$$

The tangential force, P_t, at the drum is the load, 4000 lb., which is to be lifted.

Hence we obtain from formula (**35**),

$$H = \frac{P_t V_L}{33{,}000}$$

$$H = \frac{4000 \times 157.08}{33{,}000} = 19.04 \text{ hp.} \quad \textit{Ans.}$$

(*b*) Using formula (**196**),

$$H_a = \frac{P_t V_L}{33{,}000\, e}$$

$$= \frac{4000 \times 157.08}{33{,}000 \times 0.75} = 25.39 \text{ hp.} \quad \textit{Ans.}$$

Or, by using formula (**195**),

$$H_a = \frac{H}{e}$$

$$= \frac{19.04}{0.75} = 25.39 \text{ hp.} \quad \textit{Ans.}$$

Frictional Resistance to Motion. That portion of the input energy which is used by a machine in its own operation is consumed to a large degree in overcoming the resistance to motion between two contacting parts or elements of the machine which have relative motion with each other. This resistance is known as a frictional resistance. It may be explained in a sense as due to the interlocking action between the minute projecting particles that form the surfaces of the contacting members. Therefore smooth surfaces present less frictional resistance than rough surfaces. The amount of frictional resistance is also dependent upon the action of one machine element

relative to another. When the action is one of sliding, that is when one part slides upon the other, sliding friction is involved. When one part rolls upon another, rolling friction is involved. And if some slipping occurs as the elements roll with each other, a combination of rolling and sliding friction results. Friction in general tends to resist or oppose the relative motion between the machine elements and hence energy must be expended in overcoming this resistance so that the motion may ensue or take place. In general, sliding friction between two elements in contact over a finite surface is of the greatest magnitude.

Fig. 126 (a). Solid Journal Bearing
Courtesy of Link-Belt Company, Chicago, Ill.

Lubrication. The frictional resistance set up between two machine elements in contact depends not only on the physical condition or the smoothness of the contacting surfaces but also upon the materials used, that is different materials have varying coefficients of friction. Therefore if a substance having a low coefficient of friction can be introduced between two surfaces of a relatively high coefficient of friction, the frictional resistance can be materially lowered. This method of reducing the frictional resistance is known as Lubrication and the substance introduced is known as a Lubricant.

The subject of lubricants and lubrication is of utmost importance in the design and operation of machinery. The prospective designer of machines should avail himself of all the information that he can possibly obtain on this phase of the subject. For proper lubrication means not only a smaller energy input and hence lower cost of operation, but also longer life and lower cost of repairs and replacements.

Bearing Classification. A machine part which supports another machine part and at the same time confines or constrains the relative motion of that part is called a Bearing. The relative motion between the two contacting members may be translation or rotation. In the case of translation or rectilinear motion, the bearing is termed a Slide, Guide, or Way, such as the cross-slide of a lathe, the cross-head guides of a steam engine, or the ways of a lathe or planer. Thus it has become customary to restrict the use of the word, Bearing, to those supporting machine elements where rotation is involved.

The latter can be classified in several ways. If, for instance,

Fig 126 (b) Journal Bearing with Removable Cap
Courtesy of Link-Belt Company, Chicago, Ill

the bearing supports a load that is perpendicular to the axis of rotation, it is called a Radial Bearing, while if the load acts along or parallel to the axis of rotation, it is called a Thrust Bearing. In some instances, a single bearing supports simultaneously both a radial and a thrust load. It is then known as a combination Radial and Thrust Bearing. The most common type of a radial bearing is the Journal Bearing, which is shown in Figs. 126 (a) and 126 (b). In such a bearing, the rotating element is called the Journal and the supporting element, the Bearing. The latter is a sleeve which surrounds the journal.

Journal and thrust bearings are said to be Plain Bearings when the contact they afford with the rotating element is over a finite surface and the resulting action between the elements in contact is one of sliding. To obtain point or line contact with rolling action instead of sliding and hence substitute rolling friction for sliding friction, the contacting surfaces of journal and thrust bearings

are often separated by balls or rollers, the latter being either cylindrical or conical. Such bearings as a whole are known as Anti-Friction Bearings and are known individually as Ball Bearings, Cylindrical Roller Bearings, or Conical Roller Bearings, etc.

Journal Bearings. The simplest type of a bearing to take a radial load is a plain journal bearing. Such a bearing is often

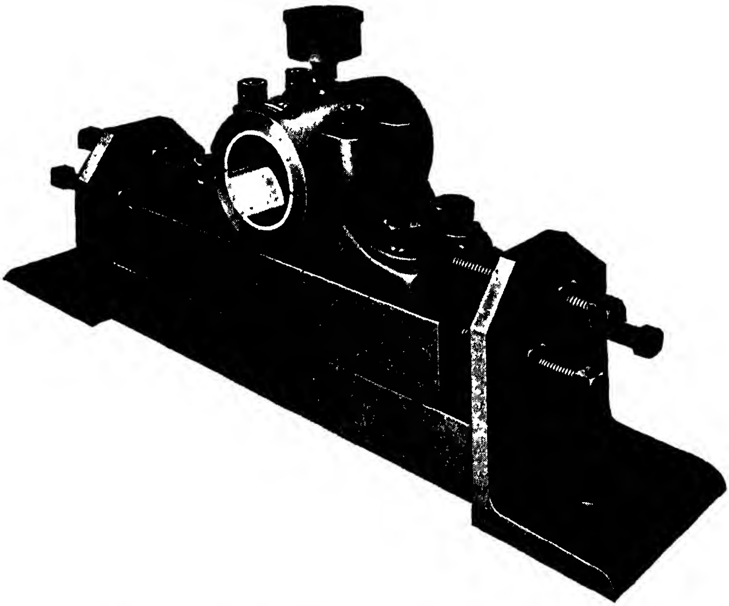


Fig. 127 Adjustable Base Plate in Use with Rigid Pillow Block
Courtesy of Link-Belt Company, Chicago, Ill.

nothing but a hole in the cast-iron frame of a machine into which a shaft is fitted. Otherwise the journal bearing may be as illustrated in Figs. 126 (a) and 126 (b). In the former the sleeve is solid; in the latter, the sleeve is split to form a cap, which provides an adjustment for wear through the introduction of shims between the cap and the base. This cap is easily removable and hence facilitates introduction of the shaft (or journal) into the bearing. For light loads and low speeds, the bearing which is generally cast iron may be unlined, but otherwise the bearing is lined with some bearing metal such as babbitt or bronze, which may be renewed when wear takes place. The journal is usually made of steel and may be hardened. Its surface is accurately finished and highly polished.

Since the axis of a bearing must coincide with the axis of the shaft, some means of adjustment must be provided in many cases as for instance when individual bearings are mounted on concrete foundations or on unfinished machine parts, or when a line-shaft is supported by independent bearings attached to the ceiling, wall, or floor of a building. The Adjustable Base Plate of Fig. 127 affords

Fig. 128. Post Hanger

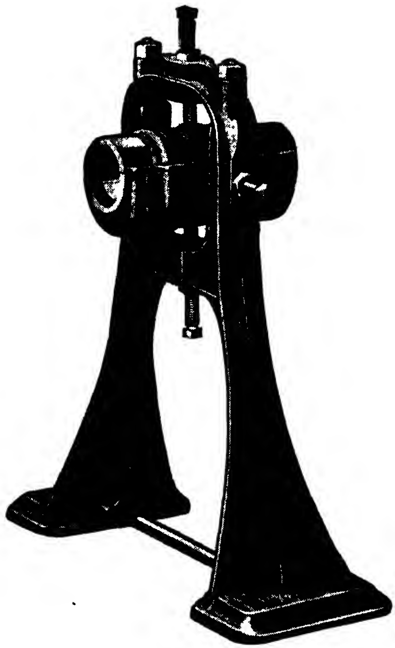


Fig 129 Dodge Four-Point Adjustable Floor Stand
Courtesy of Dodge Manufacturing Corp., Mishawaka, Indiana

through the use of wedges, as shown, both horizontal and vertical adjustment. The Post Hanger of Fig. 128 provides for vertical adjustment through slots in its frame. The Floor Stand of Fig. 129 gives a so-called four-point adjustment by means of the use of several screws. This floor stand can be inverted for use as a Drop Hanger for attachment to overhead beams or ceiling in supporting a line-shaft.

Since most of the energy consumed by a machine in its own operation is used in overcoming the frictional resistances at its bearings, and likewise since most of the wear occurring in a machine

is at its bearings, the design of the latter to permit effective lubrication is of the utmost importance. The method of lubrication and the type of the lubricant used will depend on the conditions to which the bearing is subjected and the service it is rendering.

Each of the journal bearings of Figs. 125, 126, and 127 is tapped on the upper side so that an oil or grease cup can be screwed into the tapped hole. The lubricant thus gains access to the upper part of the bearing and is carried by the rotating journal to the lower bearing surfaces.

A more adequate system of lubrication is the Ring-Oiling System which is shown in use with a journal bearing in Fig. 130.

The journal bearing as here shown is a hanger box designed by the W. A. Jones Foundry and Machine Company of Chicago for

Fig 130 Ring Oiling System of Lubrication
Courtesy of W A Jones Foundry & Machine Co , Chicago, Ill.

use with their drop hangers, bracket hangers, post hangers, and pillow blocks. Such a hanger box is babbitted, accurately bored, and faced on both ends.

In Fig. 130 one or more steel rings hang loosely over the journal and are caused to revolve by contact with it. They are drawn in this manner through a reservoir of oil located below the bearing. The oil is thus carried to the top of the journal from which point it is distributed to the bearing surfaces by the aid of oil grooves in the bearing. The Chain-Oiling System of lubrication is similar to the preceding system but uses chains hanging loosely on the journal instead of rings.

Where conditions warrant, the Pressure System of lubrication should be employed. By this method, oil is delivered from a reservoir to the bearing by means of a pump. The oil is thus supplied to the bearing under a pressure and is returned by gravity to the reservoir from which it is recirculated. Such a system generally includes oil

strainers and filters and permits the oil to dissipate or release some of the heat which it absorbed while in the bearing.

Thrust Bearings. As has been previously stated, a bearing which is subjected to a load acting parallel or along the axis of the shaft is known as a Thrust Bearing. Most horizontal shafts are subjected to a major radial load with a minor thrust load accompanying it. If the thrust load is small it can often be carried by the hubs of gears or pulleys or by collars so located on the shaft as to bear against the ends of the journal bearings. If the thrust load is large on a horizontal shaft either a thrust bearing must be used in addition to a journal bearing or a single bearing must be used which is capable of taking both thrust and radial loads. Such a combination thrust and radial bearing will be introduced shortly.

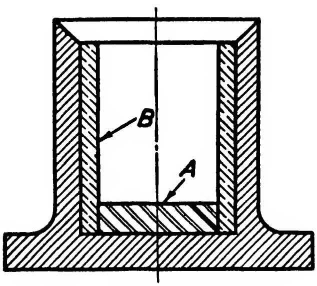


Fig. 131. Rigid Step Bearing

Fig. 132. Adjustable Step Bearing
Courtesy of Link-Belt Company, Chicago, Ill.

On the other hand, most vertical shafts are subjected to a major thrust load, and the thrust bearings which they employ at their lower ends are called Step or Pivot Bearings. A section of a Rigid Step Bearing is shown in Fig. 131. It is provided with a hardened steel disc, *A*, to take the axial thrust and a bronze or brass liner, *B*, to reduce the friction due to the radial load. An Adjustable Step Bearing for a vertical shaft is illustrated in Fig. 132. The cup which holds the shaft is rounded on the bottom so that the bearing cannot bind, and screws with lock nuts provide adjustment for the shaft alignment.

Ball and Roller Bearings. In addition to providing rolling friction in the place of sliding friction and hence lowering the frictional resistance, Ball and Roller Bearings have many other advantages over plain journal and plain thrust bearings. The ease and

simplicity of lubrication in these so-called anti-friction bearings is quite noteworthy. A supply of oil or grease placed in the housing of the bearing need not be replenished for six months to one year if the bearing is properly protected from dirt and moisture. The small amount of wear that takes place makes adjustment of the bearing unnecessary, and accurate alignment of parts may be maintained for a long time. They accommodate themselves to rather heavy overloads for quite a length of time without failure. They are extremely reliable in every way. They can easily be adapted in design to accommodate both radial and thrust loads simultaneously.

A Radial Ball Bearing is shown in Fig. 133. This bearing

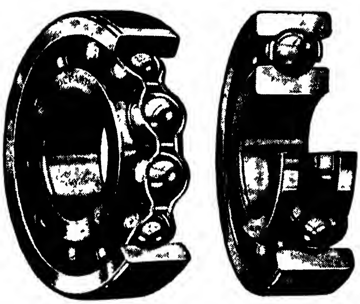


Fig. 133. Radial Ball Bearing with Single Row of Balls, for Radial Loads Only

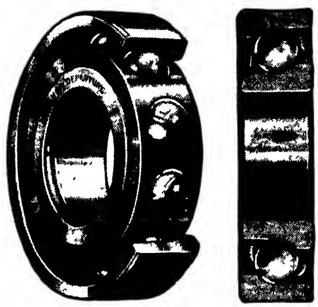


Fig. 134 Radial Ball Bearing, for One-Direction Thrust Load in Combination with Radial Load

Courtesy of New Departure Manufacturing Co., Bristol, Conn.

carries a single row of balls, which are held in a steel separator or cage. The balls are made of high carbon alloy steel. The inner and outer rings or races with which the balls make contact are provided with curvilinear raceways and are made of high carbon chrome alloy steel. This bearing is intended by its manufacturer, the New Departure Manufacturing Company of Bristol, Connecticut, to carry radial loads only. It will however resist a relatively small thrust load as well. The major parts of a radial ball bearing manufactured by the Norma-Hoffman Bearings Corporation of Stamford, Connecticut, are illustrated in Fig. 136. When a thrust load in one direction only is to be carried in addition to its radial load, the single row ball bearing of Fig. 134 may be used. The difference between the two bearings of Figs. 133 and 134 rests primarily in the outer contour of the inner race and the inner contour of the outer race. The Double Row Ball Bearing of Fig. 135 is a combination thrust

and radial bearing that will resist thrust loads in either direction along its axis together with its radial load.

Thrust Ball Bearings are shown in Figs. 137 and 138. Such bearings are used in the place of plain thrust or step bearings when it is wished to replace surface contact by point contact. It will be noticed that the bearing of Fig. 138 is provided with a lower race

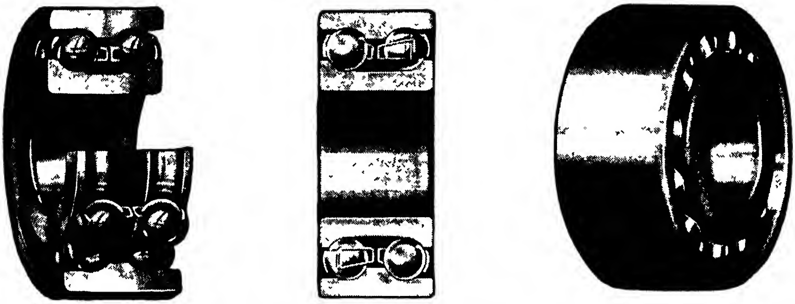


Fig 135. Double Row Type Ball Bearing
Courtesy of New Departure Manufacturing Co, Bristol, Conn.

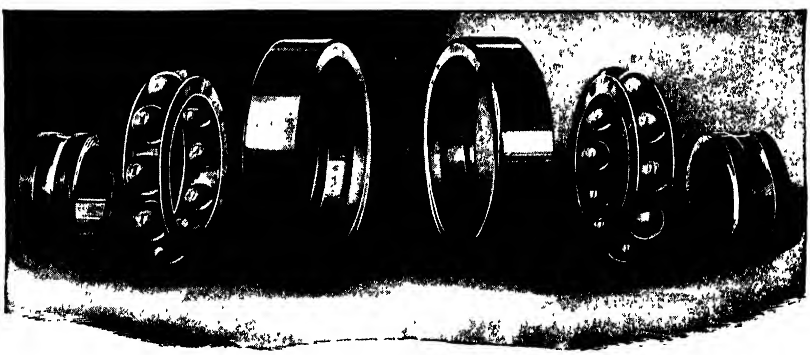


Fig 136. Major Parts of a Ball Bearing
Courtesy of Norma-Hoffman Bearings Corp, Stamford, Conn.

that has a spherical seat. This is a self-aligning feature that insures a more equal distribution of the load on the balls.

Cylindrical Roller Bearings to carry radial loads are illustrated in Figs. 139 and 140. Such bearings consist primarily of an outer and inner race, a cage or separator, and the cylindrical rollers. The bearing of Fig. 139 is of the solid roller type. It is essential that the rollers be of true cylindrical form and that their axes be maintained parallel to each other and to the axis of the shaft. Otherwise

the contact between roller and race cannot be the line contact that is necessary in this type of bearing. The bearing of Fig. 140 is unique in that it uses wound rollers in the place of solid rollers, thus introducing a flexibility that is absent with the solid roller type. The wound roller is made from a strip of steel which has a rectangular cross section. This strip is wound in a helical manner on a mandrel. Some rollers are made right handed and some left handed so that

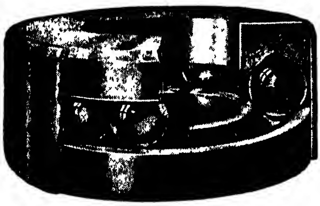


Fig 137. Thrust Ball Bearing

Courtesy of Auburn Ball Bearing Co., Rochester, New York

Fig. 138. Self-Aligning Thrust Ball Bearing with Housing

Courtesy of Norma-Hoffman Bearings Corp, Stamford, Conn

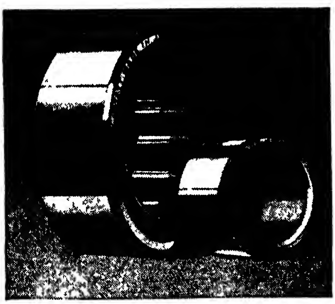


Fig. 139 Solid Roller Radial Bearing with Inner Race Removed

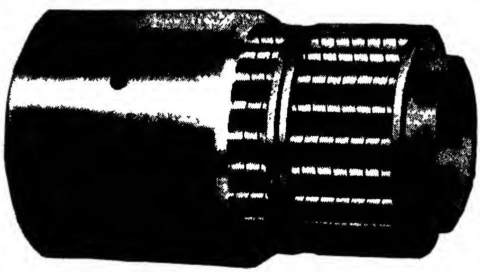


Fig. 140. Wound Roller Radial Bearing Double Roller Assembly

Courtesy of Hyatt Bearings Division, General Motors Corp, Harrison, N. J.

in assembly as clearly shown in Fig. 140 rollers of opposite hand are adjacent to each other. This provides for the distribution of the lubricant in both directions. Single roller assembly is also provided in this type of bearing.

The Conical Roller Bearing of Fig. 141 makes use of solid conical rollers which give line contact with inner and outer races of a conical form. To insure the desired pure rolling, the cones are so constructed and the bearing so assembled that the elements of each cone meet at a common point on the axis of the shaft. Adjustment for wear

may be made easily by altering the relative position of the two races. This bearing will carry both radial and thrust loads.

Frictional Work and Heating of Bearings. Attention has been called to the fact that some of the mechanical energy supplied to a machine is necessarily used in overcoming the frictional resistance that is set up between those machine parts that rub on each other. Since it is a fundamental law of Thermodynamics that energy can be neither created nor destroyed, the mechanical energy used in

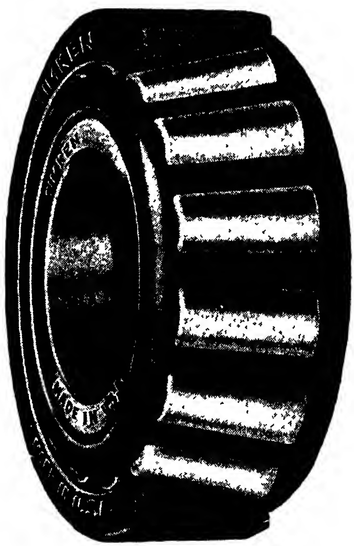


Fig. 141 Conical Roller Bearing
Courtesy of The Timken Roller Bearing Co., Canton, Ohio

overcoming the frictional resistance at bearings in general must appear after its use in some other form. So we find that this mechanical energy so used is transformed into heat energy. Thermodynamics instructs us that for every 778 foot-pounds of mechanical energy used in overcoming frictional resistance, one British Thermal Unit (B.t.u.) of heat results, or

$$778 \text{ ft.-lb.} = 1 \text{ B.t.u.}$$

It should be remembered that one British Thermal Unit is $\frac{1}{180}$ of the amount of heat required to raise the temperature of one pound of water from 32°F. to 212°F.

The heat energy thus released at a bearing must be continually

dissipated by this bearing. Otherwise the heat will become resident therein, resulting in a temperature which will break down the lubricant and result in destruction of the bearing. The heat is normally dissipated into the air surrounding the bearing but sometimes artificial cooling must be resorted to. The latter may be accomplished by the forced circulation of air or water around the bearing, or of oil through the bearing. The bearing must be so designed that the rate at which heat is released or radiated must be equal to the rate at which the heat is generated after the bearing has once reached its normal operating temperature. The latter should not exceed 180°F. for good operation. It becomes evident that good lubrication lowers the frictional resistance and hence decreases the rate at which heat is generated by the bearing.

Heat Generated by Journal Bearings. In order to demonstrate the heat generated in overcoming the frictional resistance, we shall consider the case of the journal bearing. The following notation will be used;

P = total radial load or pressure on bearing in pounds
p = load or pressure per square inch of the projected area of the bearing in pounds
L = length of bearing, in inches
D = diameter of bearing, in inches
N = r.p.m. of journal
μ = coefficient of friction
W = total work of friction in foot-pounds per minute
w = work of friction per square inch of projected area of bearing in foot-pounds per minute
Q = total heat generated by the bearing in B.t.u. per minute
q = heat generated by the bearing per square inch of projected area in B.t.u. per minute
V = rubbing velocity in feet per minute.

The total radial load, P, is assumed to be distributed over the projected area, $L \times D$, of the bearing. It follows, then, that the pressure, p, per square inch of the projected area is equal to the total load divided by the projected area or:

$$p=\frac{P}{LD} \tag{a}$$

The safe value that this unit pressure, p, may take has been shown by experience to depend on many things such as the type of service the journal bearing is rendering, the materials used in the bearing, etc. Thus for line shafts with cast-iron bearings, unit pressures of from 15 to 30 pounds per square inch are generally used, while with installations involving low speed and intermittent loads, the pressures may be from 2000 to 5000 pounds per square inch or even higher. Since the diameter of the journal (or bearing) is generally based on a consideration of strength and hence is more or less fixed for a given design, the projected area of a bearing is directly dependent on its length. Thus it would seem that by increasing the length of a bearing, practically any projected area and hence any unit pressure desired could be obtained. To a certain extent this is correct. However other factors enter into the design such as the difficulty of maintaining or establishing bearing alignment. These factors place a limit on the length of the bearing. Such limits are generally given as a ratio of the length of the bearing to its diameter, $\left(\frac{L}{D}\right)$. For instance, the accepted ratio of $\frac{L}{D}$ for the crank pin of a stationary engine is from 1 to 1.5, while in the case of standard shafting supported by self-adjusting bearings by which alignment is practically assured, the value of $\frac{L}{D}$ may be from 3 to 5.

The total frictional force on a bearing is equal to the total bearing pressure multiplied by the coefficient of friction. It therefore becomes equal to μP pounds. Since in one revolution, this frictional force or resistance moves through a distance equal to πD, the circumference of the bearing, the work done by it (or in overcoming it) per revolution is equal to $\mu P \times \frac{\pi D}{12}$ foot-pounds.

Since there are N revolutions per minute, the total work of friction is given by the formula

$$W = \frac{\mu P \pi D N}{12} \text{ ft.-lb. per min.} \tag{b}$$

Since the work of friction per square inch of projected area per minute is equal to the total work of friction divided by the projected area,

$$w=\frac{W}{LD}$$

$$=\frac{\mu P\pi DN}{12\ L\ D}$$

$$=\frac{\mu P\pi N}{12\ L}\text{ ft.-lb. per min.} \tag{c}$$

As previously stated the total heat generated at the bearing is the heat equivalent of the total work done in overcoming the frictional force. Therefore from step (*b*) as given above,

$$Q=\frac{W}{778}$$

$$=\frac{\mu P\pi DN}{12\times 778}\text{ B.t.u. per min.}$$

$$=\frac{\mu P}{778}\times\frac{\pi DN}{12} \tag{d}$$

$$=\frac{\mu PV}{778}\text{ B.t.u. per min.} \tag{e}$$

for $\frac{\pi DN}{12}=V$, the rubbing velocity in f.p.m.

It is evident that the heat generated per square inch of projected area per minute is equal to the heat equivalent of w, therefore

$$q=\frac{w}{778}$$

Therefore from step (*c*),

$$q=\frac{\mu P\pi N}{12L\times 778}\text{ B.t.u. per min.} \tag{f}$$

When bearings are well designed and properly lubricated, the coefficient of friction, μ, may be taken as about 0.02. The heat generated per square inch of projected area per minute will vary from 0.2 to 1.0.

Example. A journal bearing whose diameter is $2\frac{1}{4}$ inches is subjected to a load of 1000 pounds while rotating at 200 revolutions per min. If $\mu=0.02$ and $\frac{L}{D}=3$, find

(a) the projected area in square inches

(b) the pressure on the bearing per square inch of projected area

(c) the total work of friction in foot-pounds per minute

(d) the work of friction per square inch of projected area per minute in foot-pounds.

(e) the total heat generated per minute in B.t.u.

(f) the heat generated per minute per square inch of projected area in B.t.u.

Solution.

(a) Since $\frac{L}{D}=3,$

$$L=3D=3\times 2\tfrac{1}{4}=6\tfrac{3}{4}\text{ in.}$$

The projected area $=L\times D$

$$=6\tfrac{3}{4}\times 2\tfrac{1}{4}=15\tfrac{3}{16}\text{ sq. in. } \textit{Ans.}$$

(b) From step (a) of the preceding article,

$$p=\frac{P}{LD}$$

$$=\frac{1000}{15\frac{3}{16}}=65.8\text{ lb. per sq. in. } \quad \textit{Ans.}$$

(c) From step (b),

$$W=\frac{\mu P\pi DN}{12}$$

$$=\frac{0.02\times 1000\times\pi\times 2.25\times 200}{12}$$

$$=2356+\text{ft.-lb.} \quad \textit{Ans.}$$

(d) $$w=\frac{W}{LD}$$

$$=\frac{2356}{15\frac{3}{16}}=155.1\text{ ft.-lb.} \quad \textit{Ans.}$$

(e) $$Q=\frac{W}{778}$$

Here $W=2356$ ft.-lb.

$$\therefore\ Q=\frac{2356}{778}=3.03\text{ B.t.u.} \quad \textit{Ans.}$$

(f) $$q=\frac{w}{778}$$

Here $w=155.1$ ft.-lb.

$$\therefore\ q=\frac{155.1}{778}=0.2-\text{B.t.u.} \quad \textit{Ans.}$$

or $$q=\frac{\mu P\pi N}{12L\times 778}$$

$$=\frac{0.02\times1000\times3.1416\times200}{12\times6.75\times778}$$

$$=0.2 \text{ B.t.u., to check the above.}$$

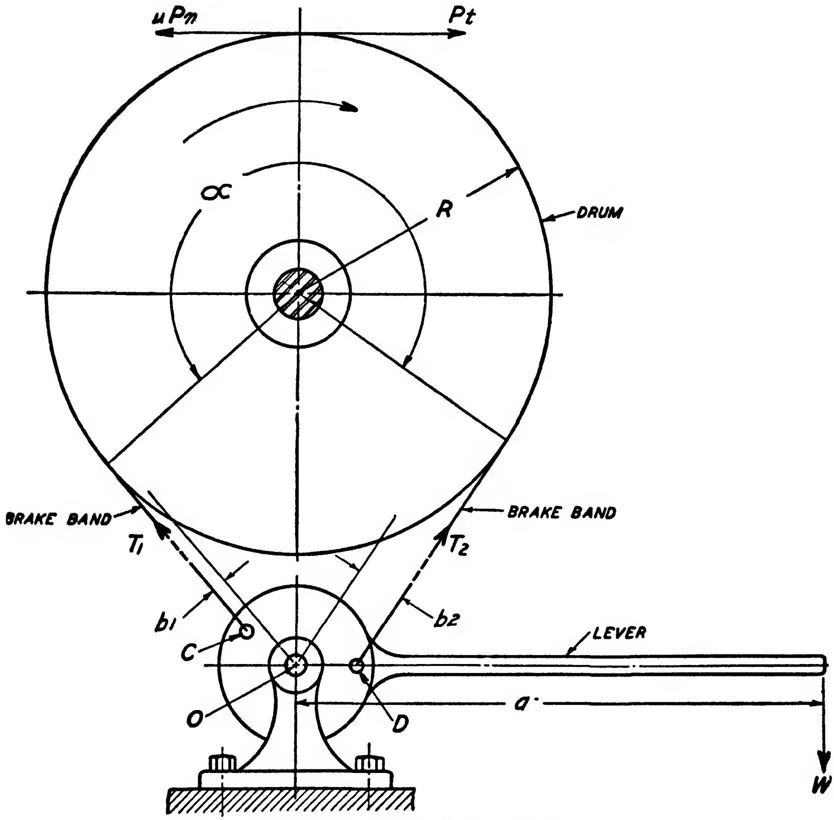


Fig. 142. Differential Band Brake

Differential Band Brake. A brake is a mechanism which is designed to regulate the speed of a machine or to stop the motion thereof through a frictional resistance applied to some rotating part. One of the more common forms is that of the Differential Band Brake which is shown in Fig. 142. It consists of a cast-iron drum around which is wrapped a steel band. The latter may be tightened against the drum by the application of a force, *W*, at the end of a lever. The fulcrum or center of rotation of the lever is at point *O*, and the

ends of the brake band are connected to the disk end of the lever by pins at C and D. Therefore as the lever is caused to rotate in a clockwise direction (in the arrangement as shown in the figure), the band is pulled against the drum, thereby setting up a frictional resistance which acts against the rotation of the drum controlling the speed of the drum or bringing it to a stop. Wooden blocks are often attached to the inside of the steel band thus giving contacting surfaces of wood on cast iron. This permits of a higher coefficient of friction and a renewable surface in that the wooden blocks are easily replaced. For the same reasons, brake linings of woven fabric are also used.

In the analysis of the band brake, the following notation will be used and all forces will be expressed in pounds and their moment arms in inches.

W = the operating force applied at end of lever.

a = length of lever, or the moment arm of the force, W, with respect to the fulcrum, O.

T_1 = total tension on the so-called tight side of band.

b_1 = moment arm of T_1 with respect to fulcrum, O.

T_2 = total tension on the so-called loose side of band.

b_2 = moment arm of T_2 with respect to fulcrum, O.

R = radius of brake drum.

α = the arc, or central angle, of the drum subtended by the brake band, in radians.

μ = coefficient of friction between band (or band lining) and the drum. (See Table XXV)

P_n = total normal pressure between band and drum.

P_t = tangential force at rim of drum.

T = torque in the drum shaft in in.-lb., or the torque to be sustained by the brake.

$e = 2.718$

As previously stated, a force of W pounds acting at the end of the lever as shown in Fig. 142, will pull the brake band against the drum, causing the band to set up between itself and drum a normal pressure, P_n. Due to this normal pressure, a frictional resistance equal to μP_n is established over the contact surfaces of band and drum for a frictional resistance is always equal to the product of the coefficient of friction and the normal pressure. To be effective

this frictional resistance, μP_n, must be equal in magnitude to the tangential force, P_t, occurring at the rim of the drum. This frictional resistance opposes the action of the tangential force, P_t, and hence acts in a direction opposite to that of P_t (see Fig. 142). To obtain P_t, we have from formula (39)

$$P_t = \frac{T}{R}$$

and from the preceding statements,

$$P_t = \mu P_n = \frac{T}{R} \tag{197}$$

The action of the brake band while in contact with the drum is identical to that of a belt wrapped around its pulley. That is, tensions T_1 and T_2 are produced in the band as shown in the figure. The relative placement of these tensions is dependent on the direction of rotation of the drum. The placement of these tensions as given in Fig. 142 is for clockwise rotation of the drum. (For counter-clockwise rotation, T_1, the larger tension, would replace T_2, the smaller tension, the latter then acting in the former location of T_1.) From formulas (156) and (157), we obtain the relationships

$$T_1 - T_2 = P_t \qquad \text{[formula (156)]}$$

and

$$\frac{T_1}{T_2} = e^{\mu\alpha} \qquad \text{[formula (157)]}$$

In the application of the latter formula to brake design, the angle of contact, α, should be assumed as from $\frac{7}{10}$ to $\frac{8}{10}$ of the circumference of the drum. Since the circumference of the drum subtends a central angle of 360 degrees or 2π radians, the angle, α, should have a value somewhere from $0.7 \times 2\pi$ to $0.8 \times 2\pi$ radians or from 4.40 to 5.03 radians.

The next step in the analysis of the band brake is to consider the moments of forces, W, T_1, and T_2 with respect to the fulcrum, O. For the condition of equilibrium upon which this consideration must be based, the summation of the moments of these forces must be equal to zero. Since W working through the moment arm, a, and T_1, working through its moment arm, b_1, tend to rotate the lever in a clockwise direction, while T_2, working through its moment arm, b_2, tends to rotate the lever in a counter-clockwise direction, the moments of W and T_1 will oppose and hence be of the opposite algebraic sign

to that of the moment of T_2. Equating the summation of these moments to zero, we obtain:

$$Wa+T_1b_1-T_2b_2=0 \tag{198}$$

from which

$$Wa=T_2b_2-T_1b_1$$

$$W=\frac{T_2b_2-T_1b_1}{a} \tag{199}$$

In examining formula (199), it should be noted that if $T_1\times b_1$ becomes equal to $T_2\times b_2$, the second member of the equation or formula will equal zero. Hence $W=0$. This means that the brake will continue in engagement or operation even after removal of any force, W, on the lever. Due to this, the brake may jam and thus introduce dangerous stresses in the various links. Therefore $T_2\times b_2$ must always be greater than $T_1\times b_1$, in a regular band brake assembly. (However this self-locking principle may be made use of as is evidenced by the Differential Back Stop manufactured by the Link-Belt Company of Chicago. With this device, a drum is free to rotate in one direction but cannot reverse its direction of rotation due to the automatic braking action that will immediately take place.)

Since the brake band is placed under tension, its cross-sectional area must be designed on the basis of the larger tension, T_1. Since the total tension in a member is equal to the involved cross-sectional area, A, multiplied by the safe unit tensile stress, we have

$$\begin{aligned} T_1&=AS_t \\ &=wtS_t \end{aligned} \tag{200}$$

in which

w=width of brake band in inches

t=thickness of brake band in inches

S_t=safe tensile stress in pounds per square inch

The brake band should also be checked for its crushing or compressive load. In such a case, the total normal pressure between the band and the drum is the compressive load on the band and the involved area is the area of contact between the band and the drum. If the band carries a lining, the safe compressive stress of the lining is the one that is involved. It is evident that formula (9) applies in this case.

Simple Band Brake. This brake is the same as the differential

band brake with the single exception that the end of the brake band that is subjected to the larger tension, T_1, is attached to the fulcrum, O. In this manner, the moment arm, b_1, of Fig. 142 becomes equal to zero. Therefore the moment of T_1 is equal to zero and formulas (198) and (199) become

$$Wa - T_2b_2 = 0 \tag{201}$$

and

$$W = \frac{T_2b_2}{a} \tag{202}$$

Otherwise the analysis of the simple band brake is identical to that of the differential type.

Example. A differential band brake like that of Fig. 142 has an angle of contact between its cast-iron drum and leather-lined steel band of 270 degrees. The brake is to sustain a torque of 4000 inch-pounds. The diameter of the drum is 16 inches and the coefficient of friction is 0.30. If the moment arms, a, b_1, and b_2 are 24 inches, 1 inch, and 5 inches, respectively, find the operating force, W.

Solution. Since $R=8$ in. and $T=4000$ in.-lb., we have from formula (197)

$$P_t = \frac{T}{R}$$

$$P_t = \frac{4000}{8} = 500 \text{ lb.}$$

and since $\alpha = 270°$,

$$\alpha = \frac{270}{360} \times 2\pi \text{ radians}$$

$$= 4.71 \text{ radians}$$

With $\mu = 0.30$ and α as just found, we have from formula (157),

$$\frac{T_1}{T_2} = e^{\mu\alpha}$$

$$\frac{T_1}{T^2} = 2.718^{0.3 \times 4.71} = 2.718^{1.41}$$

To obtain the value of $2.718^{1.41}$

$$\text{Log } 2.718 = 0.4342$$

$$\text{Log } 2.718^{1.41} = 1.41 \times 0.4342 = 0.6122$$

$$2.718^{1.41} = \log^{-1} 0.6122 = 4.1$$

Therefore

$$\frac{T_1}{T_2} = 4.1$$

from which
$$T_2=\frac{T_1}{4.1}$$

Substituting the value of P_t as previously found in formula (156)
$$T_1-T_2=P_t$$
$$T_1-T_2=500 \text{ lb.}$$

Substituting for T_2 its equal, $\frac{T_1}{4.1}$, in the above equation
$$T_1-\frac{T_1}{4.1}=500$$
$$4.1T_1-T_1=4.1\times 500$$
$$3.1\ T_1=2050$$
$$T_1=661 \text{ lb.}$$

Hence
$$T_2=\frac{T_1}{4.1}$$
$$T_2=\frac{661}{4.1}=161 \text{ lb.}$$

From formula (199)
$$W=\frac{T_2b_2-T_1b_1}{a}$$
$$=\frac{161\times 5-661\times 1}{24}=6 \text{ lb.} \quad \textit{Ans.}$$

Example. The torque in the shaft of a 40-inch brake drum is 24,000 in.-lb. A differential type of band brake is employed which uses an unlined steel band which surrounds $\frac{3}{4}$ of the circumference of the drum. The operating force is applied to a foot pedal located at the end of a lever whose length is 26 inches. The moment arms of the tensions T_1 and T_2, are $1\frac{1}{2}$ and $4\frac{1}{2}$ inches respectively. It is required to obtain the magnitude of the operating force and the width of the steel band to be used in this case. Assume $\mu=0.20$ from Table XXV, the thickness of the band $=\frac{3}{32}$ inch, and S_t for the steel band $=10{,}000$ pounds per square inch.

Solution. Here $R=20$ in. and $T=24{,}000$ in.-lb.
Substituting these values in formula (197).
$$P_t=\frac{T}{R}$$
$$P_t=\frac{24{,}000}{20}=1200 \text{ lb.}$$

TABLE XXV—Coefficients of Friction of Brakes

Material	μ
Iron on iron	0 20 to 0 25
Steel on iron	0 20
Wood on iron	0 30 to 0 35
Leather on iron	0 30 to 0 40
Asbestos on iron	0 30 to 0 40
Cork on iron	0 35

and $$\alpha=\tfrac{3}{4}\times 2\pi=4.71 \text{ radians}$$

Evaluating in formula (157) with $\mu=0.20$ and $\alpha=4.71$ radians,

$$\frac{T_1}{T_2}=e^{\mu\alpha}$$

$$\frac{T_1}{T_2}=2.718^{0.20\times 4.71}=2.718^{0.942}$$

Since $$\log 2.718=0.4342$$

$$\log 2.718^{0\ 942}=0.942\times 0.4342=0.4090$$

$$2.718^{0\ 942}=\log.^{-1}\ 0.4090=2.56$$

Therefore $$\frac{T_1}{T_2}=2.56$$

or $$T_2=\frac{T_1}{2.56}$$

From formula (156)

$$T_1-T_2=P_t$$

Therefore $$T_1-T_2=1200 \text{ lb.}$$

Substituting in the above the value of $\frac{T_1}{2.56}$ for T_2, we have

$$T_1-\frac{T_1}{2.56}=1200$$

$$2.56\,T_1-T_1=2.56\times 1200$$

$$1.56\,T_1=3072$$

$$T_1=\frac{3072}{1.56}=1969 \text{ lb.}$$

$$T_2=\frac{T_1}{2.56}$$

$$=\frac{1969}{2.56}=769 \text{ lb.}$$

Using formula (199) with $T_1=1969$ lb., $T_2=769$ lb., $b_1=1\frac{1}{2}$ in., $b_2=4\frac{1}{2}$ in., and $a=26$ in., we have

$$W=\frac{T_2b_2-T_1b_1}{a}$$

$$=\frac{769\times4.5-1969\times1.5}{26}=19.5\text{ lb. } Ans.$$

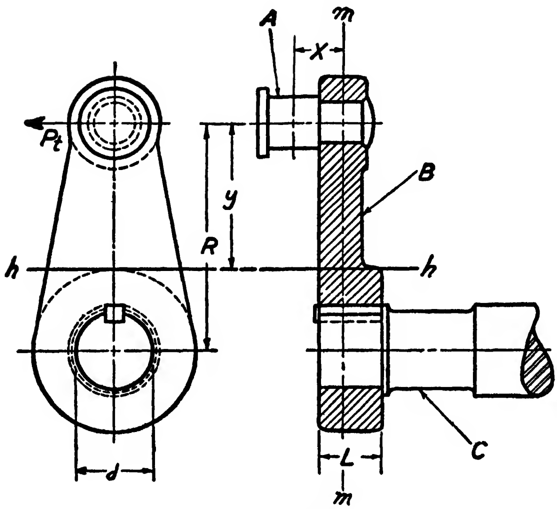


Fig. 143. Overhung Crank

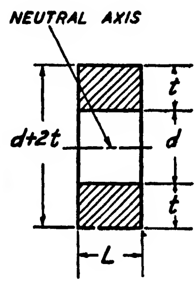


Fig. 144. Section of Crank Hub

Using formula (200) with $T_1=1969$ lb., $t=\frac{3}{32}$ in., and $S_t=10{,}000$ lb. per sq. in., we have

$$T_1=wtS_t$$

$$1969=w\times\tfrac{3}{32}\times10{,}000$$

$$w=\frac{1969\times32}{3\times10{,}000}=2.1\text{ in., say } 2\tfrac{1}{8}\text{ in. } Ans.$$

Overhung Crank. The Overhung Crank is a machine element which plays an important part in many of those machines which utilize the slider-crank mechanism such as the side-crank engine. It is shown in Fig. 143. In the figure, *A* is the crank pin, *B* the crank cheek or crank proper, and *C* is the crank shaft. The overhung crank is shrunk or forced on the crank shaft with a key inserted as an additional safeguard against relative motion between the crank and the shaft. The crank shaft and crank pin are made of steel while the crank cheek is made of cast iron.

The tangential force, P_t, introduced at the crank pin tends to twist the crank about its axis, *mm*. In so doing, it has a moment arm, x. Therefore this twisting moment, T_c, set up by P_t is given by the product $P_t \times x$, or

$$T_c = P_t\, x \tag{203}$$

This twisting moment is set up over and hence must be resisted by any section of the crank that is perpendicular to the axis, *mm*. At the same time, the tangential force, P_t, tends to bend the crank for it is in reality an end load on the crank. Hence under it the crank acts as a cantilever beam. Since the crank can be assumed as supported at the center of the crank shaft, the maximum bending moment, M_c, created by P_t upon the crank, is the product of P_t and its moment arm in bending which is the distance from its point of application to the support. Therefore we have:

$$M_c = P_t R \tag{204}$$

This maximum bending moment, M_c, occurs of course only at the section of the crank located at the center line of the crank shaft. This section is illustrated in Fig. 144. Other sections of the crank are subjected to bending moments, the magnitudes of which depend upon the moment arm through which P_t acts. Thus at section, *hh*, Fig. 143, where the crank cheek joins the crank hub, the bending moment, M_h, is equal to $P_t \times y$, and this section is subjected to the twisting moment, $P_t x$, and the bending moment, $P_t y$, simultaneously, just as the dangerous section, Fig. 144, is subjected to the twisting moment, $P_t x$, and the maximum bending moment, $P_t R$.

Now when a machine element is subjected to both bending and twisting, it must be designed so as to be strong enough to stand up safely under their combined effect. In the case of a shaft, it will be remembered that the bending moment and the twisting moment were combined into a so-called ideal or equivalent twisting moment which would create an effect on the shaft equal to the combined effects of the bending and twisting moments. This ideal twisting moment was then used in the design formula for torque, $T = S_s Z_p$. Now with such a link as a crank, the bending and twisting moments are combined into an ideal or equivalent bending moment, M_i, by introducing the values of M and T into the formula

$$M_i = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) \tag{205}$$

in which M = the bending moment in in.-lb.
T = the twisting moment in in.-lb.

(By comparing this formula with formula (104), it will be seen that for the same values of M and T, M_i is equal to $\frac{1}{2}$ of T_i). The value of M_i secured from this formula is then placed in the beam design formula, $M=SZ$, from which the dimensions of the section (or the induced stress in a given section) are obtained. For section, hh, of Fig. 143, formula (205) becomes,

$$M_i=\tfrac{1}{2}(M_h+\sqrt{M_h^2+T_c^2}) \tag{206}$$

For the dangerous section of the crank (Fig. 144), which section is along the center line of the crank shaft, formula (205) becomes,

$$M_i=\tfrac{1}{2}(M_c+\sqrt{M_c^2+T_c^2} \tag{207}$$

It becomes evident upon inspection that the ideal bending moment of formula (207) is the maximum ideal bending moment of the crank for it is based on the maximum bending moment, M_c. Therefore when M_i of formula (207) replaces M in the beam design formula, $M=SZ$, the cross section of the beam which is involved is that of the dangerous section. If M_i of formula (206) replaces M in the design formula, the cross section which is involved is that along line, hh. This is a rectangle and its section modulus, Z, may be found in Table III.

Since the section modulus for the dangerous section of a crank is not given in the table, it will be necessary to derive a formula for it. It will be seen to be a rectangular figure from which a rectangular opening has been taken away to provide for the entrance of the crank shaft into the crank. It will be noted (Fig. 144) that the neutral axis of the section is coincident with the center line of the crank shaft, that the dimension of the section parallel to the neutral axis is L, and the dimension at right angles to the neutral axis is $d+2t$. In order to obtain the rectangular section modulus of this section, we must first find its rectangular moment of inertia, I. The latter will be equal to the moment of inertia of the outer rectangle minus the moment of inertia of the inner rectangle (the opening). The general statement of the moment of inertia of any rectangle is found in Table III to be $\frac{bh^3}{12}$. For the outer rectangle, b is equal to L

and h is equal to $(d+2t)$. Substituting these values in $\frac{bh^3}{12}$, we obtain for the statement of the moment of inertia, $\frac{L(d+2t)^3}{12}$. For the inner rectangle, b of the general statement becomes L while h becomes d. This gives us $\frac{Ld^3}{12}$ for the moment of inertia of the inner rectangle. Subtracting the latter from the former, we have for the moment of inertia of our section,

$$I=\frac{L(d+2t)^3}{12}-\frac{Ld^3}{12} \tag{208}$$

From this value of I, the section modulus can be determined, for like any section modulus it is equal to the moment of inertia divided by the distance from the neutral axis to the outer fiber. The latter distance is seen to be $\frac{d+2t}{2}$ (Fig. 144), so that:

$$Z=\frac{\frac{L(d+2t)^3}{12}-\frac{Ld^3}{12}}{\frac{d+2t}{2}}$$

from which

$$Z=\left[\frac{L(d+2t)^3}{12}-\frac{Ld^3}{12}\right]\times\frac{2}{d+2t}$$

or

$$Z=\frac{L(d+2t)^3}{12}\times\frac{2}{d+2t}-\frac{Ld^3}{12}\times\frac{2}{d+2t}$$

$$=\frac{L(d+2t)^2}{6}-\frac{Ld^3}{6(d+2t)}$$

$$=\frac{L}{6}\left[(d+2t)^2-\frac{d^3}{d+2t}\right] \tag{209}$$

In the design of the overhung crank, the dimensions of the dangerous section of Fig. 144 are generally assumed in terms of the diameter of the crank shaft as follows:

$$t=\frac{d}{2} \tag{210}$$

$$L=1\times d \quad \text{to} \quad 1\tfrac{1}{4}\times d \tag{211}$$

Having made the above assumptions, the numerical value of Z is determined from formula (209). The value of M_i is then deter-

mined from formula (**207**). These values, as previously indicated, are then substituted for M and Z in the beam design formula, $M=SZ$, giving us the formula,

$$M_i=SZ \tag{212}$$

This formula is then solved for the induced or working stress, S, in lb. per sq. in.

Example. The crank-pin of an overhung crank is subjected to a force of 31,000 pounds which acts tangentially to the circular path of the crank-pin. The crank has the following dimensions: (see Figs. 143 and 144) $R=15$ in., $x=5\frac{1}{2}$ in., $t=3\frac{1}{2}$ in., $L=8$ in., $d=8$ in. It is required to find the induced stress in the crank.

Solution. Step 1. Find T_c from formula (**203**)

$$T_c=P_t\,x$$
$$=31{,}000\times5\tfrac{1}{2}=170{,}500 \text{ in.-lb.}$$

Step 2. Find M_c from formula (**204**).

$$M_c=P_tR$$
$$=31{,}000\times15=465{,}000 \text{ in.-lb.}$$

Step 3. Obtain M_i from formula (**207**).

$$M_i=\tfrac{1}{2}(M_c+\sqrt{M_c^2+T_c^2})$$
$$=\tfrac{1}{2}(465{,}000+\sqrt{465{,}000^2+170{,}500^2})$$
$$=\tfrac{1}{2}(465{,}000+495{,}273)$$
$$=480{,}136 \text{ in.-lb.}$$

Step 4. Obtain the numerical value of Z from formula (**209**).

$$Z=\frac{L}{6}\left[(d+2t)^2-\frac{d^3}{d+2t}\right]$$
$$=\frac{8}{6}\left[(8+7)^2-\frac{8^3}{8+7}\right]$$
$$=\frac{4}{3}\left[15^2-\frac{512}{15}\right]$$
$$=\frac{4}{3}\times191=254.7 \text{ in.}^3$$

Step 5. Obtain the stress, S, by using formula (**212**)

$$M_i=SZ$$

Here $M_i=480{,}136$ in.-lb. and $Z=254.7$ in.^{3}

Evaluating in the above formula,

$$480{,}136 = S \times 254.7$$

$$S = \frac{480{,}136}{254.7} = 1885 \text{ lb. per sq. in.} \quad \textit{Ans.}$$

Practical Design Procedure. It has been stressed heretofore in this text that in general it is necessary to alter, modify, or round out theoretically computed dimensions, so that they will be consistent from the standpoint of certain factors such as the type of scale used by the workman, the materials of which the parts are constructed, stock sizes or standard parts, and standardized shop equipment. Other factors which do not enter into the theoretical calculations often become involved in the design and necessitate the alteration of the theoretical proportions so that provision is made for them. Among these may be mentioned the following: minimum thicknesses for certain castings, liability of corrosion, and necessity of reboring or refinishing.

The designer should also be aware that it is practically impossible for the workman to make a machine part to an exact dimension. He should therefore specify for the benefit of the workman the variation from the given dimension that is permissible, remembering as he does so that the smaller the variation allowed, the greater the cost of manufacture. In order to arrive at the amount of variation, he must consider whether or not the dimension of the part fits either inside or around the same dimension of another part of the assembly.

Let us first consider a rod whose nominal diameter, D, is $1\frac{1}{2}$ inches. If this rod does not fit into a hole, its diameter could be somewhat more or less than 1.5 inches and still no harm would result. Suppose a variation of 0.01 inch on either side of 1.5 inches is allowed; the diameter may then be given as either

$$D = 1.50 \pm 0.01 \text{ in.}$$

or

$$D = 1.50^{+0.01}_{-0.01} \text{ in.}$$

This indicates that the diameter of the rod can be anywhere between the minimum diameter of 1.49 inches and the maximum diameter of 1.51 inches. The total variation is then 0.02 inch. This total varia-

tion permitted in the size of the part is called the Tolerance. The dimension, 1.5 inches, is said to be the basic size or nominal size. The extreme permissible diameters, 1.51 inches and 1.49 inches, are called the Limits or the Limiting Dimensions.

Let us now consider a round rod or pin (called a Shaft in machine fits) of diameter D, that is to fit inside a hole of another member. This nominal diameter of the shaft, D, is taken as the minimum diameter of the hole. It is known as the Basic Hole Size and is a very important dimension since it is the base from which the other dimensions are derived.

The manner in which the shaft is to fit in its hole must first be established. Is the shaft to fit loosely in the hole so that relative sliding may take place between them? If so, how loose may it be and still function properly? Or is the shaft to be held so rigidly in the hole by the tightness of the fit alone that no sliding is possible, that is, no motion of shaft relative to the external member containing the hole may take place? Obviously, in a fit where some freedom of motion must be made possible, the maximum size of the shaft must be less than the minimum size of the hole. Hence in such a fit if the maximum size of shaft is subtracted from the minimum size of hole, the minimum difference in dimensions is obtained. This minimum difference in dimensions of hole and shaft is called the Allowance. It is a specified amount which in the case of these so-called Working or Running fits is subtracted from the basic hole size to give the maximum shaft dimension. On the other hand, if the shaft is to be held rigidly in its hole, the dimensions of the parts must overlap, that is, the shaft must be somewhat larger than the hole. The amount by which the shaft size exceeds the hole size is known as the Interference. In these fits either pressure is resorted to in order to force the members together, or the external member is expanded by heating until it will slip over the internal member. Upon cooling, the external member contracts or shrinks, thus pressing firmly on the internal member. Such fits are known as Force or Shrink Fits. With them, the interference is in reality a negative allowance, and the selected average interference is an amount which is added to the basic hole size to give the minimum shaft dimension.

Machine Fits. The following classification of fits together with the formulas for allowances and tolerances given in Table XXVI have

been approved and recommended by the American Standards Association (A.S.A.).*

Loose Fits (*Class 1*). The loose fit, through a large allowance, permits of considerable freedom of movement between the members. Its use is recommended where accuracy is not essential as in agricultural and mining machinery.

Free Fit (*Class 2*). The free fit is made with a liberal allowance. It is used for running fits with speeds of 600 r.p.m. or over and journal pressures of 600 lb. per sq. in. or more. It may be employed in dynamos, engines, machine tools, etc.

Medium Fit (*Class 3*). This fit has a medium allowance and is used for running fits with speeds of less than 600 r.p.m. and with journal pressures of less than 600 lb. per sq. in. It is used also for sliding fits. It is employed for the more accurate machine tool and automotive parts.

Snug Fit (*Class 4*). This fit has a zero allowance and is the closest fit that can be assembled by hand. It should be employed under conditions of no perceptible shake and where moving parts are not intended to move freely under load.

Wringing Fit (*Class 5*). This fit is given zero to negative allowance (interference). It is practically a metal-to-metal fit. Parts assembled with this fit require some tapping with a hammer. This type of fit permits of selective assembly only and is not interchangeable.

Tight Fit (*Class 6*). The tight fit is made with a small negative allowance so that light pressure is necessary for its assembly. It results in a more or less permanent assembly. Fits of this type are used for drive fits in thin sections and on long fits in other sections. They are also used for shrink fits on extremely light sections.

Medium Force Fit (*Class 7*). This is a fit with a greater negative allowance than Class 6. It requires considerable pressure for its assembly. Parts thus fitted together are considered permanently assembled. It is suitable for press fits in fastening wheels, disks, or cranks, to their axles or shafts. This type of fit is also used for shrink fits on medium sections or long fits. It is the tightest fit that is recommended for cast-iron external members.

Heavy Force and Shrink Fit (*Class 8*). Still more negative allow-

*See publication *B, 4a-1925, Tolerances, Allowances, and Gages for Metal Fits.* This publication may be obtained from the American Society of Mechanical Engineers (A.S.M.E), 29 W. 39th St., New York, N. Y.

ance is provided with this type of fit. It is employed particularly for steel external members. Where heavy force fits are impractical, shrink fits are employed.

TABLE XXVI

Allowances and Tolerances

D = diameter of fit in inches

*Class	Allowance	Average Interference of Metal	Hole Tolerance	Shaft Tolerance
1	$.0025\sqrt[3]{D^2}$		$+.0025\sqrt[3]{D}$	$-.0025\sqrt[3]{D}$
2	$.0014\sqrt[3]{D^2}$		$+.0013\sqrt[3]{D}$	$-.0013\sqrt[3]{D}$
3	$.0009\sqrt[3]{D^2}$		$+.0008\sqrt[3]{D}$	$-.0008\sqrt[3]{D}$
4	.0000		$+.0006\sqrt[3]{D}$	$-.0004\sqrt[3]{D}$
5		.0000	$+.0006\sqrt[3]{D}$	$+.0004\sqrt[3]{D}$
6		$.00025D$	$+.0006\sqrt[3]{D}$	$+.0006\sqrt[3]{D}$
7		$.0005D$	$+.0006\sqrt[3]{D}$	$+.0006\sqrt[3]{D}$
8		$.001D$	$+.0006\sqrt[3]{D}$	$+.0006\sqrt[3]{D}$

*Classes 1 to 4 are for strictly interchangeable assembly Classes 5 to 8 are for selective assembly.

From A.S.A. Standard, B4a—1925

Example. A 1-inch shaft is to be designed for a Class 3 or medium running fit in its bearing. It is required to find: (*a*) the base dimension (of the hole); (*b*) the hole tolerance; (*c*) the shaft tolerance; (*d*) the allowance; (*e*) the minimum and maximum diameters of the hole (bearing); (*f*) the maximum and minimum diameters of the shaft (journal). See Fig. 145.

Solution.

(*a*) The base dimension (D of Table XXVI) is the minimum diameter of the hole. It is equal to the nominal diameter of the shaft. Therefore,

$$D = 1 \text{ in.} \quad \textit{Ans.}$$

(*b*) A medium running fit is Class 3 of Table XXVI. From the table, the hole tolerance for this class of fit is given by the formula,

$$\text{hole tolerance} = 0.0008\sqrt[3]{D} \text{ in.}$$

and therefore we have

$$\text{hole tolerance} = 0.0008\sqrt[3]{1}$$
$$= 0.0008 \text{ in.} \quad \textit{Ans.}$$

It will be noted that in the table the hole tolerance has a positive sign placed before it. This indicates that it is to be added to the base dimension to give the maximum diameter of the hole. There is no negative tolerance for the hole since the basic size is the minimum hole dimension.

(*c*) For Class 3, Table XXVI gives

$$\text{shaft tolerance} = 0.0008\sqrt[3]{D} \text{ in.}$$

It will be noted that this shaft tolerance has a negative sign placed before it in the table. This negative sign indicates that the value obtained should be subtracted from the maximum diameter of the shaft to obtain the minimum diameter of the shaft.

$$\text{shaft tolerance} = 0.0008\sqrt[3]{D}$$

$$0.0008\sqrt[3]{1} = 0.0008 \text{ in.} \quad \textit{Ans.}$$

(*d*) From Table XXVI for Class 3,

$$\text{the allowance} = 0.0009\sqrt[3]{D^2}$$

$$= 0.0009\sqrt[3]{1^2} = 0.0009 \text{ in.} \quad \textit{Ans.}$$

This is the value to be subtracted from the base dimension to obtain the maximum diameter of shaft.

(*e*) The minimum diameter of the hole is equal to the base dimension and hence in this example is equal to 1 in. *Ans.*

The maximum diameter of the hole is equal to the base dimension plus the hole tolerance and hence is equal to

$$1 + 0.0008 = 1.0008 \text{ in.} \quad \textit{Ans.}$$

(*f*) The maximum diameter of the shaft is equal to the base dimension minus the allowance. Hence in this example it is equal to

$$1 - 0.0009 = 0.9991 \text{ in.} \quad \textit{Ans.}$$

The minimum diameter of the shaft in this case is equal to the maximum diameter minus the shaft tolerance, or

$$0.9991 - 0.0008 = 0.9983 \text{ in.} \quad \textit{Ans.}$$

Example. It is required to find the minimum and maximum differences in size of hole and shaft in the preceding example.

Solution. The minimum difference in size is equal to the difference obtained by subtracting the maximum shaft diameter from the minimum hole diameter. Therefore we have

$$1.0000 - 0.9991 = 0.0009 \text{ in.} \quad \textit{Ans.}$$

(It will be noted that this difference is the so-called allowance as previously defined.)

The maximum difference in size is equal to the difference obtained by subtracting the minimum shaft diameter from the maximum hole diameter. Therefore we have

$$1.0008-0.9983=0.0025 \text{ in.} \quad \textit{Ans.}$$

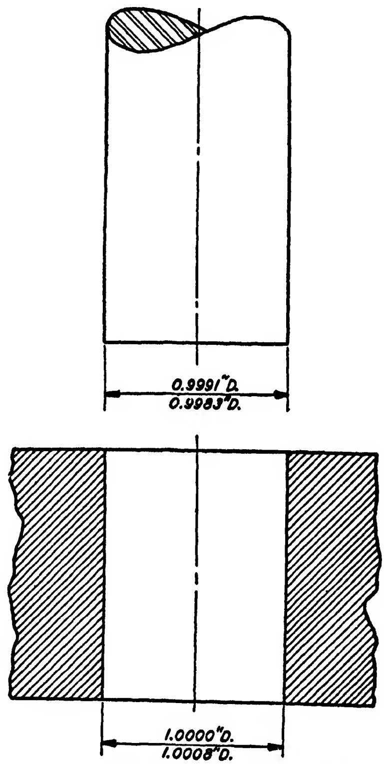


Fig. 145. Dimensions for a Medium Running Fit

Example. An 8-inch shaft is to be secured by a medium force fit to the cast-iron hub of a wheel. It is required to determine the tightest, loosest, and average fits for such an assembly.

Solution. The base dimension is the minimum hole diameter which is 8 inches in this case. Therefore,

$$D=8 \text{ in.}$$

A medium force fit is Class 7. From Table XXVI, an interference of metal is given for this class of fit instead of an allowance. Since interference is a negative allowance it is added to (rather than

subtracted from) the base dimension. From the table,

$$\text{interference} = 0.0005\ D$$
$$= 0.0005 \times 8 = 0.0040 \text{ in.}$$

This interference is added to the base or minimum hole size to obtain the minimum diameter of the shaft. Thus

$$\text{minimum shaft diameter} = 8 + 0.0040 = 8.0040 \text{ in.}$$

Table XXVI gives a positive shaft tolerance for Class 7 as follows:

$$\text{shaft tolerance} = 0.0006\sqrt[3]{D}$$
$$= 0.0006\sqrt[3]{8} = 0.0012 \text{ in.}$$

Therefore the maximum diameter of the shaft is obtained by adding the shaft tolerance since it is positive to the minimum shaft diameter. Thus

$$\text{maximum shaft diameter} = 8.0040 + 0.0012$$
$$= 8.0052 \text{ in.}$$

The shaft diameter can be stated as

$$8.0040^{+0.0012}_{-0.0000}$$

so that the upper limit produces a maximum diameter of 8.0052 in. and the lower limit produces a minimum diameter of 8.0040 in., as already given.

From the table

$$\text{hole tolerance} = 0.0006\sqrt[3]{D}$$
$$= 0.0006\sqrt[3]{8}$$
$$= 0.0006 \times 2 = 0.0012 \text{ in.}$$

The upper limit or maximum hole diameter is then the base (minimum hole diameter) diameter plus the hole tolerance, or

$$8.0000 + 0.0012 = 8.0012 \text{ in.}$$

The hole diameter can be stated as

$$8.0000^{+0.0012}_{-0.0000}$$

The tightest fit will be between the largest shaft and the smallest hole, which in this case produces a maximum difference in diameters (and therefore a maximum interference of metal) equal to

$$8.0052 - 8.0000 = 0.0052 \text{ in.} \quad \textit{Ans.}$$

The loosest fit will be between the smallest shaft and the largest

hole which will produce a minimum difference in diameters equal to

$$8.0040-8.0012=0.0028 \text{ in. } \textit{Ans.}$$

The average fit will be between the largest shaft and largest hole or between the smallest shaft and smallest hole. These will produce the average interference of metal, which is equal to

$$8.0052-8.0012=0.0040 \quad \textit{Ans.}$$

or

$$8.0040-8.0000=0.0040 \quad \textit{Ans.}$$

(**Note:** It is evident from an inspection of the preceding work that the formula for interference of metal given in Table XXVI gives the average interference.)

PROBLEMS

1. A load of 6000 pounds is lifted by a hoist whose drum has a diameter of 28 inches and rotative speed of 24 revolutions per minute. It is required to find: (*a*) the speed of lift in feet per minute, (*b*) the theoretical horsepower, H, of the hoist, (*c*) the actual horsepower, H_a, assuming the efficiency of the hoist to be 75 per cent. *Ans.* (*a*) 175.93 ft.p.m. (*b*) 32 hp. (*c*) 42.7 hp.

2. Ten horsepower are supplied to a machine from the motor that it is driven by. The machine is 80 per cent efficient. (*a*) What is the theoretical horsepower of the machine? (*b*) How many foot-pounds of useful work does the machine do per minute? *Ans.* (*a*) 8 hp. (*b*) 264,000 ft.-lb. per minute.

3. Define a bearing.

4. What is the difference between a radial bearing and a thrust bearing?

5. What is the most common type of radial bearing?

6. What are the so-called anti-friction bearings?

7. In what class of bearings is contact between the bearing and the rotating element (*a*) over a finite surface, (*b*) along a straight line, (*c*) at a point (or points)?

8. Define a lubricant.

9. Why lubricate the bearings of a machine?

10. What happens to the mechanical energy that a machine uses in overcoming the frictional resistance which is offered at its bearings?

11. A differential band brake like that of Fig. 142 has an angle of contact between its cast-iron drum and leather-lined steel band of 270 degrees. The brake is to sustain a torque of 6000 inch-pounds. The diameter of the drum is 20 inches and the coefficient of friction is 0.30. If the moment arms, a, b_1, and b_2, are 22 inches, 1 inch and $5\frac{1}{2}$ inches, respectively, find the operating force, W. *Ans.* 12.3 lb.

12. The torque in the shaft of a 30-inch brake drum is 18,000 inch-pounds. A differential type of band brake is employed which uses an unlined steel band which surrounds $\frac{7}{10}$ of the circumference of the drum. The operating force is applied to a foot pedal located at the end of a lever whose length is 24 inches. The moment arms of the tensions, T_1 and T_2, are $1\frac{1}{2}$ inches and 4 inches, respectively. It is required to obtain the magnitude of the operating force, W, and the width, w, of the steel band to be used in this case. Assume $\mu=0.20$ from Table XXV, the

thickness of the band = $\frac{3}{32}$ inch, and S_t for the steel band = 9000 lb. per sq. in. *Ans.* $W = 13.9$ lb. $w = 2\frac{7}{16}$ in.

13. The crank pin of the overhung crank of Fig. 143 is subjected to a force, P_t, of 12,000 pounds which acts tangentially to the circular path of the crank pin. The crank has the following dimensions: (see Figs. 143 and 144) $R = 9$ inches, $x = 4\frac{1}{2}$ inches, $t = 2$ inches, $L = 5$ inches, $d = 4$ inches. It is required to find the induced stress in the crank. *Ans.* 2500 lb. per sq. in. nearly.

14. Define tolerance and allowance.

15. What is the difference between a shrink fit and a force fit?

16. A journal bearing whose diameter is $2\frac{1}{2}$ inches is subjected to a load of 1125 pounds while rotating at 240 revolutions per minute. If $\mu = 0.02$ and $\frac{L}{D} = 3$, find:

(*a*) the projected area in square inches,
(*b*) the pressure on the bearing per square inch of projected area,
(*c*) the total work of friction in foot-pounds per minute,
(*d*) the work of friction per square inch of projected area per minute, in foot-pounds,
(*e*) the total heat generated per minute, in B.t.u.,
(*f*) the heat generated per minute per square inch of projected area, in B.t.u.

Ans. (*a*) 18.75 sq. in. (*b*) 60 lb. per sq. in. (*c*) 3534+ ft. lb. (*d*) 188.5 ft. lb. (*e*) 4.5 B.t.u. (*f*) 0.24 B.t.u.

17. An 8-inch shaft is to be designed for a Class 3 or a medium running fit in its bearing. It is required to find:

(*a*) the base dimension,
(*b*) the hole tolerance,
(*c*) the shaft tolerance,
(*d*) the allowance,
(*e*) the minimum and maximum diameters of the hole (bearing),
(*f*) the minimum and maximum diameters of the shaft (journal).

Ans. (*a*) 8 in.; (*b*) 0.0016 in.; (*c*) 0.0016 in.; (*d*) 0.0036 in.; (*e*) 8 in., 8.0016 in.; (*f*) 7.9948 in., 7.9964 in.

18. It is required to find the minimum and maximum differences in size of hole and shaft in the preceding problem. *Ans.* 0.0036 in., 0.0068 in.

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